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Keynes Without Nominal Rigidities

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The principal aim of this essay is to restate John Maynard Keynes's (1936) view of the economy as tending towards an equilibrium which will in general fall short of full employment. The approach outlined here, in sharp contrast to what has become standard theory, does not depend on nominal rigidities. The goods market adjusts prices according to excess demand as determined by the *IS* schedule; competitive price-taking producers adjust output on the basis of marginal profitability as determined by a comparison of price and marginal cost; and the labor market adjusts labor supply on the basis of a comparison of workers' marginal rates of substitution of goods for leisure and the real wage.

The model thus has three elements. First, there is a labor-supply equation deriving from utility maximization on the part of workers. Second, there is a goods-supply function (which doubles as the labor-demand function) deriving from profit maximization on the part of price-taking producers. Third, there is the distinctively Keynesian element, an *IS* schedule reflecting aggregate demand–the sum of consumption and investment demand–the latter determined by an exogenous money supply *via* the interest rate. There are two state variables, the real wage and the rate of capacity utilization. (Capacity utilization rather than output is used to track production since the capital stock and the labor force change over time.)

Figure 1 depicts these relationships. In principle, any two of the schedules suffice to determine equilibrium, the third over-determines the result. Pre-Keynesian economics omitted the *IS* schedule, while Keynes is generally understood to have "solved" the over-determinacy by eliminating the labor-supply schedule. More precisely, the "classicals" endogenize the *IS* schedule, so that it crosses the labor-supply and demand schedules at their point of intersection. For his part, Keynes elides labor-supply considerations early on in *The General Theory*, but returns to the issue in Ch 19. However, to say the least, nobody has claimed that Ch 19 cleanly or clearly disposes of the problem. Indeed, Ch 19 can be taken as the starting point of the present essay.[1]

The central novelty of the present essay is the interaction of statics and dynamics in an over-determined model. We appeal to a basic assumption of decentralized markets, namely that an imbalance between the demand and supply in one market, say the apple market, drives the price of apples, and that apple production responds to its marginal profitability. Pear production and

[1]The starting point of error in turning Keynes into an economics of nominal rigidities is Franco Modigliani [1944]. Modigliani famously contrasts Keynes and "classical" models solely on the basis of labor supply. According to Modigliani, the two models are identical except that the classical model assumes labor supply is a function of the real wage, whereas the Keynesian assumption is that the labor supply is perfectly elastic (up to full employment) at a given nominal wage W_0. The implication is that rigid money wages are an essential ingredient of the Keynesian story. But this is not so. Formally, the role of the labor-supply equation is to anchor the monetary to the real part of the model, and this can be done with a wide variety of normalizing equations, for instance

$$W + P = 1 \text{ or } W^2 + P^2 = 1,$$

which will deliver the central Keynesian result of an unemployment equilibrium. We do not need a labor-supply equation at all–other than labor supply≡labor demand.

price are affected only through the effects of the change in the price of apples on the demand for pears and the supply of pears. Thus in the model underlying Figure 1 disequilibrium in the goods market will affect the price of goods directly and disequilibrium in the labor market will affect the wage rate. At a point like A, the price level falls because the economy is to the right of the IS schedule, and the wage rate falls because the economy is to the left of the labor-supply schedule. If both prices and wages fall at the same percentage rate, the real wage remains constant. The point A is then a stationary point with respect to W/P. In general there will be a locus of such stationary points lying between the locus of stationary prices and the locus of stationary money wages. This locus–the black line in Figure 2–together with the locus of stationary capacity utilization–the blue line–defines the equilibrium.

This in effect reduces an over-determined system to a just-determined system. But there remains an essential difference between the conventional just-determined system and the present model. In just-determined models we normally concentrate on statics. To the extent we consider dynamics at all, it is as an afterthought; a plausible adjustment process is introduced to show stability, which is to say either that if the system undergoes a shock, it returns to equilibrium, or equivalently that the system converges to its equilibrium from an arbitrary starting point (within a neighborhood of equilibrium in the case of local stability).

In systems like the one in Figure 1, we cannot even define the equilibrium apart from the dynamic process. That is, the position of equilibrium as well as its stability depends on the dynamic process. So we have to reverse the usual procedure. In over-determined systems we start from the dynamics and let the equilibrium emerge–or not emerge–from the adjustment process as it plays itself out. What might be merely sensible in a just-determined system–to put the dynamic horse before the static cart–becomes a structural necessity in the over-determined system.

The primary focus of this essay is thus methodological. The paper develops a way of modeling *The General Theory* which is compatible with the price flexibility that normally goes along with competitive markets. Producers are price takers and the price level and the wage rate adjust to departures from the IS schedule and the labor-supply schedule. The model has room for a "neoclassical" labor-supply schedule, Keynesian aggregate demand, and optimizing, price-taking producers. I do not claim that the capitalist economies of Western Europe and North America were in Keynes's day actually very close to the textbook model of competition, not to speak of today. The point is rather that the essence of the problem Keynes addressed was not monopoly, oligopoly, or monopolistic competition–however much these "imperfections" complicate the story.

This is important because as long as Keynesian economics is tied to nominal rigidities, it is difficult if not impossible for it to be more than an economics for the short run. Prices and wages may be rigid for a time, but most economists believe that in the long run competition asserts itself and Keynes becomes irrelevant. Except for a few mavericks like Roy Harrod and Joan Robinson, Keynesians long since ceded the long run to the neoclassicals. In my view this misses the essential and enduring truth that Keynes discovered: aggregate demand matters–in the long run as well as the short. Keynes provides a basis for growth economics as well as for depression economics. I shall turn to the first after addressing the second.

A second purpose is more substantive. I will argue that whatever the limitations of the competitive straitjacket, the model does a tolerably good job of representing the very different dynamics of the agricultural and the non-agricultural economies during the catastrophic three years that inaugurated the Great Depression in the United States. Between 1929 and 1932, real output outside agriculture (as measured by private non-farm product) fell by 30 percent, non-farm employment by 22 percent, prices (wholesale industrial prices) by 22 percent, and money wages (hourly wage rates in manufacturing) by 16 percent. The agricultural sector, which accounted for 23 percent of total employment in 1929 and 27 percent in 1932, as well as 10 and 14 percent of GDP, fared very differently. Real agricultural output remained constant, as did employment, but prices farmers received fell by 55 percent, while prices farmers paid in production fell by 33 percent and wage rates fell by 28 percent. Moreover, the model offers an explanation–one that does *not* hinge on rigidities–of the elusive relationship between economic activity and real wages: although competitive profit maximization appears to imply a negative relationship between the real wage and the level of activity, the data for the depression (and, indeed, the data in general) is at best ambiguous on this point.

1 On Adjustment Processes

Since dynamic adjustment is central to the story this paper tells, it may be well to begin with this topic. Textbooks (see for example, Mas Colell *et al,* sec 17H) often distinguish between Walrasian and Marshallian dynamics, the first a price dynamics based on adjustment of price to excess supply, the second a quantity dynamics based on adjustment of output to marginal profitability.

In a partial equilibrium setting like that depicted in Figure 3, Walrasian dynamics are particularly simple. The demand curve labelled D is normally taken *à la* Walras to reflect the quantity demanded as a function of the price. Similarly for the supply curve S. Walras explained his dynamics in terms of the order books stock brokers maintain on their clients' behalf. Suppose one broker has clients who in the aggregate wish to buy 100 shares of XYZ at 61, 200 shares at 60, 300 shares at 59. Another has clients who have placed orders to sell 100 shares at 59, 200 at 60, and 300 at 61. If the market opens at 61, offers to sell will outweigh offers to buy in the ratio 3:1, with the result that the price will fall. Similarly, if the market opens at 59, offers to buy will swamp the market. In the first case there will be an excess supply of 200 and in the second case an excess supply of -200 shares. In both cases, according to Walras, the price will respond to the excess supply, falling in the first case and rising in the second. The share price will come to rest only when demand and supply are in balance, in the present case at 60. In continuous time we can represent this dynamic process by a single equation

$$\frac{\dot{P}}{P} = \theta \left[D\left(P\right) - S\left(P\right) \right] \tag{1}$$

Although Walras suggests the process takes place in real time, on the trading floor (Walras 1954, Lesson 5, Sec 42, pp 84-86), his story makes more sense as a virtual, pre-market, process than as real time, market, dynamics. As virtual dynamics, the process finesses the question of what happens to demand and supply schedules as a result of disequilibrium trading. In the present case, what happens to the demand and supply of shares at 60 if trades take place at an opening price of 61 or 59? If either demand or supply changes, 60 is unlikely to remain an equilibrium price. By assuming the adjustment process takes place before markets open and trade takes place, it becomes plausible to stipulate that nothing at all happens out of equilibrium. Mathematically we need not even specify an equation for quantity adjustment, because the only quantities that matter are the equilibrium quantities. This is a logically consistent way of solving the problem of the relationship between disequilibrium and equilibrium in a just-determined system, but not a very satisfactory assumption in terms of how the world works, and not even a logically consistent approach in the context of this essay, in which the focus is on over-determined systems.

Alfred Marshall sketches an alternative adjustment mechanism (Marshall 1948, Book 5, ch 5, pp 363-380) For Marshall, the independent variables are not prices, but quantities. Prices are dependent variables, the demand price being the amount of money an agent will pay per unit of a good as a function of the quantity purchased, the supply price being the amount of money per unit required to coax out a given supply, again as a function of quantity.[2] Marshall's story goes like this. Imagine a fish market in an era with no refrigeration (refrigeration introduces the complexity of storage and inventory). The supply for the day is determined by the day's catch, which is the result of both systematic factors (the size of the fishing fleet, the quality of the fishing grounds, and so forth) and random factors (weather, etc.). To keep matters at their simplest, abstract from the random factors and assume only an initial arbtirariness in the day's catch, labelled Q_0 in Figure 4. Today's equilibrium price is the price that clears the market, namely P_0. So the short-run quantity is determined by supply and the short-run price by demand.

How many fishermen set forth tomorrow however depends on the relationship between today's equilibrium (demand) price, and the supply price associated with today's catch. If the demand price exceeds the supply price (as at Q_0 in

[2]It is because the $P \times Q$ diagram derives not from Léon Walras but from Alfred Marshall, that we typically draw demand and supply schedules in a space in which price is represented along the vertical axis, and quantity along the horizontal, which inverts the normal representation of independent and dependent variables.

Figure 4), then fishermen expect a windfall profit for going out to sea (that is, a return over and above the minimum required to coax them out of their homes), and the quantity of fish brought to market will increase, let us suppose, to Q_1. Consequently the price of fish will fall. Tomorrow's equilibrium is $[Q_1, P_1]$. Mathematically, we can describe this process by the equations

$$\frac{\dot{Q}}{Q} = \theta \left[P - S^{-1}(Q)\right] \tag{2}$$

$$P = D^{-1}(Q) \tag{3}$$

where $D^{-1}(Q)$ and $S^{-1}(Q)$ are the inverse demand and supply functions relating prices to quantities. Observe that the process described by Eq 2 and Eq 3 constitute a "slave dynamics," by which the dynamic path is confined to one of the equations describing the stationary values of the state variables, in this case the (inverse) demand function.

Evidently there is a certain symmetry between the Walrasian price adjustment process and the Marshallian quantity process. We can reinforce this symmetry by assuming with Walras that the process described by Eq 1 takes place in real time and adding the assumption (which Walras never made) that away from equilibrium as well as at equilibrium suppliers are on their supply curve:

$$Q = S(P) \tag{4}$$

This, it should be noted, creates a slave dynamics in which the trajectory is confined to the supply schedule.

As Figures 3 and 4 are drawn, the adjustment processes converge to the equilibrium, as can be easily verified by examining the derivative of the adjustment equations, Eq 1 and 2. We have

$$G(P) = \theta [D(P) - S(P)] \tag{5}$$

$$G'(P) = \theta \left[\frac{dD}{dP} - \frac{dS}{dP}\right] = \theta [(-) - (+)] = - \tag{6}$$

for the Walrasian case and

$$G(Q) = \theta \left[P - S^{-1}(Q)\right] = \theta \left[D^{-1}(Q) - S^{-1}(Q)\right] \tag{7}$$

$$G'(Q) = \theta \left[\frac{dD^{-1}}{dQ} - \frac{dS^{-1}}{dQ}\right] = \theta [(-) - (+)] = - \tag{8}$$

for the Marshallian case. $G' < 0$ guarantees a stable equilibrium in the two cases.

Of course the system need not be stable. The equilibrium may be unstable for both adjustment processes, or it may be stable according to one dynamic and unstable according to another. Figure 5 depicts a case where Walrasian adjustment is stable but Marshallian adjustment is not:

$$G'(P) < 0 \text{ and } G'(Q) > 0.$$

To see where we are going with these stories, combine the two processes into a single process that is defined over the whole state space, not just along one of the two stationary loci. For any point $[Q, P]$ let quantity adjustment be determined by Marshallian marginal profitability

$$\frac{\dot{Q}}{Q} \equiv G(Q, P) = \theta_2 \left[P - S^{-1}(Q)\right] \tag{9}$$

and let price adjustment be determined by Walrasian excess demand

$$\frac{\dot{P}}{P} \equiv H(Q, P) = \theta_1 \left[D(P) - Q\right] \tag{10}$$

Stability depends on the trace and determinant of the Jacobian matrix

$$\begin{bmatrix} G_Q = -\theta_2 \dfrac{dS^{-1}}{dQ} & G_P = \theta_2 \\ H_Q = -\theta_1 & H_P = \theta_1 \dfrac{dD}{dP} \end{bmatrix} = \begin{bmatrix} - & + \\ - & - \end{bmatrix} \tag{11}$$

With a downward sloping demand curve and an upward sloping supply curve, the trace is negative and the determinant is positive, so the equilibrium is stable.

The economics is less straightforward than the mathematics. Suppose the starting point is $[Q_0, P_0]$ in Figure 6. With the adjustment process described by Eqs 9 and 10, we would observe the odd spectacle of a falling price coupled with rising output. For the *rhs* of Eq 10 is negative whereas the *rhs* of Eq 9 is positive. Prices fall because demand $D(P_0)$ falls short of the actual quantity supplied Q_0, but the marginal profitability of output is positive since P_0 exceeds the supply price $S^{-1}(Q_0) \equiv MC$, where MC stands for marginal cost. Perhaps the best way to understand this paradox is to partition producers into two groups, the lucky ones who are able to sell their entire output and the unlucky ones who are not. Although some producers cannot sell all their current output at the going price, this fails to dampen the general enthusiasm for expanding production among the first group, whose members receive windfall profits $[P_0 - MC]$ on the units they sell. For the second group at least, the paradox of falling price along with rising output suggests a fundamental problem with the idea of pure price-taking behavior outside of equilibrium.

We can address this problem in more than one way. One possibility is to allow demand, as well as supply, to influence quantity adjustment. At any point $[Q, P]$, we can define, for the representative consumer, a marginal rate of substitution (*MRS*) as the amount of money the consumer is prepared to give up for an extra fish. (It simplifies the argument, but nothing essential hinges on the assumption of homogeneous consumers.) The marginal consumers' surplus is measured by the difference $MRS - P$, which decreases monotonically with Q since, for given P, *MRS* is a decreasing function of the quantity of fish. The difference $MRS - P$ drives quantity adjustments by consumers. The demand curve is the locus of fish consumption at which $MRS = P$. To the left of the demand curve we have $MRS > P$, and to the right, $MRS < P$. In particular, at $[Q_0, P_0]$, we have $MRS < P_0$, which is to say that consumers are attempting to reduce purchases at the same time that fishermen are responding positively to the profit incentive $P_0 - S^{-1}(Q_0) = P_0 - MC$. So while price unambiguously falls, in accordance with Eq 10, demand and supply pull quantity in opposite directions by demand and supply considerations.

The question becomes this: which side, demand or supply, dominates outside of equilibrium? To resolve this, we invoke a "short side rule" in the spirit of Edmond Malinvaud (1977 and 1980): if, at a given price, supply exceeds demand, as at $[Q_0, P_0]$, then demand–the short side–governs. Conversely, if demand exceeds supply, then supply becomes the short side and it governs. In the present instance, we have

$$\frac{\dot{Q}}{Q} = \theta \left\{ \min \left[P - MC \right], \left[MRS - P \right] \right\} \tag{12}$$

Thus at $[Q_0, P_0]$ demand considerations would dominate, whereas at $[Q_1, P_1]$ supply would dominate. In both cases, the result would be movement in the direction of a smaller output. Starting from $[Q_0, P_0]$, price would fall, in accordance with Eq 10, and quantity would also fall, in accordance with Eq 12. At $[Q_1, P_1]$ price would rise because of the excess demand, but the quantity would fall because at the margin production is unprofitable $(P < MC)$.

We can also address the anomaly of rising output and falling price by inventing refrigeration and dropping the perishability assumption. This allows inventories to play a role in the adjustment process, which is a way of allowing demand to influence quantity adjustment indirectly. Denoting the stock of fish by V, we have inventory accumulation as

$$\dot{V} = Q - D(P) \tag{13}$$

and it is reasonable to assume that inventory accumulation has a dampening effect on producers whereas inventory depletion has an exhilarating effect. If, for simplicity, we assume away storage costs, it is plausible to re-write Eq 9 as

$$\frac{\dot{Q}}{Q} \equiv G(Q,P) = \theta_2\left[P - S^{-1}(Q)\right] - \theta_3\dot{V} = \theta_2\left[P - S^{-1}(Q)\right] + \theta_3\left[D(P) - Q\right] \tag{14}$$

Combining this equation with Walrasian price adjustment

$$\frac{\dot{P}}{P} \equiv H(Q,P) = \theta_1\left[D(P) - Q\right] \tag{15}$$

gives the Jacobian

$$\begin{bmatrix} G_Q = -\theta_2\frac{dS^{-1}}{dQ} - \theta_3 & G_P = \theta_2 + \theta_3\frac{dD}{dP} \\ H_Q = -\theta_1 & H_P = \theta_1\frac{dD}{dP} \end{bmatrix} = \begin{bmatrix} - & \pm \\ - & - \end{bmatrix} \tag{16}$$

Observe that despite the ambiguity of the sign of G_P, the sign (indeed the value) of the determinant is unchanged: the only difference between *Det* 16 and 11 is the term $\theta_3\theta_1\frac{dD}{dP}$. But this term enters the determinant twice, once with positive sign and once with negative sign. Hence the equilibrium is stable (or unstable) in the modified model defined by Eq 14 and 10 whenever it is stable (unstable) in the original model defined by Eq 9 and 10. The locus of stationary output, however, is modified. This locus is no longer the supply schedule, but the schedule labeled $\hat{S}$. The equilibrium itself is unchanged.

In this case we still might observe a falling price in conjunction with rising production, but inventory accumulation dampens the effect. Starting from $[Q_0, P_0]$, output continues to rise only until the locus $\hat{S}$ is reached.

Two final points before we conclude this preliminary discussion of adjustment processes. First it is of some historical interest perhaps that the difference between Walras and Marshall is almost certainly overdrawn. Yes, Walras did tell a purely price-adjustment story early on in the *Elements*, but once production enters into the model, the adjustment process becomes the same as Marshall's. Unless I am misreading Walras's Lesson 21 (1954, sec 208-220, pp 243-254) he lays out an adjustment process exactly like the one characterized by Eq 3 and 2.

Second, even if Walras and Marshall converge, the processes defined by Eq 14 (or 12 or 9) and 10 hardly exhaust the set of possible adjustment processes. Although it is a stretch in the context of perfect competition, we can easily imagine monopolistically competitive producers changing prices rather than quantities (allowing demand to determine quantities) in response to the marginal profitability of production. With profitability driving prices, aggregate demand drives production–as in the standard Keynesian model of the elementary text! We will return to this alternative way of conceptualizing disequilibrium in Section 8, "Path Dependence as a Characteristic of Over-Determined Systems."

