Elasticities of Substitution in Real Business Cycle Models with Home Production

by

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June 2000

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June 5, 2000

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Campbell acknowledges the financial support of the National Science Foundation.

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Updated versions of this paper can be found at `www.ny.frb.org/rmaghome/economist/ludvigson/ludvigson.html`. We thank Lawrence Christiano, Christopher Erceg, Jeremy Greenwood, Stephanie Schmitt-Grohe, Peter Rupert, Randall Wright and two anonymous referees for helpful comments, and Jeffrey Brown for excellent research assistance.

Abstract

This paper constructs a simple model of home production that demonstrates the connection between the intertemporal elasticity of substitution in market consumption (IES) and the static elasticity of substitution between home and market consumption (SES). Understanding this connection is important because there is a large body of empirical evidence suggesting that the IES is small, but little evidence on the size of the SES. We use our framework to shed light on the properties of a home production model with a low IES. We find that such a model must have three fundamental properties in order to match key aspects of aggregate U.S. data. First, the steady-state growth rate of technology must be the same across sectors. Second, shocks to technology must be sufficiently positively correlated across sectors. Third, capital must be used more intensively in the market sector than in the home sector. A home production model with these three properties can be surprisingly successful at reconciling the RBC paradigm with evidence for a low IES.

1 Introduction

Recently there has been considerable interest in modifying the standard real business cycle model to include home production. Authors such as Benhabib, Rogerson, and Wright (1991), Greenwood and Hercowitz (1991), Greenwood, Rogerson and Wright (1995), and Rupert, Rogerson and Wright (1997) have documented the importance of the home sector in the U.S. economy, and have shown that home production can improve the quantitative performance of the standard model.

In almost all these studies, households derive utility from three "goods": market consumption, home consumption, and leisure. Home consumption is considered to be a substitute for market consumption, as for example a home-cooked meal is a substitute for a meal in a restaurant. Leisure is distinct from both these forms of consumption and is modelled in a traditional manner, as time not occupied by home or market production.

In this paper we adopt a different perspective. We argue that households value their leisure time because of what they can do with it. Valued leisure is not the residual time unoccupied by production; after all, time spent in prison is unproductive, but does not generate utility in the same way as time spent at home. From this point of view, it is natural to think that valued leisure is the output from a home production function in which home or leisure time, home capital, and home technology appear just as market time, market capital, and market technology do in the market production function. Accordingly we follow Greenwood and Hercowitz (1991) and use a model in which households derive utility from two "goods": market consumption and home consumption, where the latter replaces the traditional leisure variable.

Even more importantly, we assume that home consumption enters the utility function separably from market consumption. All the home production studies to date have analyzed nonseparable specifications. While this approach has yielded many important insights, we think it is useful to consider a separable alternative. We do this for two reasons. First, the traditional real business cycle literature commonly assumes that utility is additively separable over consumption and leisure. Hansen (1985), for example, writes utility as the sum of the logarithms of consumption and leisure. Since we are treating home production as a generalization of the traditional concept of leisure, it is logical for us to modify the traditional model by simply replacing leisure with home consumption in the utility function.

Second, aggregate data offer no evidence of any important nonseparability between market consumption and labor hours (see Eichenbaum, Hansen and Singleton 1988, Campbell and Mankiw 1990, and Beaudry and van Wincoop 1996). For example, Campbell and Mankiw find that although there is substantial predictable variation in hours, it is not significantly related to predictable consumption growth as it should be if utility over leisure and consumption were additively nonseparable. This evidence suggests that consumption and nonmarket hours can be well characterized by an additively separable utility function over consumption and nonmarket time, or, more generally, over consumption and some function of nonmarket time, as would be the case in models with home production.

The properties of models with nonseparable utility over home and market consumption have been studied extensively in the literature. Greenwood, Rogerson and Wright (1995) provide a survey. Utility is typically modeled as a constant elasticity of substitution utility function over home and market consumption. It is now well known that these nonseparable models can come closer to matching the dynamic properties of U.S. data than do standard (separable) RBC models without home production. The home production models do better at matching the volatility of output, the volatility of investment, hours and consumption relative to output, and the correlation between hours and productivity.

All these studies have implicitly assumed a relatively high value for the intertemporal elasticity of substitution (IES) in market consumption. This creates two problems. First, the standard RBC model of Hansen (1985) sets the IES equal to one, so it is not clear whether the improved performance of home-production models is merely due to the increase in IES or to some other aspect of home production. Second, a high IES is inconsistent with a large and growing number of empirical estimates of the IES that are much lower, indeed close to zero. Researchers have overwhelmingly found little connection between the rate of consumption growth and *ex-ante* real interest rates; see Campbell and Mankiw (1989), Hall (1988), Attanasio and Weber (1993), Ludvigson (1999), and the international evidence in Campbell (1999).[1] In this paper, we ask whether the good performance of home-production models

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[1] Beaudry and van Wincoop (1996) provide a dissenting analysis. The evidence cited above is based on

survives when utility is modeled as additively separable over home and market consumption and when the IES is taken to be much lower than in previous studies.

In the standard real business cycle model, King, Plosser, and Rebelo (1988) pointed out that an additively separable specification for utility requires log utility for consumption to obtain constant labor supply along the balanced growth path. This restriction is undesirable because it forces the IES to equal one. We escape this restriction by introducing steady-state technological progress in the home sector. Thus we are able to study the effects of introducing an empirically plausible IES into the standard RBC model.

In our model there is a tight link between the intertemporal elasticity of substitution in market consumption and the static elasticity of substitution between home and market consumption (SES). Our assumption of additive separability implies that these two elasticities are equal. While equality follows only from additive separability, a tight positive relationship between the IES and the SES is not unique to our framework; using parameter values typically assumed, it is also a feature of the most commonly employed model in the existing home production literature. We discuss this further in Appendix A.

This positive relationship between the IES and the SES is important because there is little direct empirical evidence on the value of the SES. The existing home production literature has focused attention on the choice of value for the SES, placing virtually no emphasis on the value of the IES implied by this choice. Indeed the implied value of the IES typically goes unmentioned. Nevertheless the most popular home production specifications imply that the IES is less than one only if the SES is less than one. We argue that existing evidence for a low IES suggests that the SES is also low. By contrast, the existing home production literature assumes a high SES (and by implication a high IES), and this assumption is critical for the improvements in the quantitative performance of real business cycle models documented in the literature. To the best of our knowledge, this paper is the first to consider the properties

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studies that use postwar data at frequencies ranging from one month to one year. A possible caveat is that developed countries have experienced accelerated growth since the early 1800s, which should have increased real interest rates markedly if the IES is low; yet it is not clear that real interest rates have shown any large increase since that time. However the measurement of real interest rates in historical data is problematic, since default rates were higher and inflation rates were more variable than they have been in modern data.

of home production models when the IES is less than one.

To explore the theoretical properties of a model with time-separable preferences across home and market consumption, we use a standard representative-agent framework with isoelastic utility. Leisure time interacts with a home technology process, and possibly with home capital, to produce home goods and services, and affects utility only through its role as an input to home production. We solve the model using the analytical approach of Campbell (1994). To facilitate comparison with the existing literature, we use the solution to simulate the model's endogenous variables, comparing their relative variability and comovements with those found in aggregate U.S. data. Several special cases are studied, including a benchmark model which assigns a minimal role to the home sector, and a more general model which allows for home technological change and the use of home capital.

Our results suggest that a low-IES home production model must have three properties in order to match key features of U.S. aggregate data. First, in steady state, balanced growth requires the home and market sectors to display the same long-run growth rate of technology. Second, procyclical variation in both market hours and market consumption around the steady state requires a sufficiently positive correlation between the technology shocks to the home and market sectors. Equivalently, intersectoral productivity shocks must be small. This result contrasts with the existing literature which typically requires large intersectoral productivity shocks to improve the quantitative performance of the traditional RBC model. Third, there must be home capital but the capital intensity of the home sector must be less than that of the market sector. We find that a home production model that displays these three properties can be surprisingly successful at reconciling the RBC paradigm with evidence for a low IES.