2 Notation

$Y = output = F(K, L) \qquad F_K, F_L > 0 \qquad F_{KK}, F_{LL} < 0$
$\hat{Y} = potential\ output = \lim_{L\to\infty} F(K, L)$
$K = capital\ stock$
$L = labor$
$N = labor\ force$
$L_S = labor\ supply$
$a_0 = labor : output\ ratio = \dfrac{L}{Y}$
$a_1 = capital : output\ ratio = \dfrac{K}{Y}$
$k = capital : labor\ ratio = \dfrac{a_1}{a_0}$
$l = labor : capital\ ratio = \dfrac{a_0}{a_1}$
$\mathrm{f}(k) = labor\ productivity = F\left(\dfrac{K}{L}, 1\right) = \dfrac{1}{a_0}$
$f(l) = capital\ productivity = F\left(1, \dfrac{L}{K}\right) = \dfrac{1}{a_1}$
$\hat{a}_1 = full\ capacity\ capital : output\ ratio = \lim_{l\to\infty} [f(l)]^{-1}$
$z = rate\ of\ capacity\ utilization = \dfrac{\hat{a}_1}{a_1} = \dfrac{Y}{\hat{Y}}$
$\sigma = elasticity\ of\ substitution\ in\ production = -\dfrac{f'\mathrm{f}'}{\mathrm{f}\ \mathrm{f}''k} = -\dfrac{f'\mathrm{f}'}{f\ f"l}$
$g = rate\ of\ growth\ of\ the\ capital\ stock = \dfrac{\dot{K}}{K}$
$n = rate\ of\ growth\ of\ the\ labor\ force = \dfrac{\dot{N}}{N}$
$I = investment$
$S = saving$
$i = \dfrac{I}{K} = investment\ per\ unit\ of\ capital$
$s = \dfrac{S}{Y} = saving\ per\ unit\ of\ output$
$P = price\ of\ output\ and\ capital$
$W = wage\ rate$
$\Pi = profits = Y - WL$
$\pi = profit\ share = \dfrac{\Pi}{Y}$
$r = profit\ rate = \dfrac{\Pi}{K}$
$r^e = anticipated\ rate\ of\ profit$
$\rho = interest\ rate$
$s_\pi = propensity\ to\ save\ out\ of\ profits$
$\epsilon = \dfrac{r^e_\pi}{r^e_z}\dfrac{z}{\pi} = elasticity\ of\ substitution\ between\ profit\ share\ and\ capacity$ $utilization\ for\ a\ given\ anticipated\ profit\ rate$

3 A Simple Keynesian Model

As a preliminary, let me say just enough about production to finesse a potential problem, which is how to represent production possibilities both simply and satisfactorily within the time period that is the focus of this paper. The problem arises from the difference between long term and short term variations in factor proportions. Long period variations in factor proportions reflect possibilities for substitution between capital and labor, but short period variations are largely the result of variations in the intensity of capacity utilization. So the interpretation of a change in the capital:labor ratio or in the capital:output ratio depends on the time frame.

Although variable proportions is generally associated with long-run capital:labor substitution, I shall make use of this assumption about production for the short term. I employ a so-called "neoclassical" production function not because I believe it accurately reflects production possibilities, but because it permits us to focus on the issues of output, price, and wage dynamics in as simple a setting as possible, in order to keep the argument tractable and transparent. Variations in factor intensities should be understood as reflecting variations in capacity utilization rather than substitution of one factor for another.

We assume that the production function $F(K, L)$ exhibits constant returns to scale and diminishing factor productivities. We further assume that the limiting elasticity of factor substitution is less than unity

$$\lim_{l\to\infty} \sigma \equiv \lim_{l\to\infty} \left(-\frac{f'\mathrm{f}'}{f\ f''l} \right) < 1 \tag{17}$$

With assumption 17 and a given capital stock, output is bounded as the input of labor increases without bound. This upper bound is denoted $\hat{Y}$, where

$$\hat{Y} = \lim_{L\to\infty} F(K, L) \tag{18}$$

In intensive form, we denote this upper bound by

$$\frac{1}{\hat{a}_1} = \lim_{l\to\infty} f(l) \tag{19}$$

The picture is given in Figure 7.

Expression 18 is the maximum output obtainable with the given capital stock. This is in a sense "potential output," provided we interpret "potential" without reference to labor constraints. In the same sense we may define capacity utilization as the ratio of actual output to potential output,

$$z \equiv \frac{Y}{\hat{Y}} \equiv \frac{\hat{a}_1}{a_1} \tag{20}$$

Observe that full utilization of the labor force will generally fall far short of full capacity utilization–absent an infinite labor force.

Capacity utilization is the state variable of choice for measuring production. Unlike output, capacity utilization remains constant over time provided the capital:labor ratio remains constant.

We turn next to money and interest. To keep matters simple, I shall assume that the central bank varies the money supply to maintain a constant rate of interest. If we write (real) money supply as $\frac{M}{P}$, we have

$$\frac{M}{P} = \left(\frac{M}{P}\right)_0 e^{\int_0^t m(\tau)d\tau} \tag{21}$$

so that the rate of change of the real money supply is given by the parameter m:

$$\frac{\dot{M}}{M} - \frac{\dot{P}}{P} = m \tag{22}$$

Money demand will be taken as a function Φ of ρ, the rate of interest; K, the capital stock (as a measure of the assets that substitute for money in agents' portfolios), and Y, income. For the supply of money to equal the demand, we have

$$\frac{M}{P} = \Phi\left(\rho, K, Y\right) \tag{23}$$

and, over time,

$$m\frac{M}{P} = \Phi_\rho \dot{\rho} + \Phi_K \dot{K} + \Phi_Y \dot{Y} \tag{24}$$

Setting $\dot{\rho} = 0$, in order to maintain a constant interest rate, we have

$$m = \frac{PY}{M}\left(\Phi_K g a_1 + \Phi_Y y\right) \tag{25}$$

If we simplify the liquidity-preference function to

$$\Phi\left(\rho, K, Y\right) = \phi\left(\rho\right) + v^{-1}Y \tag{26}$$

then the equation for the rate of change of the money supply itself becomes much simpler. Denote the (real) quantity of money needed to meet the speculative demand A,

$$A = \phi(\rho) \tag{27}$$

and let m_T denote the growth rate of the supply of money required for transactions

$$\frac{M}{P} - A = \left[\left(\frac{M}{P}\right)_0 - A\right] e^{\int_0^t m_T(\tau)d\tau} \tag{28}$$

so that

$$m_T = \frac{Y}{v\left(\frac{M}{P} - A\right)} y \tag{29}$$

By virtue of Eq 26 and 27

$$Y = v\left(\frac{M}{P} - A\right) \tag{30}$$

Thus in real terms the rate of growth of transactions money $\frac{M}{P} - A$ is equal to the rate of growth of income.

With the interest rate given, the Keynesian investment demand function

$$\frac{I}{K} \equiv i = i(\rho) \tag{31}$$

and the savings function

$$\frac{S}{K} = s\frac{Y}{K} = sa_1^{-1} \tag{32}$$

together give the IS schedule. Setting desired saving and investment equal to each other, we have $i = sa_1^{-1}$ or $i\hat{a}_1 = sz$, from which it follows that in $z \times \frac{W}{P}$ space the IS schedule is defined by the equation

$$z = \frac{i}{s}\hat{a}_1 \tag{33}$$

In its simplest version, Keynes's model is completed by adding a schedule reflecting producers' equilibrium, which for Keynes as well as neoclassical economics is a situation where producers maximize profits. Under competitive conditions, the condition of profit maximization is $P = MC$, which defines the goods-supply schedule. The same schedule can be thought of as a labor-demand function since $MC \equiv \dfrac{W}{MP_L}$ under competitive conditions. Producers' equilibrium is thus characterized by

$$\frac{W}{P} = MP_L \equiv f'(l) \tag{34}$$

or since $\dfrac{z}{\hat{a}_1} \equiv f(l)$,

$$\frac{W}{P} = \frac{dz}{dl}\hat{a}_1^{-1} \tag{35}$$

The two schedules are shown in Figure 8. Equilibrium is at the point E, at which Eq 33 and 34 are simultaneously satisfied. The dynamics are straight forward. The simplest version of Marshall-Walras adjustment, Eq 9 and 10, translates into adjustment of the price level according to the excess of expenditure relative to income and the output level according to the marginal profitability of production. At this point the nominal wage is assumed constant, $W = W_0$, in accordance with Chapter 2 of the *General Theory*; labor supply is assumed to adjust automatically to labor demand.

For the real wage we have (given $W = W_0$)

$$\left(\frac{W}{P}\right)^{\bullet} = -\frac{W}{P^2}\dot{P} \tag{36}$$

and for the price level we have

$$\frac{\dot{P}}{P} = \theta_1\left(i - sa_1^{-1}\right) = \theta_1\left(i - s\hat{a}_1^{-1}z\right) \tag{37}$$

so that

$$\left(\frac{W}{P}\right)^{\bullet} = -\theta_1\left(i - s\hat{a}_1^{-1}z\right)\frac{W}{P} \tag{38}$$

which is to say that the price level rises when expenditure exceeds income, or investment demand exceeds desired saving–to the left of the *IS* schedule–and the price level falls when expenditure falls short of income, or saving exceeds investment–to the right of *IS*. The real wage moves in the opposite direction.

Responding to profit opportunities in a Marshallian manner, producers will expand output and employment according to the sign of $MP_L - \dfrac{W}{P}$. In addition, producers will adjust employment in line with changes in the capital stock. Indeed, with constant returns to scale, employers who wish to maintain a constant capital:labor ratio will adjust employment by the same percentage amount that the capital stock changes. Hence output will change by the same amount. We may write

$$\frac{\dot{Y}}{Y} = \theta_2 \left(f' - \frac{W}{P} \right) + g \tag{39}$$

Since

$$\frac{\dot{z}}{z} \equiv \frac{\dot{Y}}{Y} - g \tag{40}$$

Eq 39 becomes

$$\frac{\dot{z}}{z} = \theta_2 \left(f' - \frac{W}{P} \right) \tag{41}$$

The dynamic system is formed by joining Eq 41 and 37:

$$G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P} \right) z \tag{42}$$

$$H\left(z, \frac{W}{P}\right) = -\theta_1 \left(i - s\hat{a}_1^{-1} z \right) \frac{W}{P} \tag{43}$$

The Jacobian is

$$\begin{bmatrix} G_z = \theta_2 f \left(f'\right)^{-1} f'' + \theta_2 \left(f' - \frac{W}{P} \right) & G_{\frac{W}{P}} = -\theta_2 z \\ H_z = \theta_1 s\hat{a}_1^{-1} \frac{W}{P} & H_{\frac{W}{P}} = -\theta_1 \left(i - s\hat{a}_1^{-1} z \right) \end{bmatrix} = \begin{bmatrix} - & - \\ + & 0 \end{bmatrix} \tag{44}$$

for which the trace is negative and the determinant positive; the equilibrium is stable, with the direction of adjustment indicated in Figure 8. Observe that to the northwest and southeast of equilibrium prices and output move in opposite directions. In the first case the price level rises (real wages fall) while capacity utilization contracts, and in the second prices fall while capacity utilization expands.

4 A Neoclassical Alternative

Pre-Keynesian orthodoxy–which Keynes called "classical" while we would now call it "neoclassical"–shares one assumption with Keynes, namely, the idea of profit maximization driving goods supply and labor demand. The second determinant of neoclassical equilibrium is a labor supply function $L^S = \Psi\left(\frac{W}{P}, N\right)$, which, assuming identical workers, we may write as $L^S = \psi\left(\frac{W}{P}\right) \cdot N$. Normalizing by the capital stock, we have

$$l^S \equiv \frac{L^S}{K} = \psi\left(\frac{W}{P}\right)\frac{N}{K} \tag{45}$$

Since $z = f(l)\,\hat{a}_{1,}$ Eq 45 provides the relationship between capacity utilization and real wages consistent with being on the labor-supply schedule, namely

$$f^{-1}\left(\frac{z}{\hat{a}_1}\right) \equiv \frac{L^S}{K} = \psi\left(\frac{W}{P}\right)\frac{N}{K} \tag{46}$$

Figure 9 indicates neoclassical equilibrium E as the intersection of the profit-maximization schedule and the labor-supply schedule, which is stationary only as long as N and K are growing at the same rate. (If $n > g$, the labor-supply schedule shifts to the right; if $n < g$, it shifts to the left. Accordingly the equilibrium slides down or up the profit-maximization schedule.)

Away from equilibrium, production is governed by profitability, as in the simple Keynesian system we have just considered. The dynamics of production are given by Eq 41:

$$\frac{\dot{z}}{z} = \theta_2\left(f' - \frac{W}{P}\right) \tag{47}$$

Fixing the price level at unity, real-wage dynamics depend on excess demand or supply in the labor market. Consider a single worker. She puts downward pressure on wages if her marginal rate of substitution of goods for leisure is less than the real wage. Conversely, she will hold out for a higher wage if her MRS exceeds the real wage. Aggregating over the entire labor force, and normalizing by the capital stock, we can write, *à la* Walras, Eq 10,

$$\frac{\dot{W}}{W} = \theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} \tag{48}$$

Since the price level is constant, we have $\frac{\dot{W}}{W} = \frac{\left(\frac{W}{P}\right)^{\bullet}}{\frac{W}{P}}$, which is to say that the percentage rate of change of money wages is the same as the percentage rate of change of real wages. Consequently the complete dynamic system is given by

$$\dot{z} \equiv G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P}\right) z \tag{49}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}\right) = \theta_3 \left(MRS - \frac{W}{P}\right) \frac{N}{K} \frac{W}{P} \tag{50}$$

and the Jacobian matrix

$$\begin{bmatrix} G_z = \theta_2 f\left(f'\right)^{-1} f'' + \theta_2 \left(f' - \frac{W}{P}\right) & G_{\frac{W}{P}} = -\theta_2 z \\ H_z = \theta_3 \frac{\partial MRS}{\partial z} \frac{N}{K} \frac{W}{P} & H_{\frac{W}{P}} = \theta_3 \left(\frac{\partial MRS}{\partial \frac{W}{P}} - 1\right) \frac{N}{K} \frac{W}{P} + \\ & +\theta_3 \left(MRS - \frac{W}{P}\right) \frac{N}{K} \end{bmatrix}$$

$$= \begin{bmatrix} - & - \\ + & \pm \end{bmatrix} \tag{51}$$

Observe that the stability of equilibrium is not assured. If the income effect of changes in real wages is sufficiently strong, the labor-supply curve becomes backward bending. In terms of the matrix 51, this is to say that the partial derivative $\frac{\partial MRS}{\partial \frac{W}{P}}$ becomes a positive number, and this may be enough not only to make $H_{\frac{W}{P}}$ positive but enough to make the trace of the matrix 51 positive or the determinant negative (or both). In the case illustrated in Figure 9, in which the labor-supply curve is forward sloping, $H_{\frac{W}{P}} < 0$ and $\text{tr } 51 < 0$, $\det 51 > 0$. The equilibrium is stable.

5 A Model With Keynesian and Neoclassical Elements

We turn now to the model that incorporates both Keynesian and neoclassical features, an aggregate demand schedule, a profit-maximization schedule, and a labor-supply schedule. The picture is in Figure 10. We have

$$z = \frac{i}{s}\hat{a}_1 \tag{52}$$

$$\frac{W}{P} = f'(l) \tag{53}$$

$$f^{-1}\left(\frac{z}{\hat{a}_1}\right) \equiv \frac{L^S}{K} = \psi\left(\frac{W}{P}\right)\frac{N}{K} \tag{54}$$

Evidently all three relationships cannot be satisfied simultaneously; the system is over-determined.

To resolve the over-determination, we invoke the dynamics of the two models. We have

$$\frac{\dot{P}}{P} = \theta_1\left(i - sa_1^{-1}\right) = \theta_1\left(i - s\hat{a}_1^{-1}z\right) \tag{55}$$

$$\frac{\dot{z}}{z} = \theta_2\left(f' - \frac{W}{P}\right) \tag{56}$$

$$\frac{\dot{W}}{W} = \theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} \tag{57}$$

Combining Eq 55 and 94 gives

$$\left(\frac{W}{P}\right)^{\bullet} \equiv \left(\frac{\dot{W}}{W} - \frac{\dot{P}}{P}\right)\frac{W}{P} = \left[\theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1\left(i - s\hat{a}_1^{-1}z\right)\right]\frac{W}{P} \tag{58}$$

for which a stationary real-wage locus is defined by the equation

$$\theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} = \theta_1\left(i - s\hat{a}_1^{-1}z\right) \tag{59}$$

This locus is labeled $\left(\frac{W}{P}\right)^{\bullet} = 0$ in Figure 10. As drawn, the stationary real-wage locus lies to the right of the *IS* schedule, so the equilibrium defined by its intersection with the profit-maximization schedule is one at which prices and money wages *fall* while capacity utilization remains constant.

Let us look at the dynamics more closely. The complete dynamic system is

$$\dot{z} \equiv G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P}\right) z \tag{60}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}\right) = \left[\theta_3 \left(MRS - \frac{W}{P}\right) \frac{N}{K} - \theta_1 \left(i - s\hat{a}_1^{-1} z\right)\right] \frac{W}{P} \tag{61}$$

so the Jacobian is

$$\begin{bmatrix} G_z = \theta_2 f \left(f'\right)^{-1} f'' + \theta_2 \left(f' - \frac{W}{P}\right) & G_{\frac{W}{P}} = -\theta_2 z \\ H_z = \left(\theta_3 \frac{\partial MRS}{\partial z} \frac{N}{K} + \theta_1 s\hat{a}_1^{-1}\right) \frac{W}{P} & H_{\frac{W}{P}} = \theta_3 \left(\frac{\partial MRS}{\partial \frac{W}{P}} - 1\right) \frac{W}{P} \frac{N}{K} + \\ & \theta_3 \left(MRS - \frac{W}{P}\right) \frac{N}{K} - \theta_1 \left(i - s\hat{a}_1^{-1} z\right) \end{bmatrix}$$

$$= \begin{bmatrix} - & - \\ + & \pm \end{bmatrix} \tag{62}$$

As in the neoclassical case, there is no guarantee of stability. Provided however that the stationary $\frac{W}{P}$ locus is upward sloping (which will be the case if the labor-supply schedule is positively sloped), we have $H_{\frac{W}{P}} < 0$, and the trace and determinant have the requisite signs to guarantee stability, tr < 0 and det > 0. To keep matters simple, we shall henceforth assume $\frac{\partial MRS}{\partial \frac{W}{P}} < 1$.

The equilibrium in this case is characterized by unemployment (measured by the horizontal distance between E and the labor-supply schedule), as well as by falling prices and wages. Prices fall because of a chronic excess of income over expenditure, and wages fall because of unemployment. Producers are in equilibrium because the marginal productivity of labor is equal to the (stationary) real wage.

One important difference between the standard, just-determined, model and the present, over-determined, model is the existence of undesired inventory changes. Because in the present model desired saving and desired investment are not equal in equilibrium, inventory changes are assumed to bring actual investment in line with desired saving, at least for the case where desired saving exceeds desired investment. We assume that the accumulation of productive capital[3] is governed by

$$g = \min\left(i, s\hat{a}_1^{-1} z\right) \tag{63}$$

[3]Desired inventory changes are included in productive capital.

and that for $i < s\hat{a}_1^{-1}z$ inventories rise by the difference between desired saving and investment. Letting V denote excess inventories (remember that desired inventories are part of productive capital) and $v = \frac{V}{K}$ excess inventory per unit of capital, we have, on the assumption that savings intentions are always realized

$$\frac{\dot{V}}{K} \equiv \dot{v} + vg = s\hat{a}_1^{-1}z - g \tag{64}$$

Note the asymmetry. To the right of the *IS* schedule, undesired inventory accumulation–equal to the excess of income over expenditure–is chronic. To the left, investment desires are frustrated by the insufficiency of saving; investment is not assumed to be accommodated by inventory depletion.

Nothing fundamental changes if we modify the adjustment process to allow producers to take account of undesired inventory accumulation in their production planning. In place of Eq 41, producers adjust capacity utilization according to

$$\frac{\dot{z}}{z} = \theta_2 \left(f' - \frac{W}{P} \right) - \theta_4 \frac{\dot{V}}{K} = \theta_2 \left(f' - \frac{W}{P} \right) - \theta_4 \left(s\hat{a}_1^{-1}z - g \right) \tag{65}$$

The stationary capacity-utilization schedule is now the schedule labeled $\dot{z} = 0$ in Figure 11.

As noted, the labor-supply schedule is itself not stationary. With labor supply defined by Eq 45, we have the horizontal motion of the labor-supply schedule given by

$$\dot{z} = \psi \left(\frac{W}{P} \right) (n - g) f' \frac{N}{Y} z = a_0 (n - g) f' z \tag{66}$$

with $a_0 = \frac{L^S}{Y}$, which is to say that a_0 is evaluated on the labor-supply schedule. If the labor force is growing faster than the stock of capital, so that $n > g$, the labor-supply schedule slides down the profit-maximization schedule, pulling the locus of stationary real wages, $\left(\frac{W}{P} \right)^{\bullet} = 0$, to the right. Capacity utilization expands because the increasing downward pressure on real wages makes production more profitable.

6 Incorporating Labor-Force Growth

At the risk of complicating the model, we can formalize the movement of the labor-supply schedule by introducing a third state variable, namely, the ratio of the labor force to the (productive) capital stock, $\frac{N}{K}$. The equation of motion for the new variable is

$$\left(\frac{N}{K}\right)^{\bullet} = (n-g)\frac{N}{K} \tag{67}$$

In neoclassical theory n is exogenously given. The value of g depends on whether the economy is to the right or to the left of the *IS* schedule. To the right of the *IS* schedule, $g = i$ and to the left $g = s\hat{a}_1^{-1}z$.[4] The complete system is

$$\dot{z} \equiv G\left(z, \frac{W}{P}, \frac{N}{K}\right) = \theta_2\left(f' - \frac{W}{P}\right)z \tag{68}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}, \frac{N}{K}\right) = \left[\theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1\left(i - s\hat{a}_1^{-1}z\right)\right]\frac{W}{P} \tag{69}$$

$$\left(\frac{N}{K}\right)^{\bullet} \equiv J\left(z, \frac{W}{P}, \frac{N}{K}\right) = (n-g)\frac{N}{K} \tag{70}$$

for which in the first case ($g = i$) the Jacobian is

	z	$\frac{W}{P}$	$\frac{N}{K}$
G	$G_z = \theta_2 f (f')^{-1} f'' + \theta_2\left(f' - \frac{W}{P}\right)$	$G_{\frac{W}{P}} = -\theta_2 z$	$G_{\frac{N}{K}} = O$
H	$H_z = \left(\theta_3\frac{\partial MRS}{\partial z}\frac{N}{K} + \theta_1 s\hat{a}_1^{-1}\right)\frac{W}{P}$	$H_{\frac{W}{P}} = \theta_3\left(\frac{\partial MRS}{\partial\frac{W}{P}} - 1\right)\frac{N}{K}\frac{W}{P} +$ $\theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1\left(i - s\hat{a}_1^{-1}z\right)$	$H_{\frac{N}{K}} = \theta_3\left(MRS - \frac{W}{P}\right)$
J	$J_z = 0$	$J_{\frac{W}{P}} = 0$	$J_{\frac{N}{K}} = n - i$

$$\begin{bmatrix} - & - & 0 \\ + & - & - \\ 0 & 0 & \pm \end{bmatrix} \tag{71}$$

If $n - i > 0$, $\frac{N}{K}$ increases over time, which is to say that the number of potential workers per unit of capital continuously grows. The augmented system

[4] Observe that in the neoclassical growth model *à la* Robert Solow (1956), the sole state variable is the inverse $\frac{K}{N}$. The other variables, capacity utilization and the real wage, disappear by virtue of the assumptions that all willing workers find employment and all saving finds invetment outlets.

represented by the matrix 71 is unstable: one of the stability conditions for the 3×3 system is det $71 < 0$, which is violated when $n - i > 0$.[5] (There is an "equilibrium" at $\frac{N}{K} = \infty$, which is approached as $t \to \infty$. Change the state variable from $\frac{N}{K}$ to $\frac{K}{N}$, and the equilibrium value of the state variable is 0 rather than ∞. Indeed, this change transforms the signs of the matrix 71 from

$$\begin{bmatrix} - & - & 0 \\ + & - & + \\ 0 & 0 & + \end{bmatrix} \tag{72}$$

to

$$\begin{bmatrix} - & - & 0 \\ + & - & + \\ 0 & 0 & - \end{bmatrix} \tag{73}$$

so that det $71 < 0$, which along with the other conditions of footnote 5 insures the "stability" of this equilibrium.)

If $n - i < 0$, $\frac{N}{K}$ falls over time. Formally, the system is stable since all the conditions of footnote 5 are satisfied. But the dynamic story is more interesting

[5] If the Jacobian is

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

and we write

$$\begin{aligned} \alpha_1 &= -(a_{11} + a_{22} + a_{33}) \\ \alpha_2 &= -(a_{31}a_{13} + a_{32}a_{23} + a_{12}a_{21} - a_{11}a_{22} - a_{11}a_{22} - a_{11}a_{33} - a_{22}a_{33}) \\ \alpha_3 &= -\det\ A \end{aligned}$$

then the stability conditions are

$$\begin{aligned} \alpha_1, \alpha_2, \alpha_3 &> 0 \\ \alpha_1\alpha_2 - \alpha_3 &> 0 \end{aligned}$$

With the sign pattern

$$\begin{bmatrix} - & - & 0 \\ + & - & - \\ 0 & 0 & \end{bmatrix}$$

these stability conditions are all satisfied if and only if $a_{33} < 0$.

than the formalism. The stock of productive capital grows faster than the labor force; the downward pressure on wages abates and producers shrink output relative to capacity. As capacity utilization falls, the savings overhang (the excess of saving over desired investment) also falls, and price deflation abates. For a picture of this process, imagine the projection of the 3×3 system onto the $z \times \frac{W}{P}$ plane. As $\frac{N}{K}$ increases, the labor-supply schedule shifts to the left, and the (temporary) equilibrium rate of wage and price deflation decreases in absolute value, as does the equilibrium level of capacity utilization. The equilibrium real wage rises.

Eventually the labor-supply function crosses the *IS* schedule, and the economy is no longer in a deflationary regime. Saving rather than investment constrains the economy; we have $g = s\hat{a}_1^{-1}z < i$, and the equation of motion for $\frac{N}{K}$ becomes

$$\left(\frac{N}{K}\right)^{\bullet} = (n - g)\frac{N}{K} = (n - s\hat{a}_1^{-1}z)\frac{N}{K} \tag{74}$$

with the term in parentheses vanishing somewhere between the *IS* schedule and the vertical axis. This is the equilibrium condition in Solow (1956) with the difference that here it coexists with explicit conditions on labor demand and supply, conditions which are implicit in the original Solow model.

As long as $n - g \leq 0$, time would appear to cure all wounds. Starting from a deflationary spiral, capital accumulation shifts the labor-supply function inward and eventually transforms the deflationary spiral into an inflationary spiral that has its own equilibrium where saving provides a rate of growth of the capital stock just equal to the exogenously given rate of labor-force growth.

Time cures all wounds *if* the patient survives. As we shall see when we encounter the "Pigou Effect" further along in this essay, the social fabric might collapse before capital-stock growth restores the health of the economy. In the time that it takes for capital accumulation to move the labor-supply function sufficiently to the left that its intersection with the *IS* schedule is above the labor-demand function, the cumulative effects of price and wage deflation as well as unemployment well might undermine the legitimacy of the capitalist regime.

An alternative assumption to the assumption of an exogenously given rate of growth of the labor force is the assumption that labor-force growth is endogenous. However odd this idea may seem at first, it becomes more plausible when we shift the focus of our gaze from the *economy* to the *capitalist economy*. In a depression, as jobs dry up, people leave the capitalist sector altogether. In the early 1930s, the decades long exodus from agriculture was temporarily reversed–people who could no longer find jobs in the capitalist economy based on profit maximization at exogenously given wages went back to (or did not leave) the family based economy of the farm. Alexander Chayanov (1966) described peasant economies and economics in terms that Martin Weitzman would later (1984) formalize in a normative manner as the *share economy.* The key difference between a share economy and a capitalist economy is precisely the ability of the share economy to absorb surplus labor. The capitalist economy adds workers only so long as their marginal product exceeds the real wage; the share economy adds workers as long as average product increases, that is, as long as the marginal worker adds any output at all.[6]

In terms of Figure 10, the profit-maximizing capacity utilization for a share economy is defined not by $f' = \frac{W}{P}$, but by $f' = 0$, so the demand for labor becomes a vertical line where the marginal-productivity function intersects the horizontal axis, as in Figure 12. The equilibrium is no longer E but becomes E'. In such an economy, shifts in the aggregate demand or the labor-supply function translate into changes in the equilibrium real wage, not in equilibrium capacity utilization. This is a reasonably accurate picture of what happened in US the agricultural sector during the Great Depression

We shall return to the theory of an endogenous labor force later. For now, we shall simply deploy one formal model that provides an alternative to the exogenous labor-force model. Assume that labor force growth is determined by two factors: in equilibrium the labor force grows at the same rate as the productive capital stock, whereas out of equilibrium, the deviation from capital-stock growth is determined by the extent to which the employment rate differs from an exogenously given fraction α of the labor supply. The equation is

$$\frac{\dot{N}}{N} = \theta_5\left(\frac{L}{N} - \alpha\psi\left(\frac{W}{P}\right)\right) + g = \theta_5\left[\frac{L}{K}\frac{K}{N} - \alpha\psi\left(\frac{W}{P}\right)\right] + g = \theta_5\left(\frac{l}{\frac{N}{K}} - \alpha\psi\left(\frac{W}{P}\right)\right) + g \tag{75}$$

from which it follows that

$$\left(\frac{N}{K}\right)^{\bullet} = \theta_5\left[l - \alpha\psi\left(\frac{W}{P}\right)\frac{N}{K}\right] \tag{76}$$

The complete system is

$$\dot{z} \equiv G\left(z, \frac{W}{P}, \frac{N}{K}\right) = \theta_2\left(f' - \frac{W}{P}\right)z \tag{77}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}, \frac{N}{K}\right) = \left[\theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1\left(i - s\hat{a}_1^{-1}z\right)\right]\frac{W}{P} \tag{78}$$

[6] Indeed, as has been noted, real agricultural output remained remarkably steady in the US during the depression. The problem for American agriculture was price deflation, not output contraction. In the early '30s, because of deflation, farmers faced real interest rates approaching 30 percent per year. See footnote 10 below

$$\left(\frac{N}{K}\right)^{\bullet} \equiv J\left(z, \frac{W}{P}, \frac{N}{K}\right) = \theta_5\left[l - \alpha\psi\left(\frac{W}{P}\right)\frac{N}{K}\right] \tag{79}$$

and the Jacobian is

	z	$\frac{W}{P}$	$\frac{N}{K}$
G	$G_z = \theta_2 f\left(f'\right)^{-1} f'' + \theta_2\left(f' - \frac{W}{P}\right)$	$G_{\frac{W}{P}} = -\theta_2 z$	$G_{\frac{N}{K}} = O$
H	$H_z = \left(\theta_3 \frac{\partial MRS}{\partial z}\frac{N}{K} + \theta_1 s\hat{a}_1^{-1}\right)\frac{W}{P}$	$H_{\frac{W}{P}} = \theta_3\left(\frac{\partial MRS}{\partial \frac{W}{P}} - 1\right)\frac{W}{P}\frac{N}{K} + \theta_3\left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1\left(i - s\hat{a}_1^{-1}z\right)$	$H_{\frac{N}{K}} = \theta_3\left(MRS - \frac{W}{P}\right)$
J	$J_z = \theta_5\left(f'\right)^{-1}\hat{a}_1^{-1}\frac{N}{K}$	$J_{\frac{W}{P}} = -\theta_5\alpha\psi'\frac{N}{K}$	$J_{\frac{N}{K}} = -\theta_5\alpha\psi$

$$\begin{bmatrix} - & - & 0 \\ + & - & - \\ + & - & - \end{bmatrix} \tag{80}$$

Here the stability conditions are more complicated. Given the pattern of signs in the Jacobian 80, we have $tr\ 80 < 0$ and $\det 80 < 0$. Under these conditions a sufficient condition for stability is that the determinant $H_{\frac{W}{P}} J_{\frac{N}{K}} - H_{\frac{N}{K}} J_{\frac{W}{P}} > 0$, which is to say that, holding z constant, the stationary locus of real wages, $\left(\frac{W}{P}\right)^{\bullet} = 0$, is steeper than the stationary locus of the labor force to capital ratio, $\left(\frac{N}{K}\right)^{\bullet} = 0$. The picture is in Figure 13.

Whether or not producers take account of undesired inventory changes, the assumption that profitability drives production tacitly assumes that the economy is operating to the left of the labor-supply schedule. This is because the output-supply function is at the same time a labor-demand function, and to assume that profitability drives production and employment is implicitly to assume that labor is in excess supply. Recall the "short side" rule: if labor demand is less than labor supply, producers' preferences will dominate the preferences of the unemployed. By contrast, the space to the right of the labor-supply schedule is characterized by over-full employment, which is to say that workers are striving to work less, not more. Under these circumstances, even if it is profitable for producers to increase output, they will have trouble finding workers ready and willing to cooperate with their expansionary desires.

One way of expressing the short-side rule is to amend Eq 41 or 65 to make z respond to the minimum of the intensity with which employers' and workers' wish to change the level of output: Eq 41 becomes

$$\dot{z} = \min\left\{\theta_2\left(f' - \frac{W}{P}\right), \left[\theta_5\left(\frac{W}{P} - MRS\right)\frac{L}{N} + n - g\right]\frac{f'}{f}\right\} z \tag{81}$$

and Eq 65 becomes

$$\dot{z} = \min\left\{\theta_2\left(f' - \frac{W}{P}\right) - \theta_4\left(s\hat{a}_1^{-1}z - g\right), \left[\theta_5\left(\frac{W}{P} - MRS\right)\frac{L}{N} + n - g\right] f'\frac{L}{Y}\right\} z \tag{82}$$

Observe that, in the case illustrated in Figure 11, with equilibrium to the left of the labor-supply function, this amendment has no effect on (local) stability. For in the neighborhood of the equilibrium, the first term, $\theta_2\left(f' - \frac{W}{P}\right)$ or $\theta_2\left(f' - \frac{W}{P}\right) - \theta_4\left(s\hat{a}_1^{-1}z - g\right)$, will be vanishingly small, thus smaller than the second term, at least so long as we assume $n > g$. The situation is different if the labor-supply schedule lies to the left of the *IS* schedule. In this case the dynamics become more complicated because the equilibrium lies to the right of the labor-supply schedule, so labor-supply dynamics, rather than profitability dynamics, govern. (I shall argue anon that the notion of a labor supply schedule actually makes little sense in the context of an economy in which labor demand consistently outstrips labor supply, so this last case is of only formal interest.)

7 Fisher, Keynes, and Pigou Effects

The *IS* schedule, like the labor-supply schedule, is not stationary. So far it has been assumed that investment is a function of the nominal rate of interest ρ, which central bank policy (again by assumption) holds constant over time. But businessmen presumably are more interested in real rates of interest than in nominal rates of interest. The difference becomes important once we allow equilibrium to co-exist with changing prices and wages. With the price level changing at the rate $\frac{\dot{P}}{P}$, the real rate of interest is $\rho - \frac{\dot{P}}{P}$. (This is the so-called "Fisher Effect.") Taking the Fisher Effect into account, the investment function is $i\left(\rho - \frac{\dot{P}}{P}\right)$.[7] Under the standard assumption that i is a monotonically

[7] This makes sense if we assume that calculations of future returns involve adjusting the prices of *future* outputs and inputs in line with the *current* rate of price change, $\frac{\dot{P}}{P}$. This is to say that the Fisher Effect depends on an assumption about expectations, namely, the assumption of persistence of the current rate of price deflation (or inflation).

decreasing function of the interest rate, the value of $i\left(\rho - \frac{\dot{P}}{P}\right)$ associated with the equilibrium will be less than $i(\rho)$: $\frac{\dot{P}}{P} < 0$ so $\rho - \frac{\dot{P}}{P} > \rho$.

The effect is to move the *IS* schedule to the left. We can perhaps most easily picture this movement by assuming the starting point of the economy is on the original *IS* schedule, so that initially $\frac{\dot{P}}{P} = 0$ and investment demand $i(\rho - 0)$ is given by the original IS schedule. Consider Figure 14, in which the system

$$\dot{z} \equiv G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P}\right) z \tag{83}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}\right) = \left[\theta_3 \left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1 \left(i - s\hat{a}_1^{-1} z\right)\right]\frac{W}{P} \tag{84}$$

is replaced by the system

$$\dot{z} \equiv G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P}\right) z \tag{85}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv \left(\frac{\dot{W}}{W} - \frac{\dot{P}}{P}\right)\frac{W}{P} \equiv H\left(z, \frac{W}{P}\right) = \left\{\theta_3 \left(MRS - \frac{W}{P}\right)\frac{N}{K} - \theta_1 \left[i\left(\rho - \frac{\dot{P}}{P}\right) - s\hat{a}_1^{-1} z\right]\right\}\frac{W}{P} \tag{86}$$

Let the starting point B on the schedule $z = \frac{i}{s}\hat{a}_1$ be below the goods-supply schedule so that the dynamics immediately take the economy, $\left[z, \frac{W}{P}\right]$, to the right. Prices fall, and the *IS* schedule edges leftward. The equilibrium aggregate demand schedule is given by *IS*′, for which the real rate of interest is equal to the sum of ρ and the value of $-\frac{\dot{P}}{P}$ at equilibrium. But since the target *IS* schedule itself shifts as $\frac{\dot{P}}{P}$ changes, $\left[z, \frac{W}{P}\right]$ is in effect chasing a moving target. Whether the system is stable under these conditions depends on how rapidly the target *IS* schedule moves relative to the dynamic trajectory of the economy.

A sufficient condition for stability is that investment demand be very inelastic. Since $\frac{\dot{P}}{P} = \frac{\dot{W}}{W} - \frac{\left(\frac{W}{P}\right)^{\bullet}}{\frac{W}{P}}$, we have by the implicit function rule[8]

[8] Let

$$\left[\begin{array}{ll} G_z = \theta_2 f\left(f'\right)^{-1} f'' + \theta_2\left(f' - \frac{W}{P}\right) & G_{\frac{W}{P}} = -\theta_2 z \\ H_z = \dfrac{\left(\theta_3 \dfrac{\partial MRS}{\partial z}\dfrac{N}{K} + \theta_1 s \hat{a}_1^{-1}\right)}{1+\theta_1 i'} & H_{\frac{W}{P}} = \dfrac{\theta_3\left(\dfrac{\partial MRS}{\partial \frac{W}{P}} - 1\right)\dfrac{N}{K}}{1+\theta_1 i'} \end{array}\right]$$

$$= \left[\begin{array}{cc} - & - \\ \mp & \pm \end{array}\right] \tag{89}$$

With $\dfrac{\partial MRS}{\partial \frac{W}{P}} < 1$ by assumption, the sign of H_z and $H_{\frac{W}{P}}$ hinge on the magnitude of i'. For values sufficiently close to zero, $1 + \theta_1 i'$ is positive, so $H_z > 0$ and $H_{\frac{W}{P}} < 0$, which is to say that the system is stable. If, on the other hand, investment demand is highly elastic with respect to the rate of interest, then the signs will be reversed, and the system is unstable.

Along with the Fisher Effect, the literature speaks of a "Keynes Effect" and a "Pigou Effect." The Keynes Effect is the response of the interest rate to changes in money demand that result from changes in the price level. Changes in the price level affect first of all the transactions demand for money. The higher the price level, the more of a given stock of money is needed for transactions, and the less is available to satisfy the speculative demand for money. The consequence is a higher nominal interest rate to equilibrate money demand and supply. Our model suppresses the Keynes effect by assuming the money supply adjusts to maintain a constant nominal rate of interest.