There is an extensive labor supply literature that examines how individuals allocate time between market and home activities—Juster and Stafford (1991) provide a survey. The literature finds that nonmarket time is used productively and that the value of what is produced in the home sector is quite large. Time-use diaries indicate that time spent in housework and active leisure is roughly the same across households among developed countries (though it is higher for women than for men) and the total amount of such time represents a significant fraction of hours per week. Evidence from time-use diary data suggests a moderate convergence toward equality in market and nonmarket time of men and women from 1965 to the 1980s in several developed countries, a pattern that coincides with growing after-tax wage equality over the same period. This evidence is not directly informative about substitution elasticities, however, because changing wages have both income and substitution effects on labor supply. Nevertheless, almost all of these studies conclude that nonmarket production activities represent a quantitatively important aggregate phenomenon, and that the value of home produced output is quite large (Eisner 1988).

The rest of this paper is organized as follows. Section 2 presents the model and assumptions. Section 2.1 discusses the steady state, while section 2.2 outlines the solution procedure for studying the economy's response to technology shocks out of steady state. Section 3 presents the approximate analytical solutions, focusing on how technology shocks influence the model economy. Section 4 presents time-series simulations of the model, and compares its dynamic properties with those of the standard RBC model and the U.S. data. Section 5 concludes.

2 The Model

Consider an individual who receives utility from consumption of market goods, C, and home goods, H. This representative agent maximizes expected lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \rho^t U(C_t, H_t), \tag{1}$$

where ρ is the discount factor restricted to be between zero and one, and preferences are specified as

$$U(C_t, H_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \theta \frac{H_t^{1-\lambda}}{1-\lambda}. \tag{2}$$

The intertemporal elasticity of substitution in market consumption is given by $1/\gamma$. Output is produced in both the home and market sectors according to the following Cobb-Douglas production technologies:

$$Y_t = K_t^{1-\alpha}(A_t N_t)^{\alpha}, \tag{3}$$

and

$$H_t = D_t^{1-\beta}(Z_t(1-N_t))^{\beta}, \tag{4}$$

where Y_t is market output, K_t is market capital, D_t is household capital, and N_t is the portion of labor's endowed time allocated to market activities. A_t and Z_t are labor augmenting technological shocks to the market and home production sectors, respectively.

Two points about the preferences and technologies specified above deserve mention. First, nonmarket time is assumed to be devoted entirely to home production rather than divided between leisure and home production hours. This captures the idea that leisure is not valued for its own sake, but for what can be done with it. Second, equations (2) and (4) taken together indicate that home production equals home consumption period by period; thus investment requires the production of market goods. More specifically, if the evolution of each capital stock is denoted by

$$K_{t+1} = (1-\delta)K_t + I_{kt} \tag{5}$$

and

$$D_{t+1} = (1-\delta)D_t + I_{dt}, \tag{6}$$

where δ is the common rate of depreciation, I_{kt} is gross business investment and I_{dt} is gross household investment, the resource constraint for market output is given by

$$Y_t = C_t + I_{kt} + I_{dt}. \tag{7}$$

The first-order conditions are as follows. For market capital accumulation, the standard Euler equation holds:

$$C_t^{-\gamma} = \rho E_t C_{t+1}^{-\gamma}[(1-\alpha)K_{t+1}^{-\alpha}(A_{t+1}N_{t+1})^{\alpha} + (1-\delta)], \tag{8}$$

where the quantity in brackets is the gross marginal product of market capital, denoted R_{t+1}. For household capital accumulation a similar intertemporal condition holds:

$$C_t^{-\gamma} = \rho E_t C_{t+1}^{-\gamma}[(1-\beta)D_{t+1}^{-\beta}(Z_{t+1}(1-N_{t+1}))^{\beta}\theta\frac{H_{t+1}^{-\lambda}}{C_{t+1}^{-\gamma}} + (1-\delta)], \tag{9}$$

where we define the quantity in brackets analogously as the gross marginal product of home capital. Finally, for the allocation of labor between market and home activities there is a static first order condition:

$$\alpha K_t^{1-\alpha}A_t^{\alpha}N_t^{\alpha-1}C_t^{-\gamma} = \theta H_t^{-\lambda}\beta D_t^{1-\beta}Z_t^{\beta}(1-N_t)^{\beta-1}. \tag{10}$$

Equation (8) forms the basis for the empirical investigations cited in the introduction which test for nonseparabilities between consumption and leisure in the standard RBC model (i.e., where $H_t = (1-N_t)$). It is straightforward to derive a loglinear approximation of (8) which demonstrates an implication of the separable specification, namely that consumption growth is only a function of the expected real interest rate (the marginal product of capital), and is not related to expected hours growth: $E_t\Delta c_{t+1} \approx \mu + \sigma E_t r_{t+1}$, where σ denotes the IES. By contrast, when consumption and leisure appear nonseparably, for example if $U(C, 1-N_t) = \frac{[C_t^{\rho}(1-N_t)^{1-\rho}]^{1-\gamma}}{1-\gamma}$ a loglinear approximation of the analogous first-order condition in this case implies that there exist constants μ_0, σ_1 and σ_2 such that $E_t\Delta c_{t+1} \approx \mu_0 + \sigma_1 E_t r_{t+1} + \sigma_2 E_t \Delta n_{t+1}$. This follows from the fact that, in the nonseparable specification, a multiplicative term involving nonmarket time would appear in the first-order condition for market capital accumulation, (8), and from the fact that market hours are simply one minus nonmarket hours. Thus, empirical research cited in the introduction which finds no relation between expected hours growth and expected consumption growth is consistent with an additively separable utility function.

These same principles may be applied to RBC models with home production sectors. Note that the additively separable home production specification considered in this paper has precisely the same first-order condition for household capital accumulation (8) as does the standard RBC model. It follows that, just as in the standard RBC model, an approximate loglinear equation for consumption growth may be obtained in which expected consumption growth is related only to the expected real interest rate, again consistent with empirical evidence cited above. The presence of a home production sector does not eliminate the implication that expected consumption growth should be related to expected hours growth if utility is nonseparable. Appendix A provides more discussion on the empirical evidence for nonseparability as it relates to various home production specifications.

2.1 The balanced growth path

The driving force of steady-state growth is technological progress, and we assume that A_t, Z_t, Y_t, K_t, C_t, D_t, and H_t grow at the common gross rate, G, along a balanced growth path. It is standard in this literature to assume that market hours are constant along the balanced growth path. Although there is some evidence that market hours have declined since 1870, the decline seems to have largely ceased in the post-war period and the data suggest that hours have remained approximately constant since 1960 (Cox and Alm 1999, Cooley and Prescott 1995).

Equation (10) illustrates how hours can be constant along the balanced growth path in this model, even if utility over market consumption is not logarithmic. Taking logs and first differences of both sides, the equation implies that the following relationship holds along the balanced growth path:

$$(1-\alpha)g + \alpha g - \gamma g = -\lambda g + (1-\beta)g + \beta g, \tag{11}$$

where lowercase letters denote logs of variables. If $\beta = 1$ and there are no shocks to Z_t, the model is essentially the standard one, except that there is now steady state growth in Z. Without steady-state technological progress in the home sector, the right hand side of (10) would be constant. This would imply that the right hand side of (11) is zero, requiring $\gamma = 1$. This restriction in traditional RBC models pins down the curvature of the utility function over consumption. By contrast, steady-state technological progress in the home production sector permits a continuum of values for γ without violating balanced growth. Thus, an important advantage of our framework is that we may free up the curvature of the utility function over market consumption, and study the effects of introducing a range of values for the IES into the standard, additively separable RBC model. This advantage would be lost if we divided nonmarket time into that devoted to productive activities and leisure. Consequently, the treatment of hours is an important ingredient in our approach, allowing us to study the model's properties over a range of preferences not previously analyzed. Appendix A provides details concerning the implications of such divisibility in nonmarket time for the curvature of the utility function when utility is additively separable.