The Pigou Effect (also called the "Real Balance Effect") is more complex. It measures the impact on consumption demand of the change in value of nominal

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$$\begin{aligned} \dot{x} &= f(x,y) \\ \dot{a} &= (u-v)y = g(x,y,v) = g(x,y,u-\frac{\dot{a}}{y}) \end{aligned}$$

Then we have the Jacobian

$$\left[\begin{array}{ll} \dfrac{\partial \dot{x}}{\partial x} = f_x & \dfrac{\partial \dot{x}}{\partial y} = f_y \\ \dfrac{\partial \dot{a}}{\partial x} = g_x + g_v\left(-\dfrac{1}{y}\right)\dfrac{\partial \dot{a}}{\partial x} & \dfrac{\partial \dot{a}}{\partial y} = g_y + \left[\left(-\dfrac{1}{y}\right)\dfrac{\partial \dot{a}}{\partial y} + \dfrac{\dot{a}}{y^2}\right] \end{array}\right] \tag{87}$$

or, since $\dot{a} = 0$ in equilibrium,

$$\left[\begin{array}{ll} \dfrac{\partial \dot{x}}{\partial x} = f_x & \dfrac{\partial \dot{x}}{\partial y} = f_y \\ \dfrac{\partial \dot{a}}{\partial x} = \dfrac{g_x y}{y + g_v} & \dfrac{\partial \dot{a}}{\partial y} = \dfrac{g_y}{y + \dot{g}_v} \end{array}\right] \tag{88}$$

balances as the price level changes. In the limit, it is suggested, if prices continue to fall at an equilibrium like E or E' in Figure 14, then eventually one would be able to buy the whole GDP with a dime. At this point, so the argument goes, motivation for saving would disappear and the propensity to save would fall to zero. Thus the denominator in Eq 33 which defines the *IS* schedule

$$z = \frac{i}{s}\hat{a}_1 \tag{90}$$

falls to zero and the *IS* schedule moves to the right. Before s reaches *0*, the *IS* schedule will intersect the labor-supply schedule and the producers' equilibrium schedule at the point of their intersection, namely, the point that defines the neoclassical equilibrium. At this point full employment–defined as a job for every willing worker–co-exists with maximization of profits *and* with equality of income and expenditure. The economy simultaneously satisfies the three conditions that otherwise over-determine the equilibrium. The picture is as in Figure 15.

Arthur Pigou himself (1947, pp 187-188), and Don Pantinkin after him (1948, pp 556-557; 1965, p 339; 1987, pp 100-101) went out of their way to emphasize that before the Pigou Effect could come into play, governments would be forced to take action to offset the deflationary spiral and the massive unemployment that characterize an equilibrium like E or E' in Figure 10.[9] The reason why is that the benefits of the Pigou Effect apply to only a part of the money supply, so-called outside money. Inside money–money created by the banking system–is offset one-for-one by the liabilities of non-bank agents, and the effects of falling prices on liabilities fixed in money terms are opposite to the effects on assets: fixed liabilities become more expensive in real terms as the price level falls. But the results are not symmetric. Whereas the Pigou Effect provides a windfall to creditors, whose wealth increases as the price level falls, the Pigou Effect is nothing short of catastrophic for debtors, who face bankruptcy as the Pigou Effect works its magic. The relatively small portion of the money supply constituted by outside money is immune to these offsets since their are no corresponding liabilities (though a believer in Ricardian Equivalence might not accept the conventional distinction between outside and inside money).

The Pigou Effect thus illustrates a strange notion of the role of theory. A logical possibility with little to no relevance to the world we actually inhabit becomes the reason for arguing that *The General Theory* does not provide a coherent basis for an unemployment equilibrium!

Keynes did not address the Pigou Effect directly, but his strictures on wage flexibility as a policy to combat depression apply to the Pigou Effect as well since both the Pigou Effect and wage flexibility presuppose that a segment of the population acquiesces in its economic destruction This is what Keynes had to say about wage flexibility (1936, pp 266-267, 269):

[9] Pigou, according to William Baumol (2000, p 1n), was in some doubt about the Pigou Effect. "Dennis Robertson repeatedly told me how on passing Pigou's lair, the great man would regularly emerge, demanding 'Robertson–tell me, what is the Pigou effect?"'

> Just as a moderate increase in the quantity of money may exert an inadequate influence over the long-term rate of interest, whilst an immoderate increase may offset its other advantages by its disturbing effect on confidence; so a moderate reduction in money-wages may prove inadequate, whilst an immoderate reduction might shatter confidence even if it were practicable.
>
> There is, therefore, no ground for the belief that a flexible wage policy is capable of maintaining a state of continuous full employment... The economic system cannot be made self-adjusting along these lines.
>
> ...
>
> To suppose that a flexible wage policy is a right and proper adjunct of a system which on the whole is one of *laissez-faire*, is the opposite of the truth. It is only in a highly authoritarian society... that a flexible wage-policy could function with success. One can imagine it in operation in Italy, Germany or Russia, but not in France, the United States or Great Britain.

What is the comparative advantage of an authoritarian regime in this regard? To put it simply, repressive force. Absent the repression that authoritarian regimes can impose on the people, there would be riots in the streets before the benefits of a Pigou Effect could be reaped. The economic turmoil of debtors whose economic position was eroding before their eyes would result in political turmoil that the democratic process would be hard put to contain.[10] In view of the limited reach of the Pigou Effect, readers will perhaps indulge my decision to ignore it.

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[10] One example well illustrates how far the democratic fabric was stretched in the United States by the fall in agricultural prices of more than 50 per cent between 1929 and 1932. In response to the effects of this fall in the price level on indebted farmers, the Minnesota Legislature enacted a statute allowing mortgage debtors to postpone the repayment of their loans on the finding by a local judge of financial hardship. A perfectly reasonable reaction to hard times, the democratic process at work.

The problem is that the warp of American democracy on which the Minnesota Legislature had to weave debt relief was the Constitution of the United States, and on every canon of constitutional interpretation–language, intent of the framers, and the spirit ot the text–the Minnesota law was plainly unconstitutional.

The language of the relevant Constitutional provision prevented states from enacting any law abridging the obligation of contract (the so-called "contract clause,"). Moreover this was the clear intent of the framers (the contract clause was a reaction to the agitation in many states–including the famous Shays' Rebellion in Western Massachusetts–to provide relief to debtors in the post-Revolutionary depression). Finally, the spirit of the provision was clearly directed against populist actions on the part of states. Nonetheless, in *Home Saving and Loan v Blaisdell*, the Supreme Court of the United States upheld the Minnesota statute on the grounds that although the Depression did not create special state power, it might be the occasion for the exercise of such power. (Figure that one out if you can.)

The power at issue was the police power of the states. *Police* power? On second thought, quite right. The Minnesota Legislature had good reason to fear riots and worse if the hard pressed farmers of the state were not granted some relief.

8 Path Dependence as a Characteristic of Over-Determined Models

Up to now, we have worked in the context of a single adjustment process, in which the balance of expenditure and income drives prices, and marginal profitability drives output. This corresponds to what Sir John Hicks has called "flexprice" Keyensian economics (Hicks 1974). I believe that the flexprice model fits the *General Theory,* but this is not the model through which generations of students have made the acquaintance of Keynes in the elementary texts. Rather the canonical model of the elementary text is what Hicks called the "fixprice" model, in which demand directly drives output, and prices are more or less ignored. It is difficult to reconcile such a model with a competitive framework, and, indeed, when the process of demand driving output is justified, it is customarily through the length of order books and the size of inventories. An excess of expenditure over income, for instance, is supposed to stimulate output by increasing the backlog of orders and depleting inventories below their desired levels, signals not generally associated with the competitive framework.

We can capture this fixprice adjustment process in our models by assuming that the difference between desired investment and desired saving drives capacity utilization. But we need not assume prices are fixed once and for all. Rather, we shall assume that prices are adjusted by producers according to the marginal profitability of production. Imagine a Hicksian "week," in which prices are fixed on Monday for the duration of the week, and changed again only on the following Monday: the model is "fixprice" in the sense that during the week, from Monday to Monday, variations in product demand are accommodated by changes in output rather than by price changes. Prices change only once per week. As we take the limit of ever shorter weeks, we arrive at the continuous time model in which prices and output adjust simultaneously. It may appear anomalous that the a "fixprice" process has prices changing continuously, but in the present model, the essential difference between the fixprice and the flexprice model is not that prices are sticky in one and fluid in the other, but the manner in which demand drives prices: the fixprice process remains different from the process of the flexprice model even when the length of the Hicksian week goes to zero.[11]

With the excess of investment over saving driving capacity utilization, we have

$$\dot{z} = G\left(z, \frac{W}{P}\right) = \theta_1 \left(i - sz\hat{a}_1^{-1}\right) z \tag{91}$$

Producers will be assumed to modify prices according to the relationship between marginal revenue and marginal cost. Prices are raised when marginal

[11]It may also appear anomalous that *competitive* producers adjust prices. In the pure price-taking world, price changes are by assumption not initiated by indidvidual agents–hence the auctioneer of Walrasian theory. This anomaly would be easily set right with a dose of monopolistic competition.

cost exceeds marginal revenue (in order to discourage sales and curtail output), and prices are reduced when marginal revenue exceeds marginal cost (in order to encourage activity). We have

$$\dot{P} = \theta_6 \left(\frac{W}{f'} - P \right) \tag{92}$$

or

$$\frac{\dot{P}}{P} = -\theta_6 \left(f' - \frac{W}{P} \right) (f')^{-1} \tag{93}$$

Money wages, as before, are driven by excess supply in the labor market:

$$\frac{\dot{W}}{W} = \theta_3 \left(MRS - \frac{W}{P} \right) \frac{N}{K} \tag{94}$$

This gives

$$\left(\frac{W}{P} \right)^{\bullet} \equiv H\left(z, \frac{W}{P} \right) = \left[\theta_3 \left(MRS - \frac{W}{P} \right) \frac{N}{K} + \theta_6 \left(f' - \frac{W}{P} \right) (f')^{-1} \right] \frac{W}{P} \tag{95}$$

and the Jacobian is

$$\begin{bmatrix} G_z = \theta_1 \left(i - sz\hat{a}_1^{-1} \right) - \theta_1 sz\hat{a}_1^{-1} & G_{\frac{W}{P}} = 0 \\ H_z = \theta_3 \frac{\partial MRS}{\partial z} \frac{N}{K} \frac{W}{P} + \theta_6 \left(\frac{W}{P} \right)^2 \frac{f''}{(f')^3 \hat{a}_1} & H_{\frac{W}{P}} = \theta_3 \left(\frac{\partial MRS}{\partial \frac{W}{P}} - 1 \right) \frac{N}{K} \frac{W}{P} + \theta_3 \left(MRS - \frac{W}{P} \right) \frac{N}{K} \\ & +\theta_6 \left(f' - \frac{W}{P} \right) (f')^{-1} - \theta_6 (f')^{-1} \frac{W}{P} \end{bmatrix} =$$

$$\begin{bmatrix} - & 0 \\ \pm & - \end{bmatrix} \tag{96}$$

The system is stable since tr 96 < 0 and det 96 > 0. The picture is given by Figure 16.

Unlike the just-determined model, in which the equilibrium is independent of dynamics, in the present class of models *the dynamics determine the equilibrium itself.* Figures 9 and 16 start with identical schedules–a conventional wage schedule, a labor-demand schedule, and an IS schedule. But the equilibria in the two diagrams differ because the dynamics differ. The moral is that in over-determined systems we cannot follow the customary procedure of beginning with equilibrium, with statics, and then tacking on a more or less plausible dynamics to show whether or not the system is resilient to shocks. Rather we must start with the dynamics and let the equilibrium fend for itself. If the dynamics lead to a stable equilibrium, so be it. If not, so much the worse for equilibrium. In other words, we must focus our attention on process and let the outcome emerge as it will. What is merely sensible procedure for a just-determined model is a structural necessity in the over-determined case.

One consequence of path dependence is that the dynamics determine the shape of the stationary locus of real wages. In Figure 16 the stationary locus of real wages is of indeterminate slope since H_z may be positive or negative, and $H_{\frac{W}{P}}$ is negative. The economic significance of this indeterminacy is that the comparative statics of varying aggregate demand no longer implies a negative relationship between capacity utilization and the real wage, as in the flexprice model.

In the late 1930s, there was a lively discussion of the supposed implication of the *General Theory* that real wages should behave contracyclically. Two young Americans, John Dunlop (1938) and Lorie Tarshis (1939) launched their academic careers by casting empirical doubt on the proposition of a negative relationship between real wages and output. In reply, Keynes (1939) pointed out that the negative relationship between wages and economic activity was a consequence not of the *General Theory* but of Marshallian assumptions to which the economics profession had generally assented: the assumption of the neoclassical production function of diminishing marginal productivity; the assumption of price-taking producers, for whom profit maximization implied $MP_L = \frac{W}{P}$; and the assumption that equilibrium is characterized by profit maximization.

The easy way out of the problem is to drop the assumption of price-taking producers, but the fixprice model shows that there is another way out, namely to drop the assumption that equilibrium is characterized by profit maximization. In the fixprice model of Figure 20, unlike the flexprice model of Figure 10, there is no implication that comparisons of equilibria with different levels of aggregate demand will trace out a negative relationship between capacity utilization and the real wage. At the equilibrium E in Figure 16, producers are not on their goods supply schedule: rather, producers are continually lowering prices to stimulate demand, but their attempts to reduce prices and expand capacity utilization are frustrated by the fall in money wages, so that real wages, demand, and capacity utilization remain constant. The sequence of equilibria traced out by moving the IS schedule lies along the stationary locus of real wages $\left(\frac{W}{P}\right)^{\bullet} = 0$ and may reflect a positive or a negative relationship between $\frac{W}{P}$ and z.

9 Keynes and the Long Run

It is my contention that the enduring contribution of Keynes is a vision of capitalism in which aggregate demand matters–*in the long run as well as in the short.* This, to be sure, is a variant reading of Keynes. The conventional view is that Keynes provides (at best) a theory of the short run, and Keynes's well known quip that in the long run we are all dead adds a measure of support to the idea that Keynes himself made no claims for his theory beyond the short period. In this view, this section is on its face redundant. Keynesian economics is an economics of why the classical verities do not hold *all* the time. Keynesian economics is the search for "sand in the wheels"–sticky wages, staggered contracts, menu costs–things that interfere with the smooth operation of the Walrasian economy for some length of time. *But not forever.*[12]

I have no wish to enter into doctrinal disputes about what Keynes meant or *really* meant. Suffice it to say that, although I would dispute the notion that Keynes saw the *General Theory* as a theory only of the short run, I am ready if necessary to rescue Keynes from himself.

But how can Keynesian economics be relevant to the long run? Isn't long-run output dependent on stocks of capital, labor, technology, and resources? And aren't the last three determined exogenously and the first by the saving behavior of agents?

This conceptual problem has an empirical counterpart. If demand mattered in the long run, it could only work through a short side rule: we might observe demand failure but never excess demand. But if we were to observe demand failure in the long run, it could show up only as growing excess capacity. A long run Keynesian theory would then be *a priori* a theory of secular stagnation. One can argue that there have been periods of stagnation, but it would be hard to sell the idea that stagnation is the permanent, or even the dominant, state of the American, European, or Japanese economies.

My answer is that resources are *not* exogenous. In particular, it is illegitimate to project the idea of a fixed labor force into the long run, as neoclassical economics does with the idea of a "natural rate of growth." In the long run, I would contend, *the labor force is itself endogenous.* Demand creates its own supply (*pace* J B Say). The labor force adapts to the needs of the capitalist economy.

The key is migration. Migration of two kinds, physical and social. The

[12] This certainly seems to be the view of the New Keynesians, who have long since ceded everything but the short run to the New Classicals. The labels New Keynesian and New Classical are in any case deceptive. There is not much new in either camp, and there is not much of Keynes in New Keynesian theory. New Keynesian theory is a hi-tech version of the so-called neoclassical synthesis already enshrined in Paul Samuelson's *Economics* (1954) when I took freshman economics more than four decades ago. New Classical economics is a hi-tech version of pre-Keynesian economics, with its cmphasis on market clearing, rationality-as-maximization, etc.

The New Keynesians, like most of the old Keynesians (with the exception of some British diehards like Roy Harrod and Joan Robinson) gave up the real fight when they conceded the long run to the other side.

first refers to the literal movement of people from one place to another, and the second to the movement of workers from one sector of the economy to another, without any geographical displacement.

This will immediately provoke objections of many kinds. To those who say that migration does not augment the stock of labor, but merely moves it around, my response is that our focus should be on the development of the *capitalist* economy, or if that word is too jarring for this post Cold War age, the business sector, the sector in which plant, equipment, and jobs are concentrated, and the sector in which the relationship between desired saving and desired investment–the heart of the Keynesian vision–is most problematic. For the purpose of understanding the dynamics of capitalism, the economy of the household, or the self-proprietor economy that, in the past at least, dominated agriculture, or the subsistence economy of a Mexican village are relevant as sources and sinks, but are not usefully combined with the capitalist economy into a single entity.

To those who accept that migration may have been central to the history of capitalism, but argue that it is not relevant to the future, certainly not to the asymptotic future, my reply is that the institutional mechanisms that operated in the past to adjust labor supply to demand are alive and well as we enter the third millennium of the Common Era. Yes, the asymptote may bite some day, but the asymptotic future is the time period Keynes had in mind when he made his famous quip about the long run. The long run for which I would claim Keynesian theory is relevant is not the asymptotic future, but a period over which labor supply is not fixed, instead adjusting to demand. It is the period over which a sufficient pool of potential migrants, social and physical, exists that we will never run out of workers, whatever demand conditions might be.

Central to this vision is the rejection of the idea of *the economy.* "The economy," conceived of as a homogeneous set of institutional rules and relationships that encompass all economic activity, is not, I suggest, a useful analytic category. Instead, this essay focuses on the *capitalist* economy, a subset of the whole. Neoclassical economists rarely speak of capitalism, but rather of the market economy. However, market economy is simply too crude a classification. Markets do not in themselves set capitalism apart from other forms of economy. In particular markets do not in themselves give rise to the separation of saving and investment that is at the heart of the Keynesian paradigm.

Rather the capitalist economy is understood to be embedded in a larger economic system, from which it can draw labor if its requirements outstrip what it can provide endogenously. Moreover the domestic economy is embedded in an international system and can draw on immigrants to make up for any shortfall in the domestic labor supply. Finally, capital (and capitalism) can be exported to take advantage of labor resources *in situ.* (In principle, all these mechanisms work in both directions, but in practice there is the proverbial difference between pulling and pushing on a string.)

Historically, all three possibilities have played a role in capitalist development. Virtually every capitalist economy has drawn upon agriculture to provide workers. In the United States, agricultural employment has shrunk from $\frac{1}{3}$ to $\frac{1}{50}$ of the labor force over the last century and has shrunk by more than $\frac{2}{3}$ in absolute numbers. In Europe and Japan the numbers are similar. Capitalist agriculture may now be the rule, but the dominant economic form in the agricultural economy of a century ago was the family farm, relying for the most part on family labor. (Hired labor was for most farms a seasonal and occasional supplement to the family's resources.) The *self proprietor*, not the capitalist, was the dominant figure. Self-proprietor economies follow different rules from those followed by capitalist economies, both with respect to the determination of saving and investment and with respect to the determination of employment. We have already observed that the agricultural sector behaved very differently during the Great Depression from the way the capitalist economy behaved, the burden of adjustment falling not on output, but on prices, and through the price level on real interest rates paid on mortgage and other debt.

Even more dramatic has been the incorporation of women into the paid labor force. In the US the female participation rate grew from 20 percent a century ago to 30 percent on the eve of World War II to over 60 percent today. Again, the household is certainly an "economy," but its rules and behavior are very different from those of the capitalist sector, and it serves little purpose to model the household economy the same way we model the capitalist economy.

Immigration is a second means by which the labor supply adjusts to demand. In fact, every capitalist economy with the exception of East Asia has relied on immigration at one time or another to meet its labor needs. The US did so famously until xenophobia closed the door after World War I, and it is only recently that the door has been more than ajar. In Western Europe, the post World War II boom would have been impossible without the flow of workers from the periphery: from Southern and Southeastern Europe to Germany; from Southern Europe, North Africa, and finally sub-Saharan Africa to France; from South Asia to England; from Finland and Southeastern Europe to Sweden.

Finally, if the mountain won't come to Mahomet, Mahomet goes to the mountain: capital is exported to take advantage of plentiful and cheap labor that is brought into the capitalist system without ever leaving home.

The Marxian term for this pool of labor is the "reserve army." For our purposes it is important to emphasize that this reserve army is not a static concept, not a fixed set of people available as needed. It is a dynamic concept, corresponding to a pool of workers that is created and refashioned according to the needs of the capitalist economy. Two recent examples will illustrate the dynamic nature of the reserve army. First, the recent relaxation in restrictions on social security payments to retired Americans who have earned income. With unemployment rates hovering near 4 percent, senior citizens seemed an obvious source of recruits to the labor force. Second, there has been increasing pressure from business to relax the immigration laws, at least for categories of workers in short supply.

One way of incorporating labor-force endogeneity is the model developed in the section headed "Incorporating Labor Force Growth," with the rate of growth of the labor force depending on the level of employment and labor supply, as in Eq 76:

$$\left(\frac{N}{K}\right)^{\bullet} = \theta_5 \left[l - \alpha\psi \left(\frac{W}{P}\right) \frac{N}{K} \right] \tag{97}$$

A more far reaching alternative is to remove the labor-supply equation from the model altogether. In place of a labor-supply equation, let money wages be determined by a *conventional*, or *target*, wage rate, *and* the power of workers to maintain this rate[13]. Here I am again taking a leaf from the book of Marx since the conventional wage bears more than a passing resemblance to the Marxian subsistence wage. Obviously, subsistence, if it is understood to mean *biological* subsistence, is not an issue for today's workers, but then Marx and the classical economists before him (Adam Smith and David Ricardo in particular) never understood subsistence to mean biological subsistence. Marx, like his classical forebears, stressed the historical, moral, and social elements that determined subsistence (1865, p 57 and 1867, p 171). I would emphasize two of these elements, class power and public opinion. Class power needs little comment–I would only caution against identifying the power of the working class solely with the power of trade unions. The organization, militancy, and cohesion of the working class may be correlated with the power of trade unions, but the two are not the same thing.

Public opinion influences both the agenda of the working class, and the feasibility of achieving its agenda. In the 19^{th} century, the issue was whether workers were entitled to a wage which would allow them to eat bread made of refined wheat flour rather than bread made of coarse grains, regardless of the tastes of individual workers between white and rye bread. In the 20^{th} century the issue has become whether workers should be able to own automobiles and houses, again regardless of the tastes of individual workers between these and competing goods. In the 21^{st} century the issue may become whether workers should be able to own vacation cottages by the lake. The success of workers' attempts to increase (or maintain) a share of output adequate to purchase these consumption bundles has in the past depended in no small part on whether the general public accepts workers' claims as legitimate. In this respect the future will most likely resemble the past.

Of course the other side is not without resources, both in terms of power and public opinion. The ease or difficulty with which production and jobs can be moved from one part of the country to another, or from one country to another, has had a great bearing on the conventional wage–as American workers have learned over the past quarter century. And the extent to which the climate

[13] In the real world, characterized by technical change, the argument is more realistically formulated in terms of a conventional wage *share* rather than a rate. But in models which abstract from technical change, like the ones in this essay, there is nothing lost by formulating the argument in terms of the wage rate.

of public opinion is favorable to workers' demands depends on the resources, organization, and shrewdness which the business community shows.

For the purposes of this essay, I will summarize the determinants of the relative power of labor and business in terms of a single parameter θ_7 which measures the relative speed with which the gap between the actual wage and the conventional wage is closed:.

$$\frac{\dot{W}}{W} = -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P} \right)^* \right] \tag{98}$$

The greater is θ_7, the greater the relative power of labor.[14] The basic model becomes

$$\dot{z} = G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P}\right) z \tag{100}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}\right) = \left\{ -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P} \right)^* \right] - \theta_1 \left(i - s\hat{a}_1^{-1} z\right) \right\} \frac{W}{P} \tag{101}$$

and the picture is given in Figure 17 .

Figure 17 illustrates what I would regard as the dominant long run tendency of the capitalist economy, namely the tendency of expenditure to outrun income ($i - s\hat{a}_1^{-1}z > 0$ in the present model) and for the price level therefore to rise. At an equilibrium such as E in Figure 17 , the pressure on prices coming from the demand side is exactly balanced by the pressure workers put on money wages; the real wage remains constant.

Observe that a higher conventional wage goes along with a *lower* equilibrium level of capacity utilization: as the horizontal line representing the equation $\frac{W}{P} = \left(\frac{W}{P}\right)^*$ moves upward in Figure 17 , the constant real wage locus $\left(\frac{W}{P}\right)^{\bullet} = 0$ moves upward and thus intersects the $MP_L = \frac{W}{P}$ schedule further to the left. (Evidently long run deflation, with continually falling wages and prices falling at the same rate, is a logical possibility. As a purely logical matter, the difference between the two regimes hinges on the position of the conventional wage schedule relative to the intersection of the IS schedule and

[14] A more realistic wage-adjustment equation would include a term reflecting the power of labor to maintain real wages in the face of price increases. In place of Eq 98 , wage dynamics would be reflected by

$$\frac{\dot{W}}{W} = -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P} \right)^* \right] + \theta_8 \frac{\dot{P}}{P} \tag{99}$$

In the interest of keeping the argument as simple as possible, this refinement is omitted in the discussion of the Keynesian long run.

the $MP_L = \dfrac{W}{P}$ schedule: if the conventional wage is sufficiently low, the long run equilibrium E lies to the right of the IS schedule. But the strictures entered on the political feasibility of continuing deflation during the discussion of the Pigou Effect hold in spades over the long run.)

The Jacobian is

$$\begin{bmatrix} G_z = \theta_2 f\left(f'\right)^{-1} f'' + \theta_2 \left(f' - \dfrac{W}{P}\right) & G_{\frac{W}{P}} = -\theta_2 z \\ H_z = \theta_1 s\hat{a}_1^{-1} \dfrac{W}{P} & H_{\frac{W}{P}} = -\theta_7 \left[\dfrac{W}{P} - \left(\dfrac{W}{P}\right)^*\right] - \theta_1 \left(i - s\hat{a}_1^{-1} z\right) - \theta_7 \dfrac{W}{P} \end{bmatrix} =$$

$$\begin{bmatrix} - & - \\ + & - \end{bmatrix} \tag{102}$$

We have $tr\ 102 < 0$ and $\det\ 102 > 0$, so the equilibrium is stable.

10 Distribution and the Keynesian Long Run

An aspect of the Keynesian long run that has received considerable attention in the heterodox literature, beginning with Joan Robinson (1956, 1962), is the reciprocal relationship between profitability and growth. On the one hand, high profits induce greater investment demand. On the other hand, high profits generate greater saving The metaphor is Keynes's (1930, p 139): profits are like the widow's cruse which, for the prophet Elijah's sake, God causes to refill as its oil is used up (1 Kings 17:8-16).

My preferred version is to make investment depend on the difference between the anticipated rate of profit, r^e, and the rate of interest ρ, .

$$\frac{I}{K} \equiv i = i(r^e - \rho) \tag{103}$$

(A more general form of the investment equation is $i(r^e, \rho)$, but since we will continue to regard ρ as fixed by central bank policy, the more general form would add little.)

I deliberately use the term "anticipated rate of profit" instead of "expected rate of profit" because "expected" suggests a mathematical expectation, and behind that, a probability calculus over future returns. The more vague notion of anticipation is compatible with the uncertainty that I, along with Keynes, take to be at the heart of the investment decision (Keynes 1936, ch 11; Marglin 2000). I should note here an ambiguity in the literature that also begins with Joan Robinson, namely, whether profit anticipations refer to the set of investment projects under consideration or to a more general notion of profitability on the entire capital stock. To me it makes more sense to take the argument r^e as a measure of general profit anticipations, letting the shape of the investment function depend on the flow of new projects.

In what follows I will assume r^e depends on two variables, the current profit share, π, and the current rate of capacity utilization, z. This is a shorthand for the more complex process by which profit anticipations are undoubtedly formed in reality, for which distributed lags on past values of π and z might be more appropriate. The functional dependence

$$r^e = r^e(\pi, z) \tag{104}$$

is not to be confused with the mathematical identity $r = \pi z \hat{a}_1^{-1}$. To force this form on the anticipated rate of profit is to make unwarranted restrictions on the derivatives of r^e with respect to π and z. With

$$r^e = \pi z \hat{a}_1^{-1} \tag{105}$$

we have $r^e_\pi = z\hat{a}_1^{-1}$ and $r^e_z = \pi\hat{a}_1^{-1}$, so that the elasticity of substitution between profit share and capacity utilization ϵ for any anticipated profit rate is unity:

$$\epsilon \equiv \frac{r^e_\pi}{r^e_z}\frac{z}{\pi} = 1 \tag{106}$$

which is to say that the impact of the current profit share and the current rate of capacity utilization on the anticipated rate of profit are exactly symmetric. In practice, the impact of a change in today's profit share or capacity utilization on profitability anticipations for tomorrow can be expected to depend on the mind-set of the capitalist class. If businessmen are already optimistic about growth prospects and capacity utilization but pessimistic about the profit share, then a change in today's profit margins and (consequently) today's profit share can be expected to have a relatively high impact, as compared with the impact of greater capacity utilization. By contrast, if producers are confident about the profit share but doubtful about capacity utilization, then the relative impact of a change in the profit share may be relatively weak. In any case there is no good reason to prejudge the issue by assuming the reaction symmetry implied by Eq 105.

With respect to saving, the long run Keynesian framework is more classical than Keynesian, certainly if Keynesian means the models of the *General Theory.* It was a central point of the framework of Joan Robinson, Nicholas Kaldor, and a whole generation of heterodox economists who studied at the University of Cambridge in the quarter century after World War II that the propensity to save out of profits is higher than the propensity to save out of wages. The simplest version of the so-called Cambridge Equation which captures this distributional effect on the savings rate is to assume that a fraction s_π of profits are saved and that all wages are consumed.[15] This gives the saving function as

$$\frac{S}{K} = s_\pi r = s_\pi \pi z \hat{a}_1^{-1} \tag{107}$$

The IS schedule is given by $I = S$, or $i(r^e - \rho) = s_\pi \pi z \hat{a}_1^{-1}$. Its shape in $z \times \frac{W}{P}$ space depends on the relative responsiveness of investment and saving to the profit share and capacity utilization. The slope is given by

$$\left(\frac{d\frac{W}{P}}{dz} \right)_{IS} = -\frac{i' \left\{ r^e_\pi \left[\left(-\frac{W}{P} \right) \left(\frac{f - f'l}{fzf'} \right) \right] + r^e_z \right\} - s_\pi \left(f'\hat{a}_1 \right)^{-1} \left(f' - \frac{W}{P} \right)}{-i' r^e_\pi f^{-1} l + s_\pi l} \tag{108}$$

In the neighborhood of the labor-demand schedule, $f' - \frac{W}{P}$ becomes vanishingly small, and we have

$$\left(\frac{d\frac{W}{P}}{dz} \right)_{IS} = \frac{i' \left(-r^e_\pi \frac{\pi}{z} + r^e_z \right)}{i' r^e_\pi f^{-1} l - s_\pi l} \tag{109}$$

Suppose, for the moment, that, with z fixed, the marginal responsiveness of investment to the rate of profit is less than the marginal responsiveness of saving:

$$i' r^e_\pi f^{-1} - s_\pi < 0 \tag{110}$$

Amit Bhaduri and I (Marglin and Bhaduri 1990) have called Condition 110 the "Robinsonian Stability Condition" since this is a requirement of stability in the models of Joan Robinson (and others, such as Donald Harris [1978], John Roemer [1978] and myself [Marglin 1984]) which take the rate of profit as a state variable, that is, which do not break down the profit rate into a profit share and a rate of capacity utilization. As long as the Robinsonian Stability Condition holds, the denominator of the right hand side of Eq 109 is negative. The sign of the numerator depends on the elasticity of substitution between π and z for fixed r^e. If this elasticity is less than unity, so that

$$-r^e_\pi \frac{\pi}{z} + r^e_z > 0 \tag{111}$$

This is to say that the responsiveness of anticipated profit to capacity utilization is relatively strong. In this case the numerator is positive, and the IS schedule is negatively sloped where it intersects the labor-demand schedule. If the

[15] For a discussion of the more elaborate saving models that have emerged in the Cambridge tradition, see *Growth, Distribution, and Prices* (Marglin 1984), ch 6-7, 17-18.

inequality in Expression 111 is reversed, which implies a weak responsiveness of investment to z, the IS schedule is positively sloped.

The borderline case $-r^e_\pi \frac{\pi}{z} + r^e_z = 0$, which in effect assumes that r^e depends only on r, makes it easy to see what is going on. By continuity, this IS schedule must be horizontal where it intersects the labor-demand schedule. In this case, the schedules relating investment and saving to the rate of profit are independent of the level of capacity utilization. So, given the real wage, we can map a *unique* equilibrium rate of profit to corresponding levels of capacity utilization. That is, for each level of $\frac{W}{P}$, we can map r^*–the r such that desired saving and investment are equal–to z. The first quadrant of Figure 18 shows investment and saving schedules which satisfy the Robinsonian Stability Condition. The schedules in the second quadrant relate the rate of profit to the rate of capacity utilization for various levels of the real wage. Each schedule starts at the origin since without production there is no profit. The maximum r on each of these schedules takes place where $f' = \frac{W}{P}$. Reading across from the equilibrium r^* in the first quadrant, there are three possibilities. On schedules like those corresponding to $\left(\frac{W}{P}\right)_0$ and $\left(\frac{W}{P}\right)_1$ there are *two* levels of z for which $r = r^*$. The maximum r on these schedules is greater than the equilibrium r^*, so there are z's on both sides of the maximum that correspond to r^*. On schedules like $\left(\frac{W}{P}\right)_3$ there is *no* level of z for which $r = r^*$. With a sufficiently high real wage, profits are always insufficient to generate enough saving to cover investment demand. In between, there exists a unique schedule, corresponding to $\left(\frac{W}{P}\right)_2$ in the picture, for which the maximum value is r^*, which is to say that the *only* level of z that generates an equilibrium rate of profit–in terms of balancing investment demand with desired saving–is the profit maximizing level of capacity utilization. Figure 19 illustrates the model for this case.

Analysis reveals the equilibrium at E to be stable. We have

$$\dot{z} = G\left(z, \frac{W}{P}\right) = \theta_2 \left(f' - \frac{W}{P}\right) z \tag{112}$$

$$\left(\frac{W}{P}\right)^{\bullet} \equiv H\left(z, \frac{W}{P}\right) = \left\{-\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P}\right)^*\right] - \theta_1 \left(i - s_\pi \pi z \hat{a}_1^{-1}\right)\right\} \frac{W}{P} \tag{113}$$

so the Jacobian is

$$\begin{bmatrix} G_z = \theta_2 f\left(f'\right)^{-1} f'' + \theta_2 \left(f' - \frac{W}{P}\right) & G_{\frac{W}{P}} = -\theta_2 z \\ H_z = -\theta_1 i' \left[r^e_\pi \left[\left(-\frac{W}{P}\right)\left(\frac{f - f'l}{fzf'}\right)\right] + r^e_z \right] & H_{\frac{W}{P}} = -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P}\right)^*\right] - \theta_1 \left(i - s\hat{a}_1^{-1} z\right) \\ +\theta_1 s_\pi \left(f'\hat{a}_1\right)^{-1}\left(f' - \frac{W}{P}\right) & -\theta_7 \frac{W}{P} - \theta_1 \left(-i' r^e_\pi f^{-1} l + s_\pi l\right) \end{bmatrix} \tag{114}$$

$$= \begin{bmatrix} - & - \\ 0 & - \end{bmatrix} \tag{115}$$

for which tr 115 < 0 and det 115 > 0. [16]

11 Growth Empirics and Keynesian Economics

These are highly simplified models, but, simplified though they might be, they provide a framework for understanding some of the empirics of growth that

[16] Observe that the Robinsonian Stability Condition is no longer necessary for stability once we break down the profit rate into its components, profit share and capacity utilization. Suppose the Robinsonian Stability Condition is violated so

$$i' r^e_\pi f^{-1} - s_c > 0 \tag{116}$$

which is to say that at the margin investment is more responsive than saving to changes in the profit rate. Suppose further that the Strong Accelerator Condition is also violated, so that

$$-r^e_\pi \frac{\pi}{z} + r^e_z < 0 \tag{117}$$

Then sufficient conditions for stability are, first, that capacity utilization responds more rapidly to marginal profitability of production than real wages respond to departures from the conventional wage and imbalances between expenditure and income; and, second, that the $\left(\frac{W}{P}\right)^\bullet = 0$ locus is flatter than the $\dot{z} = 0$ locus. Algebrically, these conditions are

$$\theta_2 f\left(f'\right)^{-1} f'' - \theta_7 \frac{W}{P} - \theta_1 \left(-i' r^e_\pi f^{-1} l + s_c l\right) < 0 \tag{118}$$

which guarantees tr 114 < 0 and

$$\left(\frac{d\frac{W}{P}}{dz}\right)_{\left(\frac{W}{P}\right)^\bullet = 0} \equiv -\frac{H_z}{H_{\frac{W}{P}}} > \left(\frac{d\frac{W}{P}}{dz}\right)_{\dot{z}=0} - \frac{G_z}{G_{\frac{W}{P}}} \tag{119}$$

guarantees det 114 > 0.

would be otherwise difficult to fit into a Keynesian perspective. How for instance do we incorporate the idea that wage pressure–profit squeeze–has an important role to play in the determination of economic activity? In the standard Keynesian model, the real wage is a thermometer rather than a thermostat. The real wage is determined endogenously, so it makes no sense to talk about the effects of higher real wages on any variable of the system.[17] Thus it is difficult to address the issue of profit squeeze in the standard Keynesian framework

Yet, the failure of capitalism in the rich countries to sustain the high growth rates of the first quarter century after World War II is frequently, if not universally, attributed to profit squeeze. According to one story, it was the prosperity of the early postwar period, particularly the prosperity of the 1960s that led to a growth in wage claims in excess of productivity (Eichengreen 1996, Glyn, Hughes *et al* 1990). The US economy has made major structural transformations since that time, with the result that wage growth has not threatened profits for some time, but elsewhere the argument that high real wages squeeze profits retains its force.

The present models allow us to examine the effects of higher wages on capacity utilization and, *via* saving and investment, the effects on growth. Consider the impact of a higher conventional wage on the equilibrium in Figure 19 . A higher conventional wage displaces the $\left(\frac{W}{P}\right)^{\bullet} = 0$ schedule upwards, and thus reduces the equilibrium levels of z. Since the new equilibrium level of the real wage is higher than it was before, the profit share is also lower and investment and saving both fall, a picture consistent with the stylized facts of postwar growth, particularly in post 1960s Europe.

A second issue that these models illuminate is the "Now you see it, now you don't" character of the Phillips Curve. To be sure, models with an endogenous labor force are not good vehicles for examining the relationship between the rate of unemployment and the rate of inflation. By assumption, unemployment plays no role in the present class of models. But if we interpret the Phillips Curve in its more general sense of a *positive* relationship between the rate of capacity utilization and the rate of inflation, then we can easily fit the Phillips curve into the long run Keynesian model.

In the present model, the Generalized Phillips Relationship, defined as the case in which z and $\frac{\dot{W}}{W}$ move in the same direction, holds provided the disturbance to equilibrium comes from the demand side, that is, from a displacement of the IS schedule. To see this, suppose the investment demand function moves outward, either because the anticipated rate of profit associated with given levels of π and z increases, or because the amount of investment associated with given levels of r^e increases. Then the IS schedule in Figure 19 moves downward, pulling the equilibrium value of z downward along the labor-demand schedule. As the gap between the IS schedule and the conventional wage schedule widens,

[17]This is true not only of the *General Theory*, but also of the Keynsian growth models of Joan Robinson (1962, 1965) and her followers (for example, Marglin [1984], ch 4).

the equilibrium rate of inflation increases, which is to say that z and $\frac{\dot{W}}{W}$ move in the same direction. That is, the Generalized Phillips Relationship holds.

By contrast, suppose the disturbance to equilibrium comes from a fall in the conventional wage, which is to say, from the supply side. Then, as in the previous case, the equilibrium level of z increases. But now the gap between the IS schedule and the conventional wage narrows, so $\frac{\dot{W}}{W}$ falls. So the Generalized Phillips Relationship fails to hold. Translated into the experience of the postwar era, this suggests that as long as the conventional wage remained constant[18], and inflation was driven by aggregate demand, the Generalized Phillips Curve held. Beginning in the late '60s, when changes in the conventional wage began to drive inflation, the Generalized Phillips Curve no longer held, and this before the oil shocks came to dominate supply-side considerations.

Finally, these long run Keynesian models allow us a different view of some of the implications of the "new growth economics" pioneered by Paul Romer (1986). The most striking difference between the new growth economics and the old–the neoclassical growth model introduced by Robert Solow (1956)–is that the new growth economics rejects the Solovian inevitability of a fall in the growth rate as the per capita capital stock builds up to its steady state level. In Romer's perspective, growth is not constrained by labor resources, capital does not exhibit diminishing returns, and the growth rate can continue undiminished as capital accumulates on a per capita basis.