Two other aspects of equation (11) are worth noting. First, no matter what the values of α, β, γ, and λ, balanced growth requires the steady-state growth rates of A_t and Z_t to be the same. Second, no matter what the values of α and β, a restriction necessary for balanced growth is $\lambda = \gamma$. We impose this from now on. Although this introduces two new restrictions in order to match the evidence for balanced growth, we believe that these new conditions—which allow us to free up the curvature of utility over market consumption—are more desirable than maintaining the traditional RBC constraint of logarithmic utility since, unlike the log utility specification, they are not directly contradicted by empirical evidence.

Along the balanced growth path, (8) becomes

$$G^{\gamma} = \rho R, \tag{12}$$

where,

$$R_{t+1} \equiv (1-\alpha)\left(\frac{A_{t+1}N_{t+1}}{K_{t+1}}\right)^{\alpha} + (1-\delta). \tag{13}$$

R_{t+1} is the gross marginal product of market capital, equal to a constant, R, along the balanced growth path. Note that equation (12) pins down the value of ρ, given G, R, and γ.

By combining (12), (8), and (3), the steady-state output to capital ratio can be obtained:

$$\frac{Y}{K} = \left(\frac{A_t N}{K_t}\right)^{\alpha} \approx \frac{r+\delta}{(1-\alpha)}, \tag{14}$$

where the approximate equality arises from setting $R \approx 1 + r$. Because the first order condition for market capital accumulation is the same as in the standard RBC model, (14) is the standard result for the output to capital ratio.

Equations (12) and (9) can be combined to yield the steady-state ratio of home production to home capital:

$$\frac{H}{D} = \left(\frac{Z_t(1-N)}{D_t}\right)^{\beta} \approx \frac{(r+\delta)H_t^{\gamma}}{(1-\beta)\theta C_t^{\gamma}}. \tag{15}$$

From (10), the steady-state ratio of home to market capital is

$$\frac{D}{K} = \frac{(1-N)\alpha(1-\beta)}{N\beta(1-\alpha)}. \tag{16}$$

Equation (16) implies that the steady-state ratio of home to market capital is equal to the steady-state ratio of home to market hours, if the share of market capital in market output is the same as the share of home capital in home output.

Finally, equation (16), along with (5), (6), and (7) together imply that the steady-state market consumption to market output ratio is:

$$\frac{C}{Y} = 1 - \frac{\alpha}{\beta}(1-\beta)\left(\frac{g+\delta}{r+\delta}\right)\frac{(1-N)}{N} - \frac{(g+\delta)(1-\alpha)}{r+\delta}. \tag{17}$$

By combining (17), (14), and (16), an expression for steady-state C/D can be obtained. This can be equated with the ratio $C/D = \frac{H/D}{H/C}$, implied from (15), which explicitly links A/Z and $(1-N)/N$ given θ. It is difficult to know how to calibrate θ. Fortunately, it is much easier to calibrate N, and by considering a number of special cases for A/Z, we can leave the constant θ undefined in modeling fluctuations. We discuss these cases along with calibration assumptions next.

2.2 Fluctuations around steady state

Away from steady state, the model consists of a system of nonlinear expectational equations. To solve this model we use the analytical technique of Campbell (1994), which seeks an approximate solution by transforming nonlinear equations into loglinear difference equations. Each equation is loglinearized around steady state ratios of variables given above, so that variables in logs represent deviations from steady state. Below, we review the procedure only briefly, and refer the reader to Campbell (1994) for details.

Before solving the model, a number of parameter values must be chosen. Two difficult parameters to set are β and the ratio A/Z. Given that there is little direct evidence on the values of these parameters, we consider six special cases. The first four of these compare symmetric with asymmetric specifications of capital shares and technology shocks across the home and market sectors. These four cases are given as follows: Case 1: $\beta = 1$, $\alpha < 1$, $Z_t = 1$; Case 2: $\beta = 1$, $\alpha < 1$, $Z_t = A_t^{\alpha}$; Case 3: $\beta = \alpha$, $Z_t = 1$; Case 4: $\beta = \alpha$, $Z_t = A_t$. In all the cases market technology A_t varies stochastically.

These cases cover a range of possibilities. Case 1 minimizes the role of the home production sector by eliminating both home capital and innovations in home technology; thus, it is most similar to the standard RBC setup. The only difference from the standard RBC model is that home technology grows deterministically in steady state. Thus we call this the *deterministic home technology model.* With the further assumption that the IES $\equiv 1/\gamma = 1$, this case is observationally equivalent to Hansen's (1985) divisible labor model with log utility over consumption and leisure. We will refer to Case 1 with the IES $\equiv 1/\gamma = 1$ as the *standard RBC model.*

Case 2 allows fluctuations in home technology, perfectly correlated with market technology, but assumes that home capital does not enter the household production function. We will refer to this case as the *stochastic home technology model.* To understand intuitively how home technology might improve when market technology improves, consider the example of the development of the Internet; this appears to have accelerated US productivity growth in the late 1990's, but it is quite plausible that it has also improved the ability of nonmarket time to produce utility (modelled here as home output) through home shopping, Internet chat, Web surfing, and so forth. Alternatively, consider the effect of oil price shocks in the 1970's (not literally technology shocks, but often treated as such in simple two-factor RBC models). High oil prices reduced the productivity of market hours; they may also have reduced the productivity of non-market hours by reducing the affordability of travel and the comfort of home temperatures during the winter.

Case 3 adds home capital, with the same capital share as market capital, but assumes only nonstochastic growth in home technology; we will refer to this case as the *home capital model.* Finally, Case 4 restricts both the technology shock and the share of capital to be the same across the home and market production functions. We will refer to Case 4 as the *symmetric home production model.*

Note that the symmetric home production model is the analogy to the stochastic home technology model when we allow for home capital in the home production function. In each case, a one percent change in A_t translates into an α percent change in technology in the home and market production functions. Thus, shocks to technology affect the home and market sectors symmetrically. We call Cases 2 and 4 *symmetric technology shock* cases and compare them with Cases 1 and 3, in which technology shocks create changes in relative productivity across the home and market sectors.

Cases 3 and 4 both assume that the capital intensity of home production is the same as the capital intensity of market production ($\beta = \alpha$). Thus we call Cases 3 and 4 *symmetric capital share* cases. While the assumption of symmetric capital share is appealing in its simplicity, it implies an unreasonably low value for the steady state consumption-output ratio in equation (17), equal to about 0.25 when $\alpha = 2/3$. This problem does not arise when $\beta = 1$, for in that case $C/Y = .75$, a value that is slightly higher than the U.S. data, but still plausible. To handle this problem while still allowing for the presence of capital in home production, we consider two additional cases which set $\beta = .92$, a value that allows (17) to roughly match the mean consumption-output ratio of 0.667 in postwar U.S. data.[2] These two cases are: Case 5 (the *labor-intensive home capital model*): $\alpha = 2/3$, $\beta = 0.92$, $Z_t = 1$; Case 6 (the *labor-intensive home production model*): $\alpha = 2/3$, $\beta = 0.92$, $Z_t = A_t$. Cases 5 and 6 imply that capital is used as an input to household production, but far less intensively than in Cases 3 and 4. For all six cases, we assume that log deviations from steady-state technological progress follow a first-order autoregressive process, $a_{t+1} = \phi a_t + \varepsilon_{t+1}, 0 \leq \phi \leq 1$.

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[2] This value for the share of labor in home production is also used in Benhabib, Rogerson and Wright (1991). Adding home production to the RBC framework provides an extra degree of freedom, allowing the researcher to match the empirical consumption-output ratio in order to set β.

We discuss the model's solution in each of these cases below.

Parameters in these models are calibrated at quarterly rates as follows. The steady state growth rate g is set to 0.005 (2 percent at an annual rate), the steady state real interest rate, r, is set equal to 0.015 (6 percent at an annual rate); α, labor's share in the market production process, is set to 0.667 as discussed above; the discount rate, δ, is set equal to 0.025 (10 percent at an annual rate), and N, the steady state allocation of hours to market activities is taken to be 1/3. We allow for the IES and ϕ to take on a range of values, discussed below. Table 1 summarizes the parameter values as they are calibrated for each case.

In the models with home capital, we further assume that capital can be re-allocated between the home and market sectors within the period. This assumption implies that the gross marginal products of home and market capital (defined implicitly by the two intertemporal first order conditions (8) and (9)) are equated within the period, and allows us to define a single summary capital stock state variable, $F_t \equiv K_t + D_t$, rather than having each capital stock enter the model separately. Defining a single capital stock state variable greatly simplifies the analytical solution procedure.[3]

An analytical solution to the system of nonlinear equations is sought by transforming the model into a system of approximate loglinear expectational difference equations. As before, lower case letters denote logs of variables. In Cases 1 and 2, this procedure yields a loglinear solution for the log deviation from steady state as a function of the two state variables, k_t and a_t, equal to:

$$v_t = \eta_{vk} k_t + \eta_{va} a_t \tag{18}$$

for $v_t = c_t, k_{t+1}, n_t, y_t, h_t$,and where η_{yx} denotes the partial elasticity of y with respect to x, assumed constant. Similarly, for Cases 3–6 the procedure yields a solution for the log deviation from steady state as a function of the two state variables, f_t and a_t :

$$v_t = \eta_{vf} f_t + \eta_{va} a_t, \tag{19}$$

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[3] When each capital stock enters the problem separately, the analytical solution procedure requires solving a pair of quadratic equations for the elasticity of market consumption with respect to each capital stock. This makes the problem intractable since the solution to this highly nonlinear system has at least four roots.

for $v_t = c_t, f_{t+1}, k_t, d_{t,} n_t, y_t$, and h_t. The elasticities are complex functions of the parameters in the model and the steady-state ratios of variables discussed above. An appendix, available upon request, gives the complete analytical solutions for each case.[4]

A property our model shares with the standard RBC model is that elasticities with respect to the current period capital stock ($\eta_{.k}$ or $\eta_{.f}$) depend on the IES—and therefore on the elasticity of substitution between home and market consumption—but not on the persistence parameter in the technology process (see Campbell 1994). This is because elasticities with respect to the capital stock measure the effect on current variables of an increase in capital, holding fixed the level of technology.

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[4] This appendix may also be found on the web at www.ny.frb.org/rmaghome/economist/ludvigson/ludvigson.html.

3 Elasticities and their Interpretation

3.1 General properties of the model

The model we have presented has two important properties. First, the static elasticity of substitution between home and market consumption (SES) is equal to the intertemporal elasticity of substitution (IES).[5] This model yields a one-to-one correspondence between willingness to substitute market consumption over time, and willingness to substitute between market and home produced goods. It is important to recognize that a tight positive relationship between the IES and the SES is not an unusual aspect of our framework. In fact, as Appendix A demonstrates, for the most popular nonseparable home production specification studied in the literature, the relationship between the IES and the SES is virtually identical to the one-to-one mapping implied by this model for values of the IES less than or equal to one. Appendix A provides details. Values of the SES typically assumed in the home production literature are as high as 5, implying that the IES is well above unity, at variance with a large number of empirical estimates cited above which suggest that the IES is close to zero. Although there is little direct empirical evidence on the size of the SES, Appendix A illustrates that it may be identified by setting it in accordance with the implied value of the IES, a parameter for which we do have direct empirical evidence. This approach is adopted here. Thus we extend the existing home production literature by considering models in which the IES and the SES are valued below one.

A second property of the general model concerns the behavior of market hours as market and home consumption become highly complementary. As the IES approaches zero, even though the agent becomes infinitely averse to shifting the ratio of market to home consumption, market hours can change; in fact they adjust passively to insure a fixed ratio of home to market consumption.

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[5] The static elasticity of substitution between home and market consumption is defined as $\frac{\partial \ln(H_t/C_t)}{\partial \ln(P_h/P_c)}$, where P_h/P_c is the shadow price of home goods, equal to $\frac{U_2(C_t,H_t)}{U_1(C_t,H_t)}$.

3.2 The effects of home and market technology shocks

In this section, we consider how innovations to market and home technology influence consumption, labor supply, output, and the capital stock. These effects are given by partial elasticities with respect to a_t. We focus our discussion on these elasticities, though elasticities with respect to capital—which do not depend on the specification of technology shocks—are also provided in the tables for reference. Table 2 reports elasticities for Cases 1 and 2, the deterministic and stochastic home technology models in which capital is not used in home production. Table 3 reports elasticities for Cases 3 and 4, the home capital and symmetric home production models in which capital is used in the home sector as intensively as it is in the market sector. Table 4 reports results for Cases 5 and 6, the two models that allow for capital to be used less intensively in the home sector.

In all the tables we consider a range of values for the IES and the persistence parameter ϕ. The IES is set equal to $0, 0.2, 1$, and 5; thus we include a high IES as used in other home-production studies, the unit IES of the standard RBC model, an empirically plausible low IES of 0.2, and the extreme case of IES $= 0$ which illustrates various theoretical properties of low-IES models. The persistence parameter ϕ is set equal to $0, 0.95$, and 1; a persistence of 0.95 nicely captures the properties of long-lasting but less than permanent technology shocks.

The left-hand panel of Table 2 gives consumption, capital, employment, and output elasticities for the deterministic home technology model, Case 1. This model generalizes the standard RBC model by freeing up the curvature in the utility function over consumption. As a result, it is worthwhile elaborating on several features of this case. First, as already noted, when the IES $= 1$, this case collapses to the standard Hansen RBC model with divisible labor and log utility over consumption and leisure. Hence the elasticities in the IES $= 1$ column of the left-hand panel of Table 2 are the same as those given in Campbell (1994), which uses the analytical technique employed here to solve the standard model.

Second, the elasticity of consumption with respect to a positive technology shock, η_{ca}, is increasing in persistence for low IES, but decreasing for high IES. When the IES is low, substitution effects are weak and the agent responds primarily to the positive income effect of a technology shock which increases with its persistence. When the IES is high, substitution effects are important. A non-persistent technology shock with $\phi = 0$ has no substitution effect because it increases output today but does not change the expected future marginal product of capital. A persistent technology shock, however, increases the interest rate and motivates a large substitution into consumption tomorrow; hence consumption elasticities can be very small or even negative when the IES is high and technology is persistent.

Third, when the IES is sufficiently small, the response of labor supply to a positive technology shock (η_{na}) is negative. Equivalently, consumption and hours are negatively correlated since η_{ca} is positive for small values of the IES. This counterfactual prediction can be understood by referring back to (10), setting $\beta = Z_t = 1$, and $H_t = (1 - N_t)$. Equation (10) shows that if the real wage is constant, the marginal utility of consumption will be perfectly correlated with the marginal disutility of labor hours. As consumption rises the marginal utility of consumption falls, requiring a decrease in the marginal disutility of work, or an increase in leisure, and producing a negative correlation between market hours and consumption. Procyclical variation in the real wage offsets this effect, but is not strong enough to reverse it if marginal utility declines rapidly. A small IES implies rapidly declining marginal utility and thus a negative correlation between hours and consumption.