The long run Keynesian model accomplishes the same thing by a different route. Because the labor force is endogenous (rather than technology, as in the new growth economics), the Keynesian model, like the canonical endogenous growth model *à la* Romer, need not exhibit diminishing returns to capital accumulation. Indeed, I would argue that over the long sweep of the history of capitalism, and particularly in the last century, the endogeneity of the labor force has been more important than the endogeneity of technology in allowing the rich countries to sustain high rates of growth.

12 Path Dependence in the Long Run Model

In the long run as well as the short run model, we need not confine ourselves to a single dynamic process. The long run analog of the short run fixprice model retains the idea that capacity utilization responds directly to an excess of income over expenditure. We have

$$\dot{z} = G\left(z, \frac{W}{P}\right) = \theta_1 \left(i - s_\pi \pi z \hat{a}_1^{-1}\right) z \tag{120}$$

[18]Observe that in a world characterized by continual technical change, it makes more sense to interpret the conventional wage as a *share* of product rather than as a rate.

As before, producers will be assumed to modify prices according to the relationship between marginal revenue and marginal cost. Prices are raised when marginal cost exceeds marginal revenue (in order to discourage sales and curtail output), and prices are reduced when marginal revenue exceeds marginal cost (in order to encourage activity). We have

$$\dot{P} = \theta_6 \left(\frac{W}{f'} - P \right) \tag{121}$$

or

$$\frac{\dot{P}}{P} = -\theta_6 \left(f' - \frac{W}{P} \right) (f')^{-1} \tag{122}$$

With regard to money wages, however, the long run differs from the short. There is no labor supply schedule, rather money wages are driven by the gap between actual and conventional wages:

$$\frac{\dot{W}}{W} = -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P} \right)^* \right] \tag{123}$$

This gives

$$\left(\frac{W}{P} \right)^{\bullet} \equiv H\left(z, \frac{W}{P} \right) = \left\{ -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P} \right)^* \right] + \theta_6 \left(f' - \frac{W}{P} \right) (f')^{-1} \right\} \frac{W}{P} \tag{124}$$

With Robinsonian Stability and a unitary elasticity of substitution between π and z,

$$i' r_\pi^e f^{-1} - s_\pi < 0 \tag{125}$$

$$-r_\pi^e \frac{\pi}{z} + r_z^e = 0 \tag{126}$$

the Jacobian is

$$\begin{bmatrix} G_z = \theta_1 \left(i - s_\pi \pi z \hat{a}_1^{-1} \right) + \theta_1 i' \left\{ r_\pi^e \left[\left(-\frac{W}{P} \right) \left(\frac{f - f'l}{fzf'} \right) \right] + r_z^e \right\} & G_{\frac{W}{P}} = +\theta_1 \left(-i' r_\pi^e f^{-1} l + s_\pi l \right) \\ -\theta_1 s_\pi \left(f' \hat{a}_1 \right)^{-1} \left(f' - \frac{W}{P} \right) & \\ H_z = +\theta_6 \left(\frac{W}{P} \right)^2 \frac{f''}{(f')^3 \hat{a}_1} & H_{\frac{W}{P}} = -\theta_7 \left[\frac{W}{P} - \left(\frac{W}{P} \right)^* \right] + \theta_6 \left(f' - \frac{W}{P} \right) (f')^{-1} \\ & - \left[\theta_7 + \theta_6 (f')^{-1} \right] \frac{W}{P} \end{bmatrix} =$$

$$\begin{bmatrix} + & + \\ - & - \end{bmatrix} \tag{127}$$

Now tr 127 < 0 requires

$$\theta_1 i' \left\{ r^e_\pi \left[\left(-\frac{W}{P} \right) \left(\frac{f - f'l}{fzf'} \right) \right] + r^e_z \right\} < \left[\theta_7 + \theta_6 \left(f' \right)^{-1} \right] \frac{W}{P} \tag{128}$$

which is to say that real wage adjustment must be more sensitive to $\frac{W}{P}$ than capacity adjustment is sensitive to z. Assuming this to be the case, stability hinges on det 127 > 0, which is to say that the stationary locus of real wages must be steeper than the stationary locus of capacity utilization. Figure 20 pictures a stable equilibrium.

A comparison of Figures 19 and 20 immediately reveals the consequences of path dependence in the long run model. Move the conventional wage schedule upwards to reflect an attempt by workers to gain a larger share of the pie, and the outcome in the flexprice model is a higher equilibrium real wage, and, corresponding to the higher wage, lower capacity utilization. Try the same exercise in the fixprice model pictured in Figure 20 , and the result is to *reduce* equilibrium wages while increasing the rate of capacity utilization! Hence our understanding of whether workers' attempts to raise real wages will succeed depends not only on the static structure of our models, but on how we specify the dynamics

13 Conclusions

We have covered a great deal of ground in this essay, but the conclusions are relatively simple and straightforward. In the first place it is my contention that one makes enormous progress in understanding Keynes's *General Theory* by jettisoning the static framework in which the argument has been framed, at least since Hicks's classic statement (1937). Building on Hicks, Franco Modigliani (1944) set Keynesian economics down a path in which Keynes's argument becomes nothing more than the replacement of the standard assumption of flexible money wages with the assumption of rigid money wages. Keynes's exposition gave ammunition to this point of view, but in my judgment Keynes's commitment to rigid money wages was nothing more than an expositional strategy, not a very good one in the light of the subsequent turn that his followers took. I would certainly agree with Keynes's critics that, to say the least, the argument of Chapter 19, in which he finally relaxes the assumption of a given money wage, is hardly a model of clarity.

By recasting the argument in dynamic terms and redefining equilibrium in terms of an equilibrium rate of price and wage changes, we can incorporate the two elements that define neoclassical equilibrium, a labor-supply function, and a labor-demand function, and the two elements that (taking the rate of interest as given) define Keynesian equilibrium, the labor-demand function and the aggregate demand schedule, which in our simplified context becomes the IS schedule that has played a central role in Keynesian economics since Hicks (1937). The problem is that with only two state variables, the real wage and capacity utilization, three relationships (the labor-demand schedule is common to neoclassical and Keynesian formulations) over-determine the system, at least in a static view of the model. By redefining equilibrium in terms of equal percentage changes in prices and wages, we can resolve the over-determinacy of the static model.

The main virtue of this reformulation is that it allows us to get away from the pervasive view that nominal rigidities are the essence of Keynes's theory. Rigidities stemming from monopolistic competition, trade unions, menu costs, to mention only a few of the usual suspects, exist, but these do not get at the heart of the problem. It is not my intention to deny nominal rigidities, but rather to emphasize that eliminating these rigidities would not eliminate the problem of aggregate demand that is the core of Keynes's *General Theory.*

This paper argues that the role of aggregate demand is not limited to the short run. The major difference between the short and the long run in my view is that in the long run the labor force is endogenous. This endogeneity plays the same role in the long run as excess capacity and unemployment play in the short, allowing aggregate demand a central role in the determination of economic activity.

The main lessons of this essay are thus methodological. We should give up static thinking of the kind that makes dynamics an afterthought. We ought instead to begin with dynamics, with process, and let the processes themselves determine whether the system gravitates to an equilibrium. This counsel makes sense even in just-determined systems, but in just-determined systems logic does not compel sensible modeling. In over-determined models, it is not just a matter of good sense: *we can't even define the equilibrium apart from dynamics.*

Once dynamics become an essential element of the model rather than an afterthought, it becomes more important to consider the institutional basis of dynamic systems instead of requiring merely that the dynamics exhibit a surface plausibility. Take the simple question posed in the first section of this paper, "On Adjustment Processes": How do producers react to disequilibrium? Do they change price or output? Such a simple question, such an old question: it's about time we began seriously to answer it. Such old and simple questions take on a new importance once we commit ourselves to a dynamic view.

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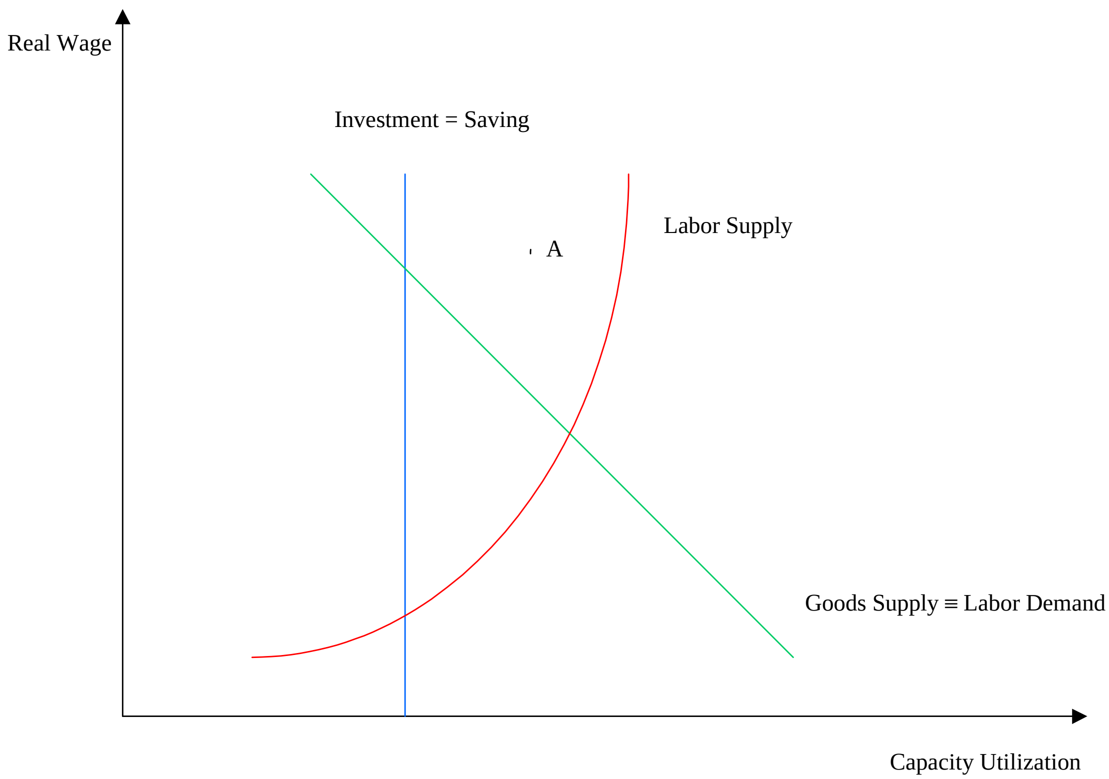

Figure 1: Aggregate Demand, Goods Supply, and Labor Supply Over-Determine the System

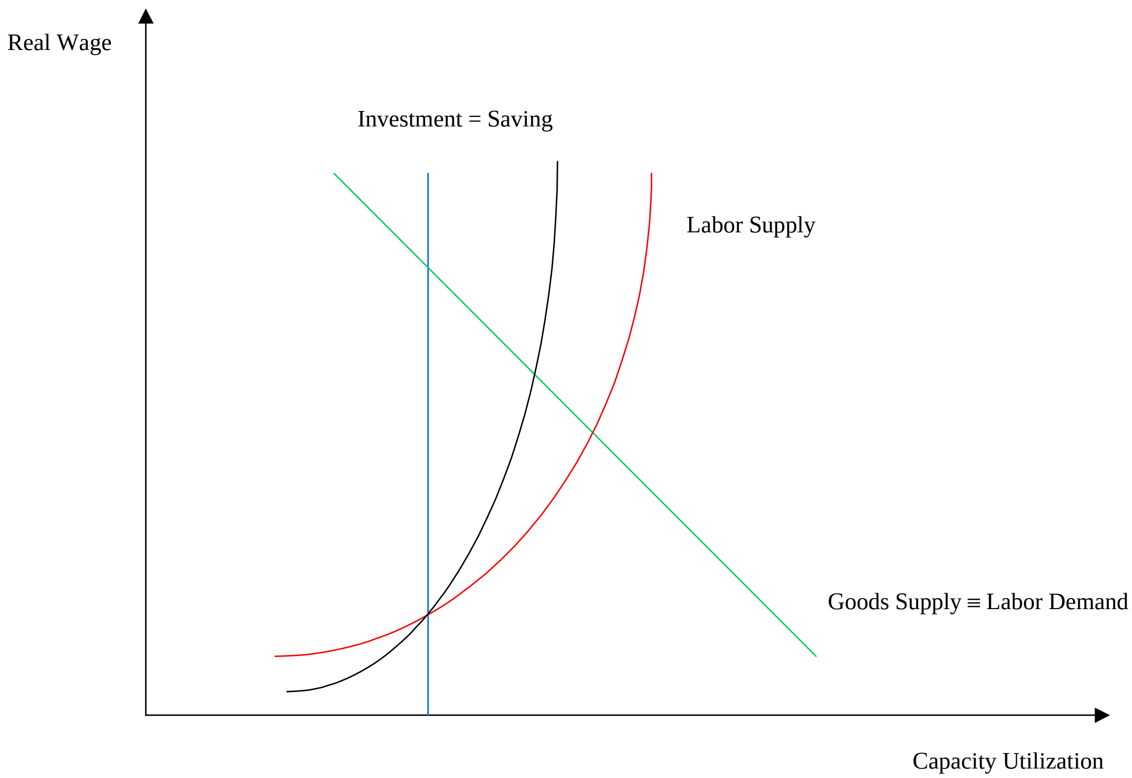

Figure 2: Equilibrium Determined by the Stationary Locus of Real Wages (the black line) and the Stationary Locus of Capacity Utilization (the green line)

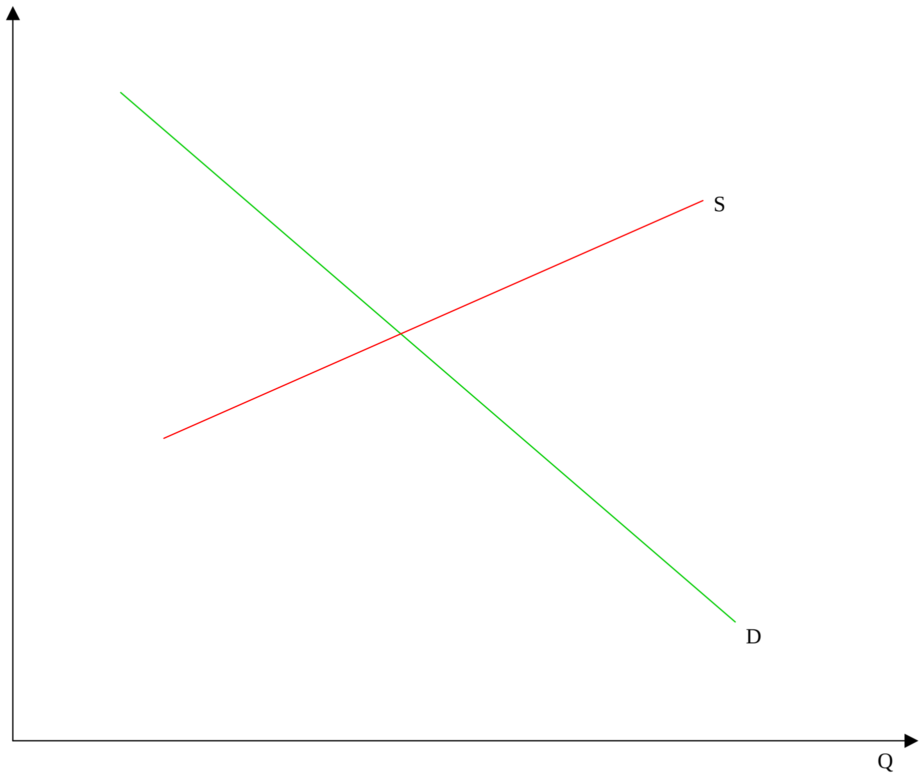

Figure 3: Demand and Supply as Functions of Price, *à la* Walras

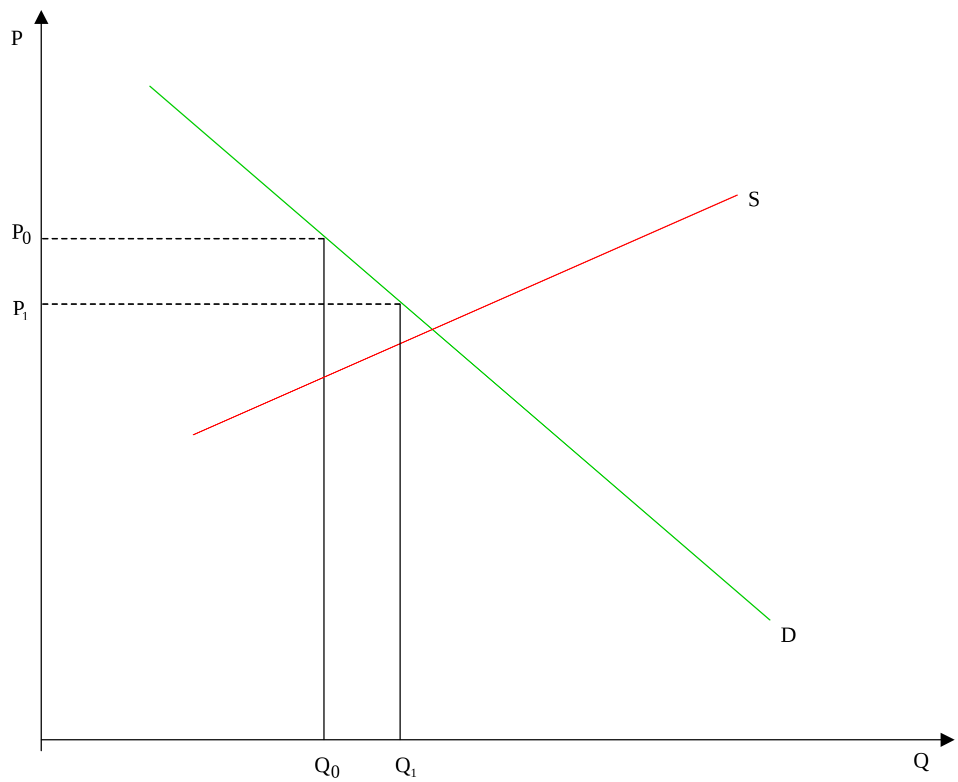

Figure 4: Marshallian Adjustment: Increase in Production From Q_0 to Q_1 Causes Price to Fall From P_0 to P_1

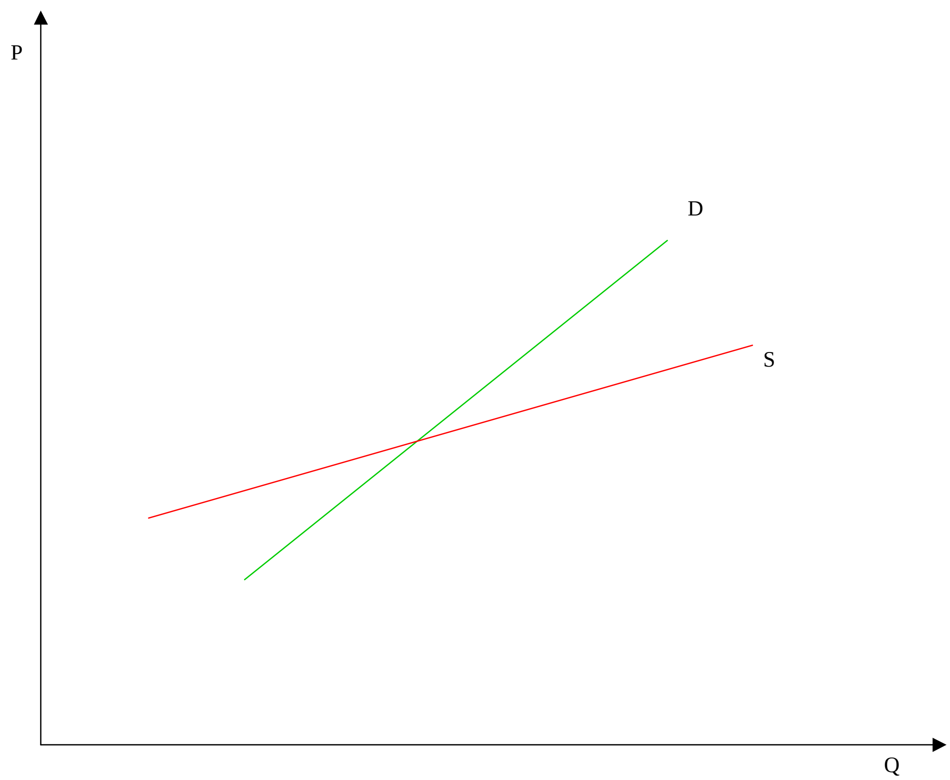

Figure 5: Walrasian Adjustment is Stable, Marshallian Adjustment is Unstable

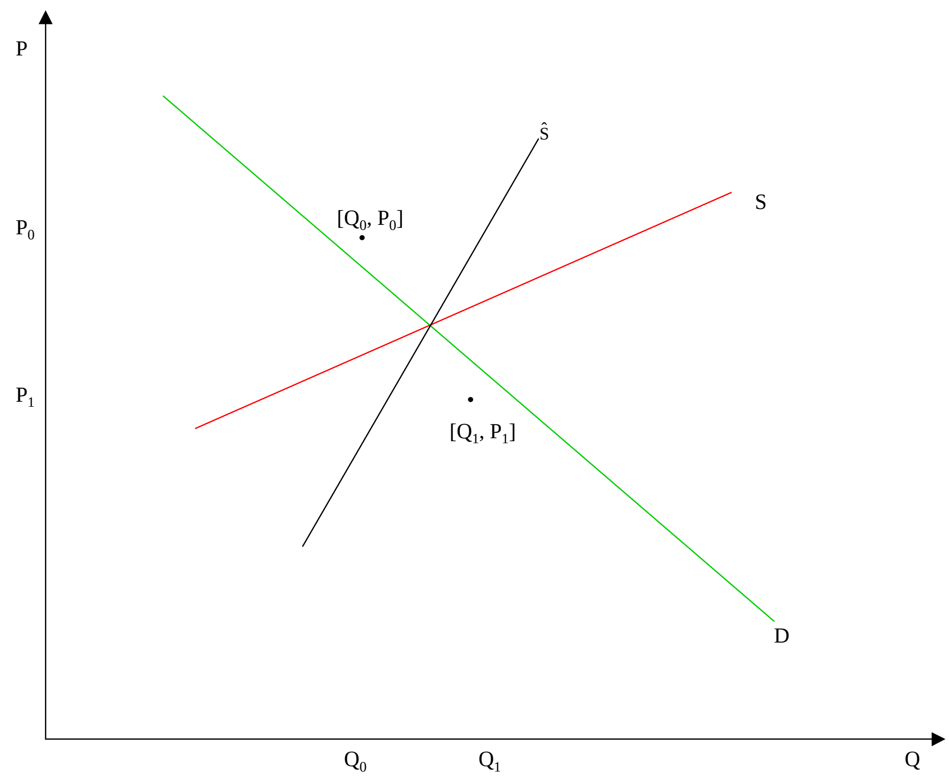

Figure 6: Demand Considerations Dominate at $[Q_0, P_0]$ and Supply Considerations Dominate at $[Q_1, P_1]$. $\hat{S}$ Becomes the Stationary Locus of Output With Inventories

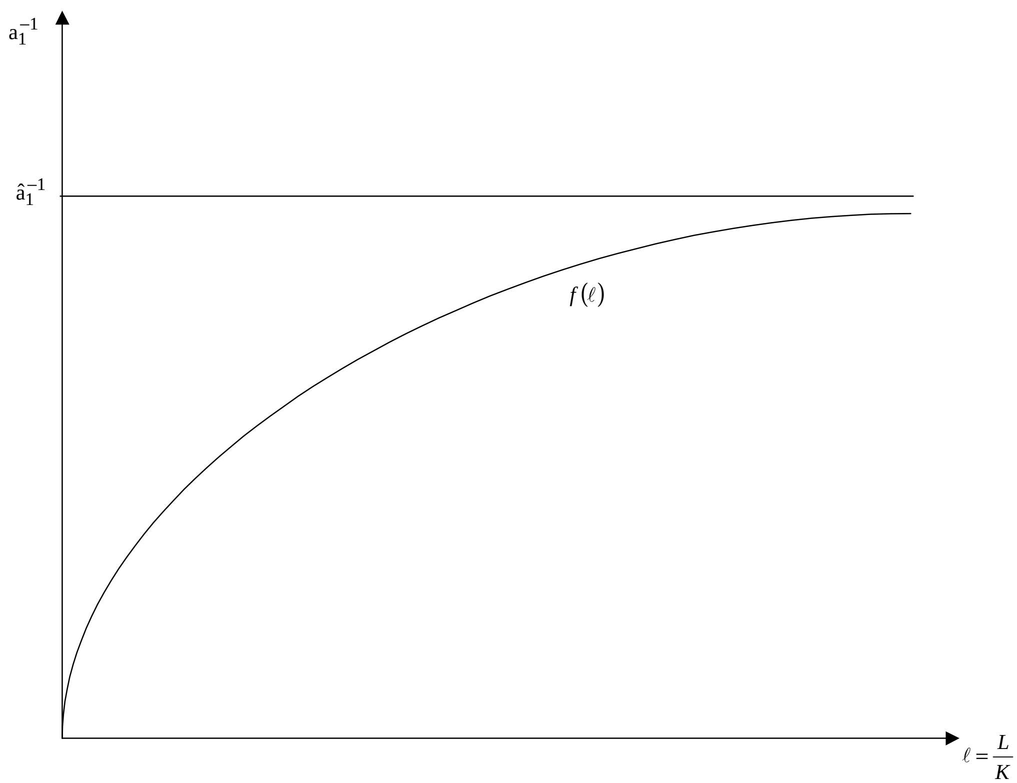

Figure 7: Output:Capital Ratio as a Function of Labor:Capital Ratio

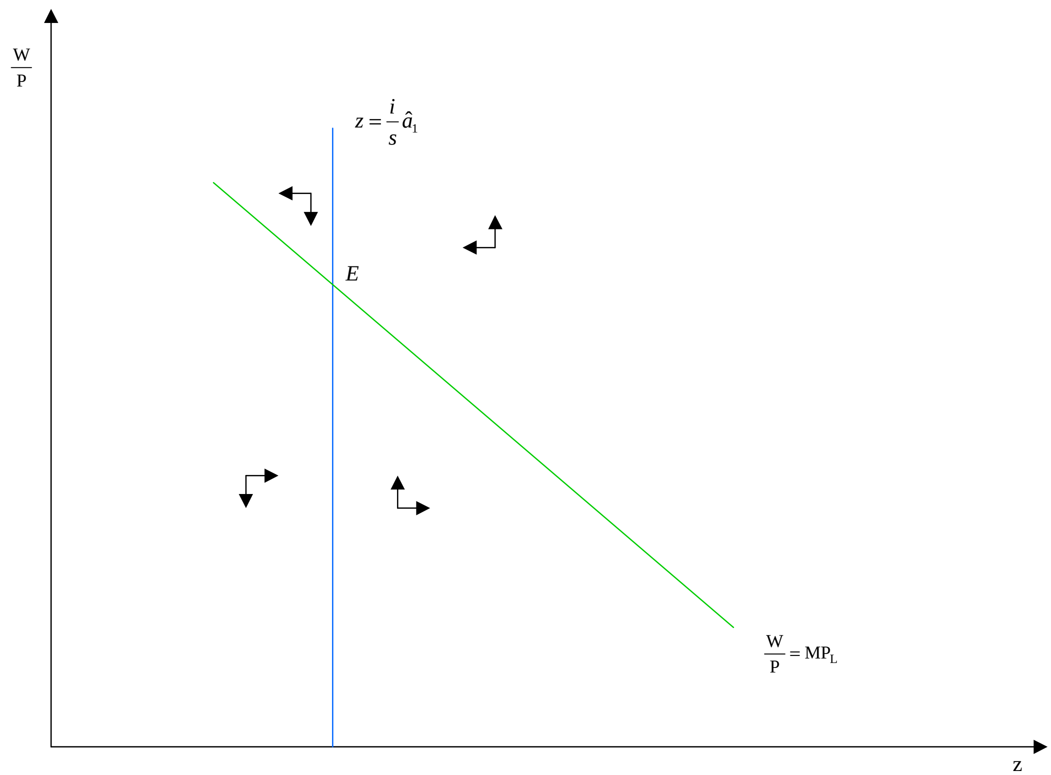

Figure 8: E is the Equilibrium in the Simple Keynesian Model Consisting of an IS Schedule and a Goods-Supply Schedule

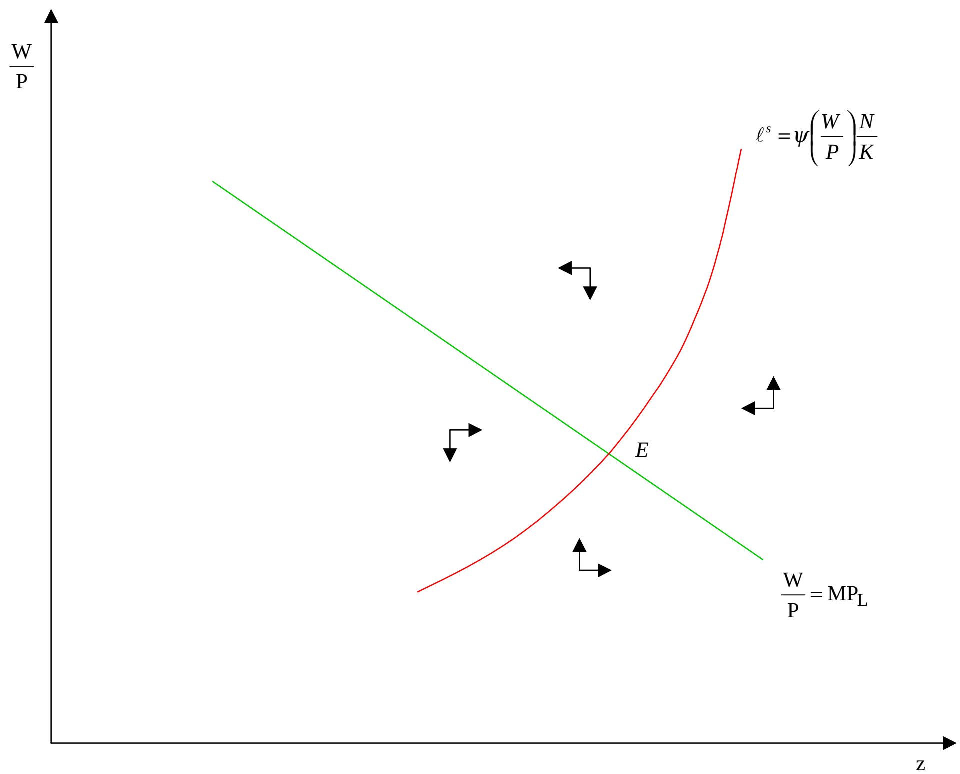

Figure 9: A Simple Neoclassical Model Consisting of a Labor-Supply Schedule and a Goods-Supply Schedule.

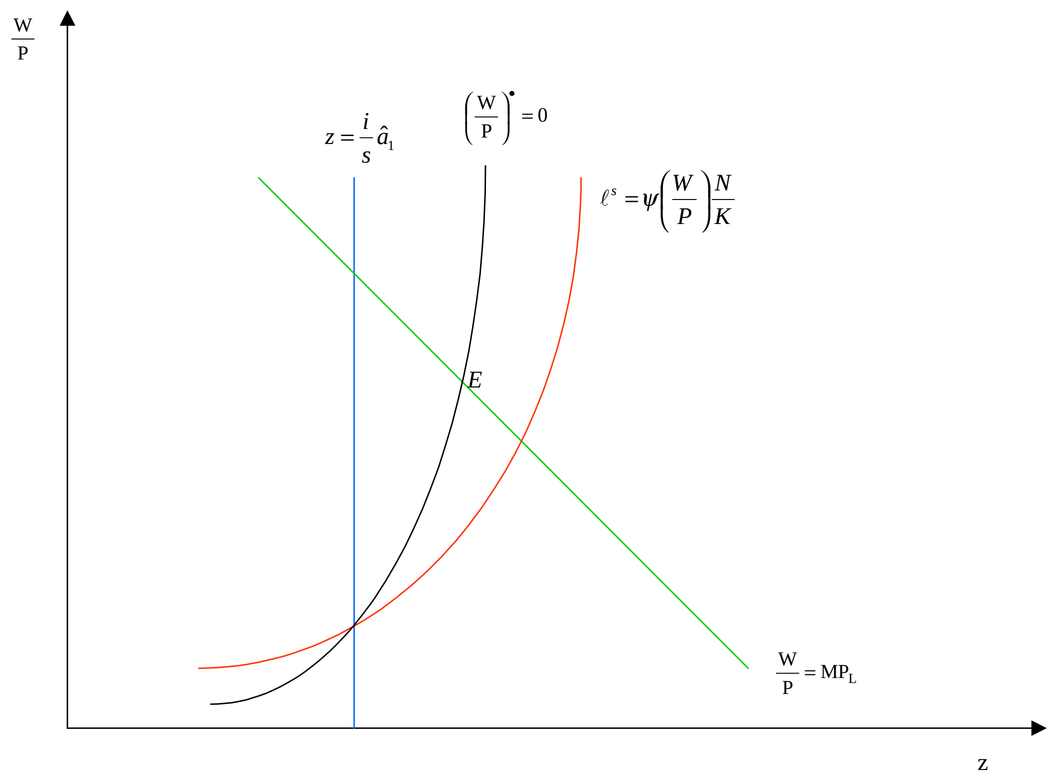

Figure 10: A Model With Keynesian and Neoclassical Elements–IS, Goods-Supply, and Labor-Supply Schedules. Equilibrium (E) is Determined by the Intersection of the Stationary Locus of Real Wages (the black line) and the Stationary Locus of Capacity Utilization (the green line).

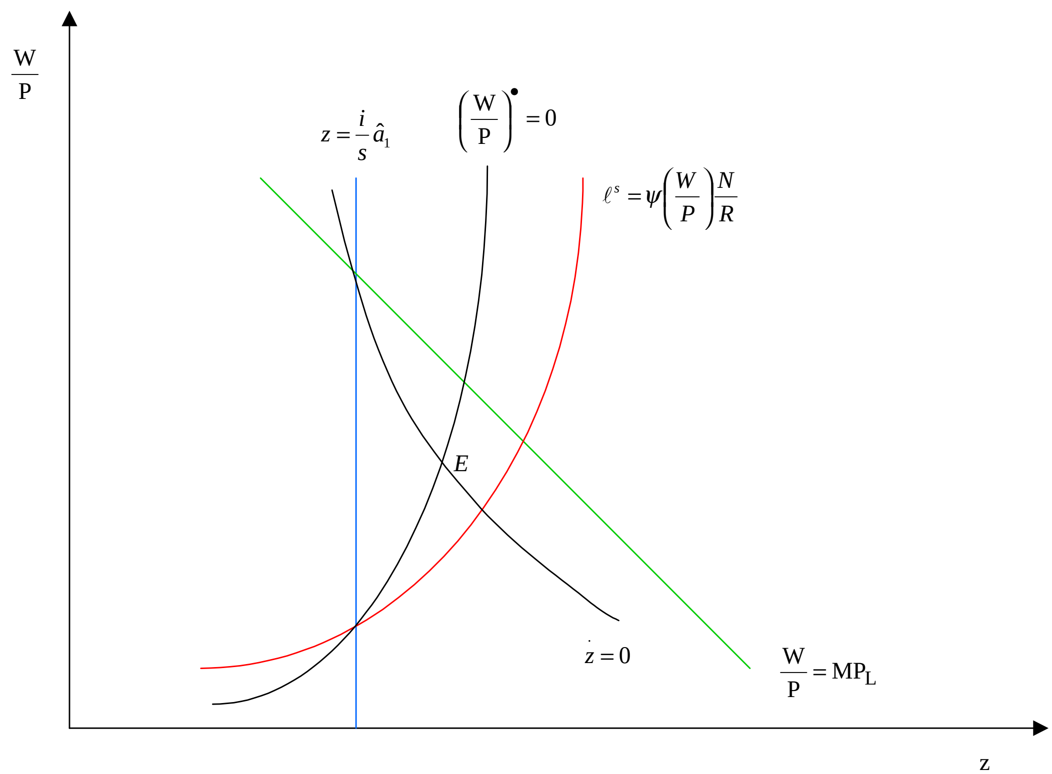

Figure 11: Equilibrium in a Model with Undesired Inventory Accumulation

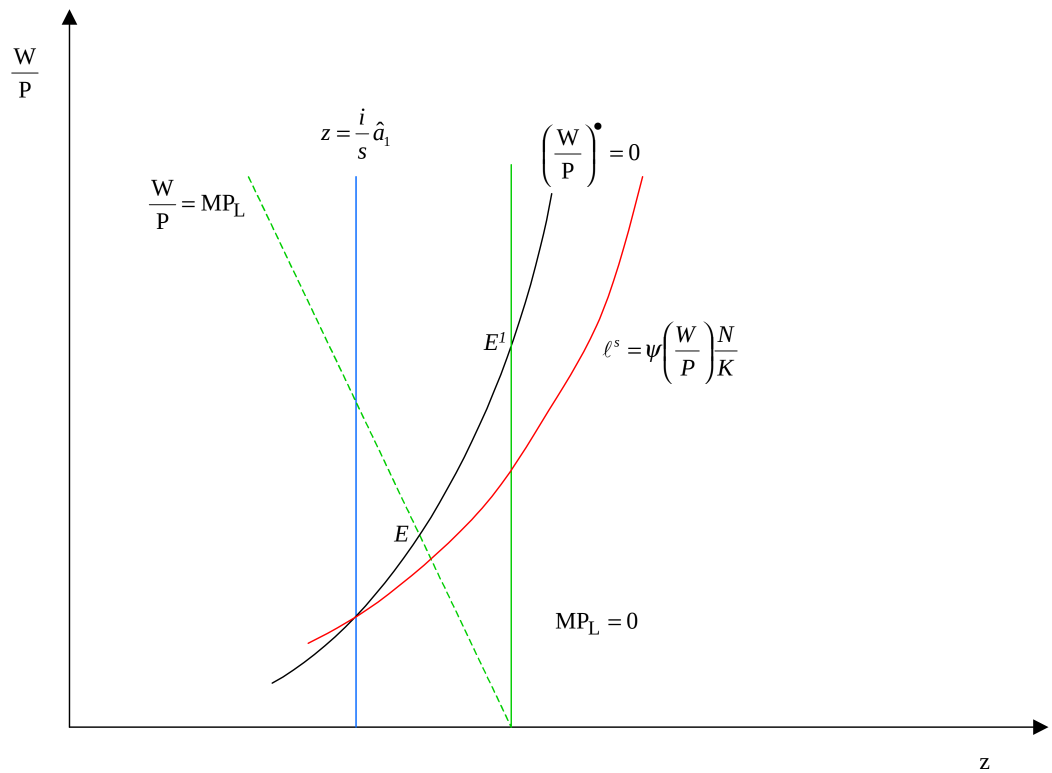

Figure 12: Equilibrium (E') in a Share Economy such as a Family-Farm Economy. The Goods-Supply Schedule Becomes Vertical where $MP_L = 0$