This result illustrates a fundamental difficulty with the standard RBC model with a low IES: it predicts that market hours will be countercyclical. Table 2 shows how symmetric shocks to home technology can remedy this problem. The right-hand panel shows the elasticities for Case 2, the stochastic home technology model, when $\beta = 1$, but $Z_t = A_t^{\alpha}$. In this case, shocks to technology affect the home and market sectors symmetrically. This change has no effect on labor supply elasticities when the IES $= 1$; it increases the elasticities for IES < 1 and reduces them for IES > 1. For low IES, the impact of the change is large enough to make labor supply elasticities positive unless the persistence of technology shocks is extremely close to one. Thus symmetric home technology shocks make it possible to obtain procyclical labor supply in a real business cycle model with a low IES.

The underlying mechanism for this effect is that home technology shocks, like market technology shocks, have both income and substitution effects. The substitution effect draws labor into home production, while the income effect pushes labor away from home production.

When the IES is low, the income effect dominates and home technology shocks offset the shift to home production that is caused by market technology shocks. On net market hours show a tendency to increase because technology shocks stimulate investment (particularly when $\phi < 1$) and market output, unlike home output, can be used to accumulate capital.

Put another way, a home production model with a low IES requires that relative productivity shocks between the market and home sectors should be *small*, because such shocks tend to produce an inverse relation between market hours and market consumption. This finding contrasts with the results of previous home production studies which emphasize the importance of *large* relative productivity shocks across sectors in order to improve the quantitative performance of the standard model. The reason for the difference, as we have already emphasized, is that previous studies have assumed that the IES and SES are high, so substitution effects dominate income effects. Table 2 shows that with a high IES, market hours are particularly procyclical when there is relative productivity variation across sectors.

Table 3 reports results for the two cases in which capital is used in the home sector with the same intensity as in the market sector. As in Table 2, the left-hand panel has deterministic home technology, so that market technology shocks are intersectoral shocks, while the right-hand panel has stochastic home technology perfectly correlated with market technology.

Case 3, the home capital model, is reported in the left-hand panel of Table 3. The value of the IES has several notable effects on the elasticities in this case. The elasticities of market consumption, market output, and market hours are generally increasing in the IES; the more willing individuals are to substitute both intertemporally and intratemporally, the larger are the effects on the economy of a technology shock to the market sector. Specifically, when the IES (equivalently, the SES) is very large, a persistent technology shock induces a very large substitution into market consumption and market output, and out of home consumption. This pattern is the opposite of the deterministic home technology model, Case 1 (which, like Case 3, has no technology shocks but, unlike Case 3, omits home capital). This indicates that, with the share of home capital in home production as large as it is in the home capital model, intersectoral substitution effects dominate intertemporal substitution effects, whereas the opposite is true when home production uses no capital.

The right-hand panel of Table 3 shows elasticities for Case 4, the symmetric home production model. In this case, both sectors utilize capital and technology in the same proportion, and shocks to technology across sectors are perfectly correlated. This implies that $\eta_{ka} = \eta_{na}$; capital shifts across sectors in the same proportions as labor, to maintain the appropriate capital-labor ratio in each sector. As in Table 2, and for the same reasons, the addition of symmetric home technology shocks to the model increases the low-IES labor supply elasticities, making them positive for all but the most persistent technology shocks.

Table 4 gives the elasticities for Cases 5 (the labor-intensive home capital model), and 6 (the labor-intensive home production model). These cases differ from those in Table 3 only in the value for the share of home capital, $1 - \beta$, which is set equal to 0.08, rather than 1/3. Turning first to the results for the labor-intensive home capital model in the left-hand panel of Table 4, note that the consumption and labor supply elasticities follow a pattern similar to that of the deterministic home technology model in Table 2. The consumption elasticities decline when IES rises above one, indicating that intertemporal substitution effects dominate intersectoral substitution effects, while the labor supply elasticities are rising in IES. This finding is not surprising since the labor-intensive home capital model differs from the deterministic home technology model only in setting $\beta = 0.92$ rather than one. In addition, many of the labor supply elasticities in the labor-intensive home capital model are negative for low values of the IES, implying that this model shares the counterfactual prediction of the deterministic home technology model that market hours will often be countercyclical when the IES is less than one.

The solutions for the labor-intensive home production model, presented in the right-hand panel of Table 4, look like a hybrid of the solutions for the stochastic home technology model in the right-hand panel of Table 2 and the symmetric home production model in the right-hand panel of Table 3. Recall that in each of these models, technology shocks influence both the home sector and the market sector, but labor is used more intensively in home production in the home technology model (which is 100 percent labor-intensive because it has no home capital) than in the symmetric home production model (where capital represents one-third of home production). The labor-intensive home production model has a capital intensity in home production that lies between that of the home technology model and the symmetric home production model.

In one important respect the labor-intensive home production model differs from the two models for which it is a hybrid. When the IES is very large, the labor supply elasticities are negative while the home capital elasticities are positive. This occurs because a given change in labor-augmenting technology has a larger impact on the marginal product of labor in home production than it does in market production, a situation that motivates a strong substitution into nonmarket hours and a reallocation towards home capital. This effect is absent from the home technology model and the symmetric home production model, where shocks to technology affect the home and market sectors symmetrically.

Finally, notice that the labor-intensive home production model does not display the counterfactual prediction that market hours are always countercyclical when the IES is sufficiently small. Instead, for small values of the IES, the labor-intensive home production model produces positive labor supply elasticities for all values of ϕ except in the limit as ϕ approaches one. The intuition for this result is the same as before and concerns the fact that a technology shock produces little intersectoral productivity variation. Because a technology shock affects *both* home and market production, a positive technology shock leads to an increase in both home and market consumption, and therefore does not reduce the marginal utility of market consumption relative to home consumption as much as in the models for which technology influences only the market sector. Thus the labor supply elasticities in the labor-intensive home production model are much larger when the IES is low (and income effects are strong) than in models where there are strong intersectoral shocks.

4 Simulation Results

The elasticities discussed above summarize how the model's properties change with key parameter values. The results indicate that the solution is very sensitive to the assumed values of the IES and ϕ. To gain further understanding into the model's predictions at empirically plausible values of the IES and ϕ, and to compare them with those of other RBC and home production models, it is useful to undertake simple time-series simulations of the model. We can then carry out the exercise typically performed in the RBC literature of asking how well moments from the simulated data match those from the U.S. data.

We focus on the model's properties when the IES is set equal to empirically plausible levels. A survey of the many studies cited above which estimate this parameter suggests that it is well below one, and in many cases close to zero. Therefore, we consider 0.20 to be a conservative value for this parameter, and we use it in the simulations reported below.

We choose parameters for the technology process that are fairly standard.[6] In particular, we assume that the AR(1) process for the log technology is given by $a_t = 0.95a_{t-1} + \epsilon_t$, where ϵ_t is normally distributed with a standard deviation equal to 0.007. Using allocation rules implied by the elasticities reported above, 100 simulations of 150 periods each are computed. Each simulation consists of a random sample of 150 realizations of ϵ_t, which is then used to compute the values of each of the other variables in the model using the decision rules reported above. The simulated data are Hodrick-Prescott filtered before computing any statistics, again following the home production literature.

Panel A of Table 5 gives selected moments from U.S. quarterly data over the period 1959:1-1996:4[7] for the following log real variables: output, y; consumption, c; investment, i; average productivity, w; market capital, k; and market hours, n. For each of these variables, the table gives the percent standard deviation in the variable relative to the percent standard deviation of y, and the cross correlation of the variable with y. Data details are given in

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[6] Our parameter choice for the variance of technology shocks coincides with Benhabib, Rogerson, and Wright (1993), Greenwood and Hercowitz (1993) and Greenwood, Rogerson and Wright (1995), while our choice of ϕ is consistent with Benhabib et al and Greenwood et al.

[7] This sample period applies for all series except the capital stock, for which the most recent data runs from 1959:1-1994:1.

Appendix B.