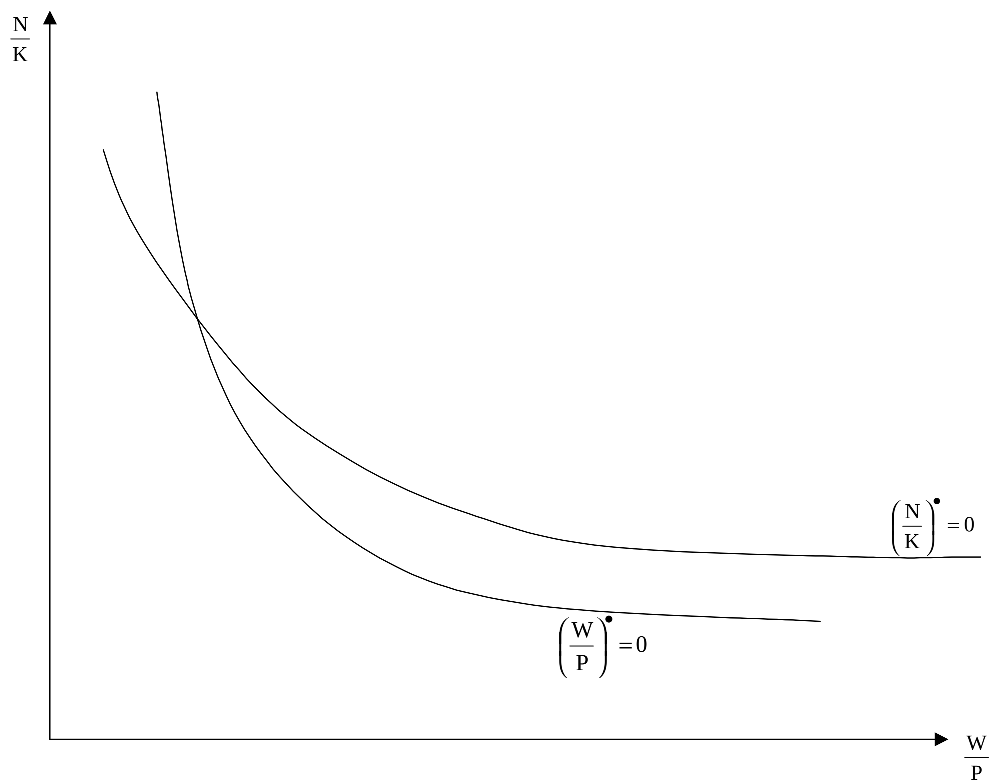

Figure 13: $\left(\frac{W}{P}\right)^{\bullet} = 0$ is Steeper than $\left(\frac{N}{K}\right)^{\bullet} = 0$, Which Guarantees Stability With an Endogenous Labor Force

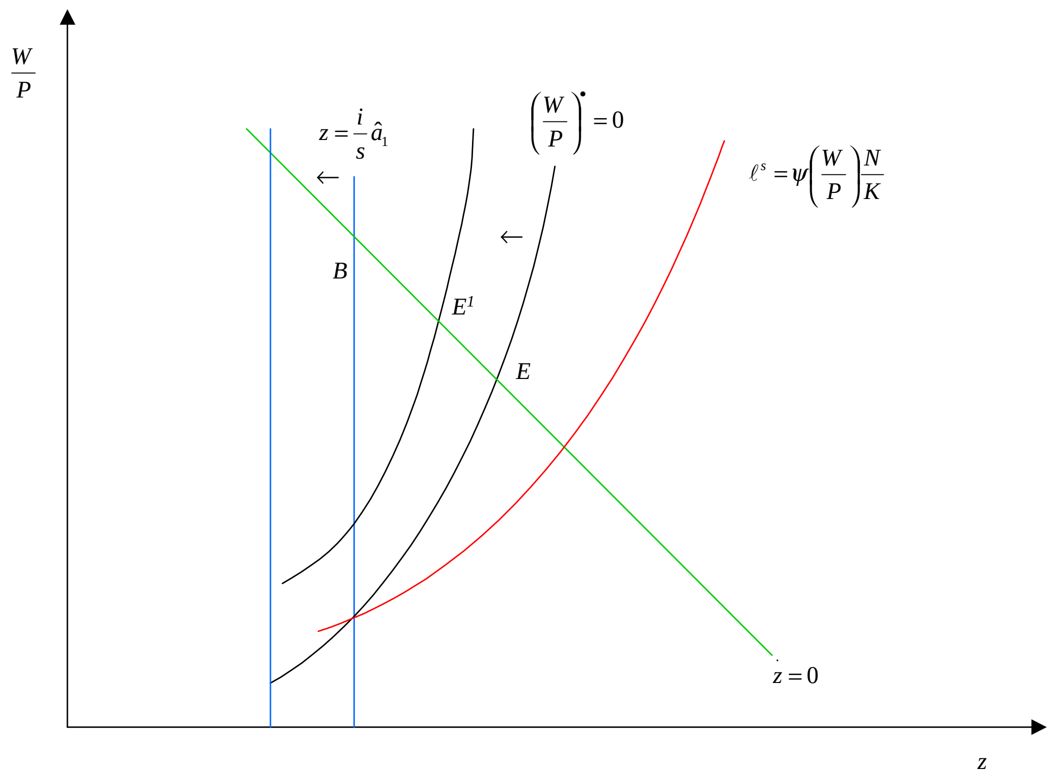

Figure 14: Equilibrium With the "Fisher Effect." The Economy Chases a Moving Target as Changes in the Real Interest Rate Shift the *IS* Schedule

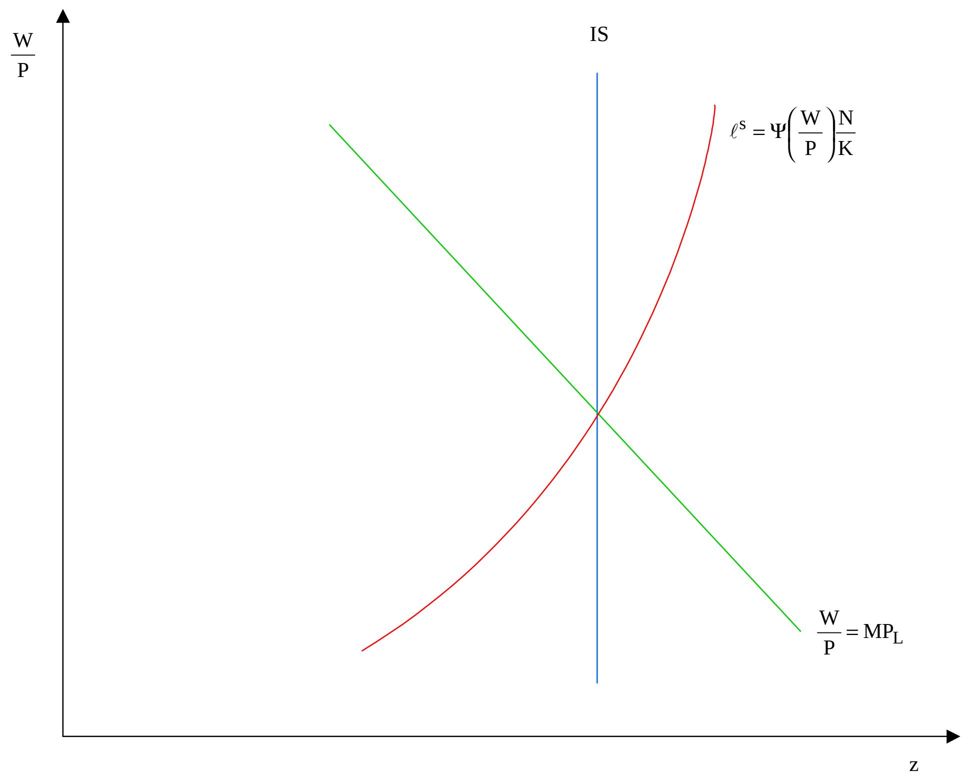

Figure 15: Equilibrium With the Pigou Effect: Keynesian and Neoclassical Conditions are Satisfied Simultaneously

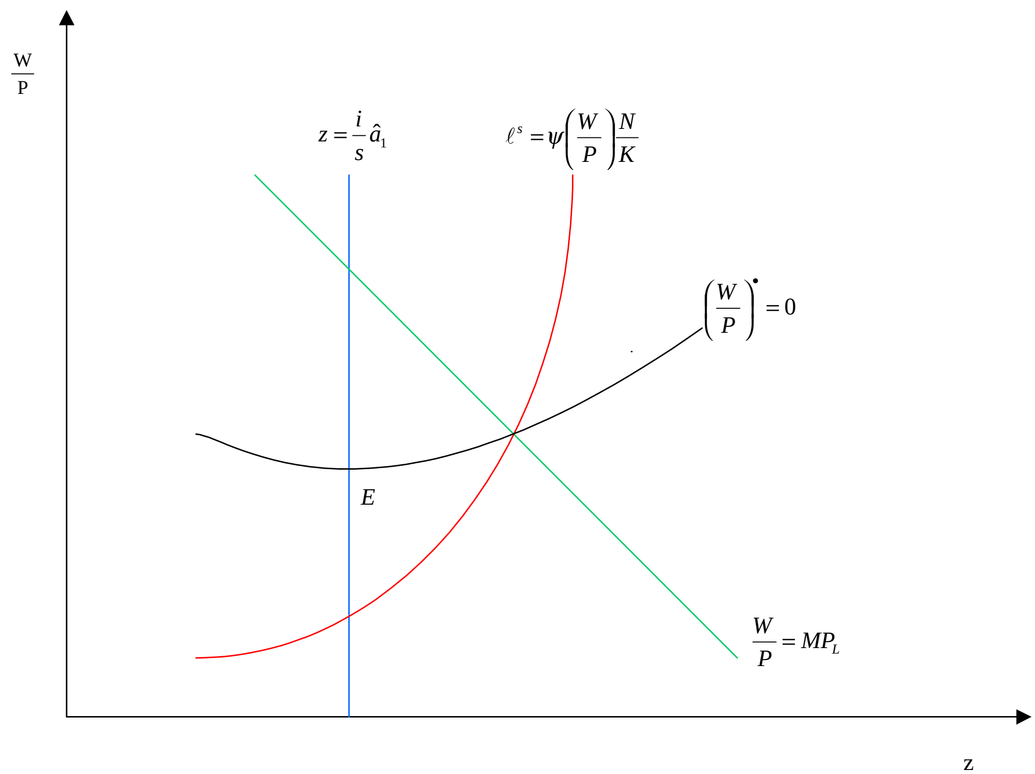

Figure 16: "Fixprice" Equilibrium: The Stationary Locus of Real Wages (the black line) is Determined by Goods-Supply (the green line) and Labor-Supply (the red line) Schedules. The Stationary Locus of Capacity Utilization is the IS Schedule (the blue line).

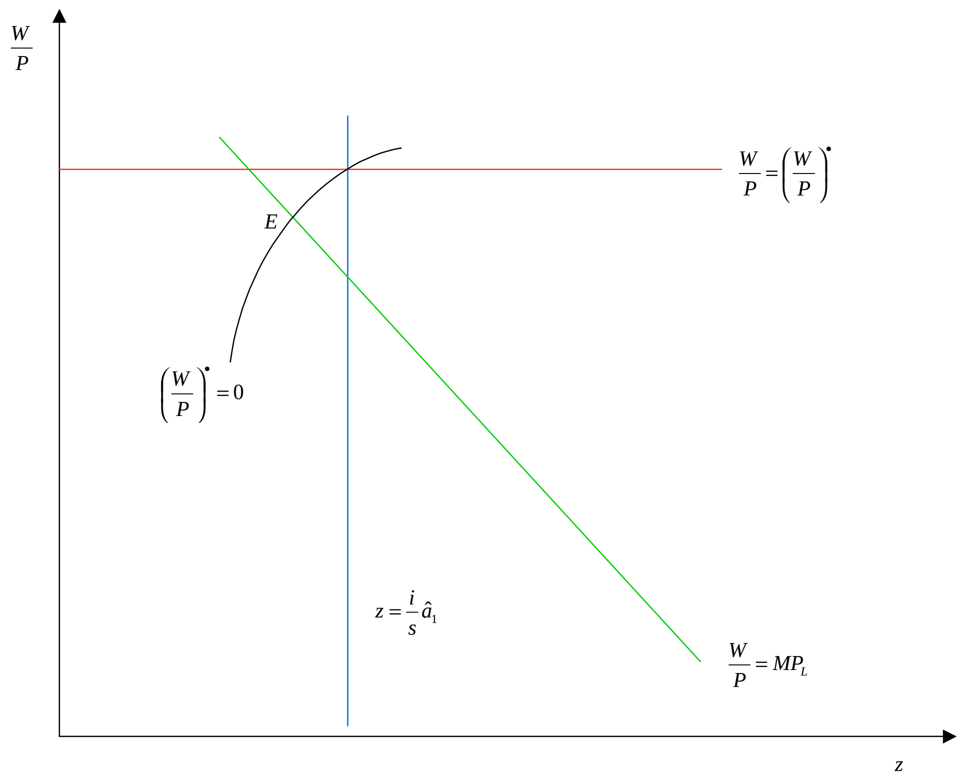

Figure 17: Long Run Keynesian Model With IS, Goods-Supply, and Labor-Supply Schedules.

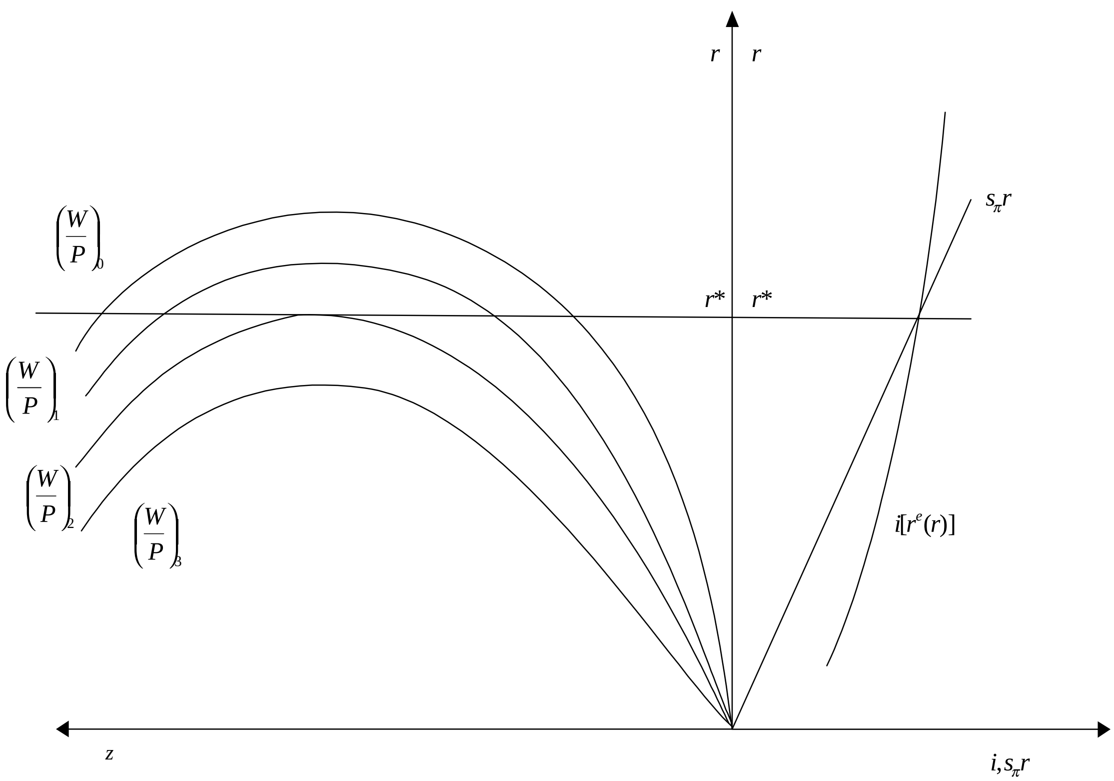

Figure 18: Construction of the mapping $r^* \rightarrow z$.

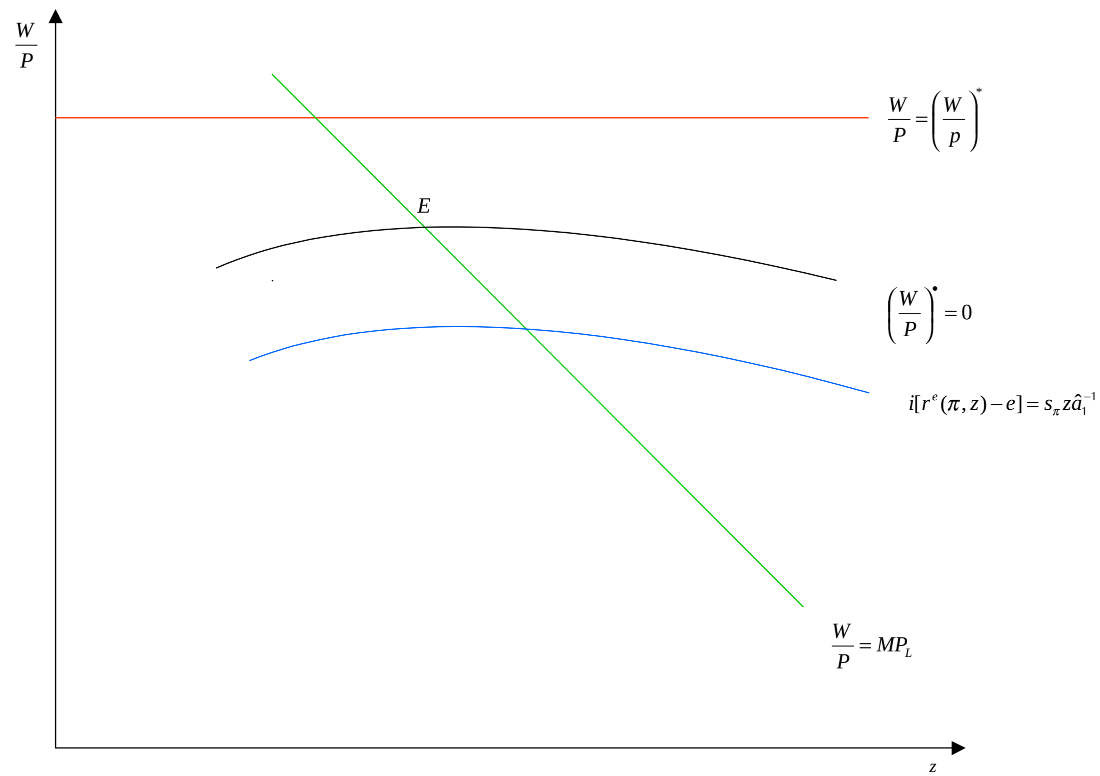

Figure 19: Equilibrium in a Long-Run Keynesian Model With Saving Determined by Profit

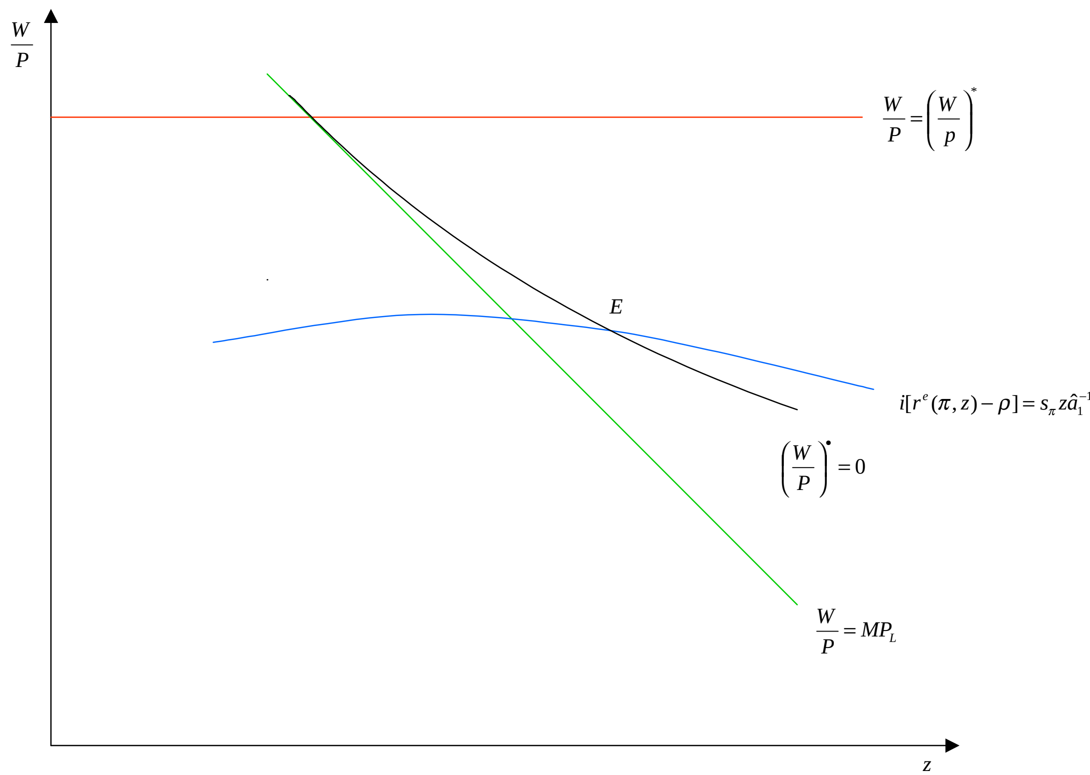

Figure 20: Long-Run Keynesian Model With Profit-Determined Saving and "Fixprice" Dynamics