Panel B uses simulated data to summarize the cyclical properties of the standard RBC model (Case 1 with IES = 1). The panel reveals several well-known discrepancies between the model's predictions and key aspects of the U.S. data. These discrepancies can be summarized as follows: compared to the data, output is not volatile enough; relative to output, consumption and hours are not volatile enough; relative to output, investment is too volatile, and productivity (w) is too highly correlated with output. Existing home production studies have documented significant improvements in the standard model's performance, along all of these dimensions, as the result of explicitly incorporating a household sector into the standard model (e.g. see Benhabib, et al., 1991). Next, we ask whether those improvements are maintained in our model with a low intertemporal elasticity of substitution in consumption.

Panels C-H of Table 5 show statistics computed from the simulated data for Cases 1–6. Note that, for these results, Case 1 is simply the standard model with IES = 0.2 instead of IES = 1.

Turning first to the results for Cases 1 through 4 (Panels C-F), Table 5 reveals that none produce results that represent a clear improvement over the standard model's performance. Instead, for these cases, generalizing the standard model to include home technological progress and a low intertemporal substitution elasticity appears to significantly deteriorate its quantitative performance along several dimensions. For example, in every case, the volatility of consumption relative to output is smaller, and the correlation of productivity with output is higher than in the standard model. Both the deterministic and stochastic home technology models, Cases 1 and 2, produce investment that is too volatile relative to output, whereas the home capital model and the symmetric home production model, Cases 3 and 4, produce too little investment volatility. Furthermore, Cases 1 and 3 have output less volatile than the standard model, and only Case 4 yields output that is more volatile than the standard model.

The existing home production literature documents that models with a household sector perform significantly better than the RBC benchmark along all of these dimensions. As previously noted, however, the most popular of these specifications implies that the IES is greater than one by imposing a value for the SES that is greater than one. Yet the RBC benchmark to which these models are compared is the standard model which fixes the IES at unity, making it difficult to determine how much of the documented improvement is due to the inclusion of a home production sector, and how much is due to the higher IES assumed in the home production framework. Our specification allows us to both fix the value of the IES at a common level across the RBC benchmark and home production model, and to give the IES an empirically plausible value. Thus, although the first four home production models considered in Table 5 perform worse than the standard model with log utility over consumption and hours, we do not believe the latter is an appropriate benchmark to which models of home production with low IES should be compared. Instead, we argue that Case 1, which minimizes the role of the home production sector but permits a more empirically plausible IES than the standard model, is the relevant benchmark.

Table 5 shows that, relative to Case 1, the symmetric technology shock cases (Cases 2 and 4) represent improvements along several dimensions over the low IES benchmark. For example, Case 1 predicts a counter-factual negative correlation between market hours and output. This is consistent with the negative labor supply elasticities (η_{na}) found in Table 2 when IES $= 0.2$. By contrast, the symmetric technology shock cases yield procyclical market hours and also match the data more closely in the relative volatility of hours and wages.[8]

Nevertheless, some problems remain with the low IES models, even for those models in which technology shocks across sectors are symmetric. The most notable difficulty is the volatility of investment relative to output: in the home technology model, investment is too volatile, while in the symmetric home production model it is not volatile enough. In both models, consumption is still too smooth relative to output. The last two panels of Table 5 demonstrate, however, that these difficulties with the relative volatility of investment and consumption are largely ameliorated in the models for which there is home capital and a realistic value for the steady state consumption-output ratio (equal to 0.667). The fact that investment is not volatile enough in the symmetric home production model is a mere artifact of the unrealistically low steady state consumption-output ratio (equal to 0.25) implied

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[8] The home capital model, Case 3, also delivers procyclical market hours for IES $= 0.2$ (although not for lower values of IES), but it does not deliver any improvement in the relative volatility of both hours and wages over the low IES benchmark.

by setting $\alpha = \beta = 2/3$ (recall the discussion in Section 2.2). Since output is the sum of consumption and investment, the standard deviation of y is a weighted average of the standard deviation of c and i, with the weights equal to the relative shares of c and i in y. When consumption is a very small share of output, as it is in the symmetric capital share models Cases 3 and 4, investment is very smooth since consumption is far less volatile than output. By contrast, when consumption is a large share of output, for example equal to 0.75 as it is in both the standard model and the home technology model, investment must be quite volatile since consumption comprises the largest fraction of output and is far less volatile than output. Finally, when the consumption share is set to 0.667 to match the aggregate data, as it is in the labor-intensive models, investment can have about the right amount of volatility and can still account for the fact that consumption is smoother than output.

In summary, the labor-intensive models both yield improvements over the low IES benchmark along the investment volatility dimension. In fact, the labor-intensive home production model produces a value for the relative volatility of investment that is very close to that found in the data. Nevertheless, of these two labor-intensive models, it is clear that the labor-intensive home production model—which has technology shocks that are perfectly correlated with those of the market sector—performs much better than the labor-intensive home capital model, where technology shocks are uncorrelated across sectors. Significantly, unlike all the other models, the labor-intensive home production model (Case 6) represents an improvement along all the dimensions discussed above over both the low IES RBC benchmark (Case 1) and the standard RBC model. The labor-intensive home production model delivers more volatile output than these benchmarks; relative to output, the model delivers less volatile investment, more volatile market hours, and comparable consumption volatility. Furthermore, the labor-intensive home production model produces a correlation between market hours and output that is not only positive but close to empirical values. This finding is striking in light of the general difficulty of reconciling the RBC paradigm with empirical evidence for a low IES.

Given these successes of the labor-intensive home production model, we further explore the persistence of output for that model, along with its comovement with other variables, in Table 6. The table presents cross-correlations of model variables with output in Panel B;

for comparison, these correlations are given for the U.S. data in Panel A. Output from the model economy does display persistence, consistent with the U.S. data, though it is clearly not as persistent as the latter. Lagged consumption and investment both covary positively with output in the model economy, though this covariation dies out more quickly than in the data; labor hours also move positively with output at the first lead and lag, but the comovement is too small at farther leads and too large at farther lags, whereas the opposite is true for productivity. Thus, the match between the model economy and the data is fairly good, but clearly imperfect.

In summary, the simulation results presented in this section demonstrate that the additively separable model of home production studied here, with a low value of the IES and technology shocks that are perfectly correlated across sectors, can yield significant quantitative improvements over the standard RBC model which has a higher IES. This improvement is even more dramatic relative to a *low* IES RBC benchmark that specifies the same value for the IES as in the home production model. Nevertheless, some features of the data, such as the comovement of hours and productivity with output, remain imperfectly captured by the home production model considered here. Constructing a framework that preserves the quantitative features of a low IES home production model while improving upon the cyclical properties of hours and productivity remains a topic for future research.

5 Conclusions

Little evidence is available to calibrate several key parameters in models with home production. One such parameter is the static elasticity of substitution between home and market consumption (SES). Yet theoretical models in the existing household production literature typically assume that home and market consumption are highly substitutable. Our strategy for calibrating the SES is to calibrate the intertemporal elasticity of substitution (IES) instead, making use of the positive relation that exists between the two parameters. In doing so, we rely on a large body of empirical evidence which suggests that the value of the IES is substantially below unity.

The framework studied in this paper allows us to explore several possible generalizations of the standard real business cycle model. A minimal generalization de-emphasizes the role of the home production sector, but relaxes the restriction of the standard model that the IES must be one to permit balanced growth. The most general specification incorporates a complete home production function with household capital and stochastic shifts in household technological progress. The value of the IES has a critical effect on the time-series properties of all these models.

While previous studies have concentrated on home production models with high values for the IES, we have explored the properties of a home production model with a low IES. We develop a low IES benchmark by introducing steady-state technological growth into the home sector of an otherwise standard, time-separable real business cycle model.

A long-standing difficulty in real business cycle theory is that preferences that have rapidly declining marginal utility of consumption make it difficult to capture the cyclical properties of consumption and hours worked, and often lead to a deterioration of the model's quantitative performance along several other dimensions. Our results demonstrate how these problems can be resolved, and provide three key insights about the underlying structure of a home production model with a low IES. First, freeing up the curvature of the utility function while maintaining balanced growth requires that the home and market sectors display the same long-run rate of technological progress. Second, when the IES is low, the cyclical behavior of market hours is not well captured in a home production model with a high degree of intersectoral productivity variation. In contrast to models which impose higher values for the IES, intersectoral technology shocks are not helpful because they make market hours and market consumption move inversely. Third, to match many cyclical properties of the data, capital must be present in the home sector but must be used more intensively in the market sector than in the home sector. A home production model that incorporates these three properties can be surprisingly successful at reconciling the real business cycle paradigm with evidence for a low IES.

The home production model proposed here is radically different from the existing literature in several respects. Conceptually, home production is thought of as the production of valued leisure. The model has separable utility across market and home production; it has very weak substitution effects, so household behavior is dominated by income effects; and it has stochastic home technology that moves in parallel with market technology. The first of these properties is consistent with econometric evidence that time-varying market hours do not affect the marginal utility of market consumption. The second property matches evidence that the intertemporal elasticity of substitution in market consumption is close to zero. The third property is plausible if leisure activities benefit from the same technological advances—in travel, telecommunications, and computing, for example—that affect market production. Thus we believe that the model is an appealing alternative to existing models of home production.

Appendix A: Implications of Alternative Specifications

I. Nonseparabilities and home production

The evidence cited above that there is little relation between expected consumption growth and expected hours growth can shed light on whether there may be important nonseparabilities between home and market consumption in RBC models with a variety of home production specifications. To see this, note that home production models are simply generalizations of standard RBC models, where nonmarket time is (at least in part) productive rather than unproductive. For example, in the case of the home production model considered in this paper, utility derived from nonmarket time is replaced by utility derived from some function of nonmarket time, where this function is given by $H_t = D_t^{1-\beta}(Z_t(1-N_t))^{\beta}$. In this case, the first-order condition (8), which must hold in equilibrium, is precisely the same as that in the standard RBC model with additively separable utility over consumption and leisure. Expected consumption growth is again unrelated to expected hours growth, consistent with the evidence cited in the introduction. By contrast, like their nonseparable RBC counterparts, nonseparable home production models will always imply that expected consumption growth should be related to some function of expected hours growth. For example, given the utility function, $U(C, H_t) = \frac{(C_t^b H_t^{1-b})^{1-r}-1}{1-r}$, and combining this with a particular functional form for H_t (e.g., $H_t = D_t^{1-\beta}(Z_t(1-N_t))^{\beta}$) it is immediately evident from a loglinearization of the appropriate first order condition for market capital accumulation in this model, that expected consumption growth will again be related to the expected growth rate in market hours, as well as, in this case, the expected growth rates of home capital and home technology. Similar expressions relating expected consumption growth with expected growth in market hours may be obtained for other nonseparable home production specifications. Thus, the presence of a home production sector does not eliminate the implication that expected consumption growth should be related to expected hours growth if utility is nonseparable between home and market consumption. Accordingly, evidence that there is little relationship between expected consumption growth and expected hours growth is suggestive of a separable utility function, regardless of whether the model under consideration is a standard RBC model with utility specified over consumption and leisure, or a more general RBC model with home production in which utility is specified over home and market consumption.[9]

II. Divisibility of nonmarket time

Consider the separable utility function $U(C_t, H_t, L_t) = \frac{C_t^{1-\gamma}}{1-\gamma} + \theta_1 \frac{H_t^{1-\lambda}}{1-\lambda} + \theta_2 \frac{(1-N_{mt}-N_{ht})^{1-\varepsilon}}{1-\varepsilon}$, where $L_t \equiv 1 - N_{mt} - N_{ht}$ is leisure, N_{mt} is market time and N_{ht} is home time devoted to productive activities. For the problem using this utility function, the single static first-order condition (10) would be replaced by two first-order conditions, one for the choice of market hours and one for the choice of nonmarket production hours:

$$\alpha K_t^{1-\alpha} A_t^{\alpha} N_{mt}^{\alpha-1} C_t^{-\gamma} - \theta_2 (1 - N_{mt} - N_{ht})^{-\varepsilon} = 0,$$

$$\theta_1 H_t^{-\lambda} \beta D_t^{1-\beta} Z_t^{\beta} N_{ht}{}^{\beta-1} - \theta_2 (1 - N_{mt} - N_{ht})^{-\varepsilon} = 0.$$

It is clear that, since each of these equations must hold in equilibrium, along any balanced growth path with constant market and nonmarket hours, the parameters γ and λ must both equal unity and we are back in the world of traditional RBC models in which the curvature of the utility function is restricted by balanced growth. Such a specification would be far more restrictive than the one studied in this paper, which allows the parameters γ and λ to take on a continuum of values ranging from zero to infinity.

III. Relation between the IES and SES

This section shows that the positive relation between the SES and IES is not an unusual aspect of our framework. Consider the most popular preference specification in the home production literature, for example the one found in Benhabib et al. (1991) and McGrattan et al. (1993). This specification is a nonseparable model with leisure, home consumption, and market consumption entering a nonseparable CES utility function of the form $U =$

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[9] If leisure is introduced explicitly in a nonseparable home production model, with leisure, $L_t = 1 - N_{mt} - N_{ht}$, expected consumption growth will then be related to expected leisure growth, in addition to expected market hours growth, and expected home capital and technology growth. Again, the presence of a home production sector does not eliminate the implication that expected consumption growth should be related to expected hours growth if utility is nonseparable between home and market consumption.

$u(\widetilde{C})v(L) \equiv \widetilde{C}^{b(1-r)}L^{(1-b)(1-r)}/(1-r)$, where L is leisure and $\widetilde{C}$ is a composite consumption good consisting of market and home consumption equal to $[aC^e + (1-a)H^e]^{1/e}$. In this model the SES equals $1/(1-e)$.

Defining the IES in this case as $-u_c/(u_{cc}C)$, it is straightforward to show that

$$\text{IES} = \frac{-\partial\widetilde{C}/\partial C}{[(b(1-r)-1)\widetilde{C}^{-1}(\partial\widetilde{C}/\partial C)^2 + \partial^2\widetilde{C}/\partial C^2]C}.$$

The relationship between the IES and the SES depends on the value of r; the studies which use this specification set $r = 1$.[10] Using the steady state values of C, H, market hours, h_m, and home hours, h_n, it is straightforward to compute the value of the IES implied by the value of the SES. Figure A1 illustrates this relationship using the parameter values and the values of a and b consistent with steady state allocations given in Benhabib et al., (1991). Note that the value of the IES approaches zero only when when the SES is also close to zero. Moreover, for a range of values of the IES less than one, the nonseparable CES model implies virtual equality between the IES and the SES.[11]

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[10] McGrattan, Rogerson and Wright (1991) estimate that r is close to 1. McGrattan, Rogerson and Wright (1993) estimate a higher r (about 5), but because the standard error is large, they cannot reject the hypothesis that $r = 1$ and so they too keep r at unity. Greenwood, Rogerson and Wright (1993) and Benhabib, Rogerson and Wright (1991) also set $r = 1$.

[11] We are aware of only two studies which attempt to estimate the value of e in the home production model specified above. McGrattan et al. (1993) use aggregate data, and Rupert et al. (1995) use household level data from the Panel Study of Income Dynamics (PSID). It should be noted that neither of these studies estimate values for e that are nearly as large as 0.8; the former study estimates $e = 0.385$, while the latter study finds a very small (and imprecisely estimated) value of e for single men, and a statistically significant but small value of e for single women.

Appendix B: Data

This appendix reviews the data used to compute the summary statistics in the first panel of Table 5. All series are per capita, measured at quarterly frequency, seasonally adjusted, and chain weighted in 1992 dollars, except where otherwise noted.

Consumption

Consumption is the sum of personal consumption expenditures (PCE) on nondurables and services, excluding expenditure on housing services, 1959:3-1996:4. Source: Bureau of Economic Analysis (BEA).

Investment

Total investment series is defined as residential and non-residential investment plus personal expenditure on consumer durables. Source: Bureau of Economic Analysis (BEA).

Hours

This series is aggregate hours of all wage and salary workers in non-agricultural industries, in millions. These data are monthly and converted to quarterly averages over the period 1959:1-1997:2. Source: Bureau of Labor Statistics.

Capital Stock

This series is the constant-cost net stock of fixed nonresidential structures and equipment, in billions of 1987 dollars from 1959-1994 at annual frequency. The data are linearly interpolated to quarterly frequency. Source: Bureau of Economic Analysis.

Output

The output series is constructed as consumption plus investment, following Benhabib et al., (1991).

Productivity

Average productivity (proportional to the real wage with Cobb-Douglas technology) is output divided by hours, defined above.

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Figure B1

Relationship Between IES and SES

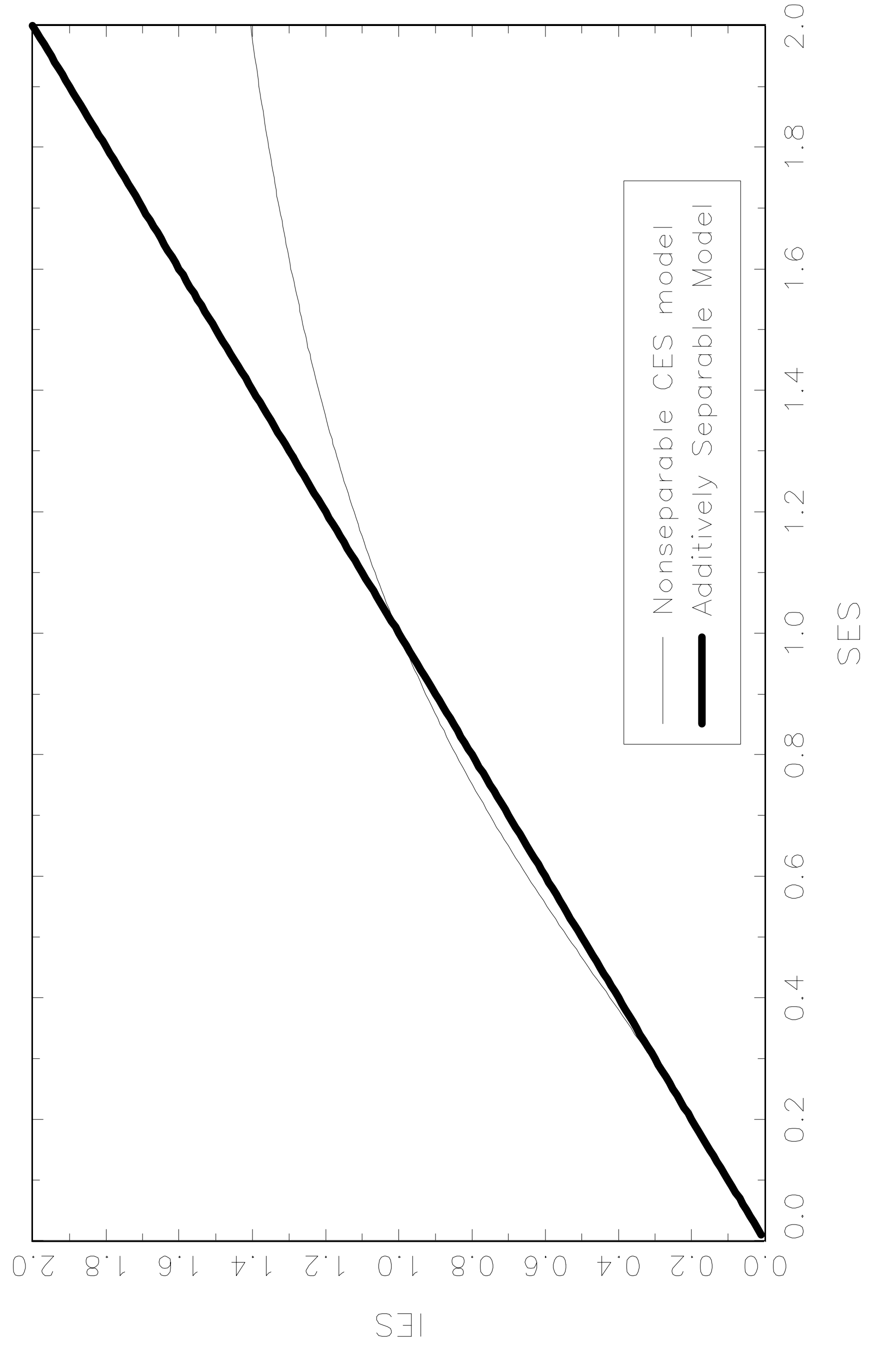

Table 1

Calibrated Parameter Values

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6
α	2/3	2/3	2/3	2/3	2/3	2/3
β	1	1	2/3	2/3	0.92	0.92
Z_t	1	A_t^{α}	1	A_t	1	A_t
g	0.005	0.005	0.005	0.005	0.005	0.005
r	0.015	0.015	0.015	0.015	0.015	0.015
N	1/3	1/3	1/3	1/3	1/3	1/3
δ	0.025	0.025	0.025	0.025	0.025	0.025

Notes: α is the share of labor in market production; β is the share of labor in home production; Z is the home technology shock; A is the market technology shock; g is the steady state growth rate (quarterly rate); r is the steady state real interest rate (quarterly rate); N is the steady state allocation of hours to the market sector; δ is the home and market capital depreciation rate (quarterly rate).

Table 2

Elasticities With Respect to Capital and Technology For Cases 1 and 2

		IES				IES			
		0	0.2	1	5	0	0.2	1	5
Elasticities		Case 1: Deterministic home technology model				Case 2: Stochastic home technology model			
η_{ck}	All ϕ	0.04	0.21	0.54	1.19	0.04	0.21	0.54	1.19
η_{kk}		1.00	0.97	0.94	0.92	1.00	0.97	0.94	0.92
η_{nk}		-0.08	-0.26	-0.24	0.22	-0.08	-0.26	-0.24	0.22
η_{yk}		0.28	0.16	0.17	0.48	0.28	0.16	0.17	0.48
η_{hk}		0.04	0.13	0.12	-0.11	0.04	0.13	0.12	-0.11
η_{ca}	$\phi = 0$	0.00	0.02	0.07	0.23	0.01	0.04	0.07	0.12
η_{ka}		0.08	0.09	0.13	0.17	0.18	0.17	0.13	0.09
η_{na}		-0.01	0.20	0.71	1.43	1.32	1.11	0.71	0.25
η_{ya}		0.66	0.80	1.14	1.62	1.55	1.41	1.14	0.84
η_{ha}		0.00	-0.10	-0.36	-0.72	0.01	0.11	0.31	0.54
η_{ca}	$\phi = 0.95$	0.05	0.17	0.29	-0.36	0.12	0.30	0.29	-0.18
η_{ka}		0.07	0.06	0.09	0.25	0.16	0.11	0.09	0.13
η_{na}		-0.11	-0.06	0.45	1.70	1.09	0.65	0.45	0.39
η_{ya}		0.60	0.63	0.97	1.80	1.39	1.10	0.97	0.93
η_{ha}		0.05	0.03	-0.22	-0.85	0.12	0.34	0.76	0.47
η_{ca}	$\phi = 1$	0.32	0.37	0.46	-7.0	0.75	0.65	0.46	-0.36
η_{ka}		0.00	0.01	0.06	0.29	0.00	0.02	0.06	0.15
η_{na}		-0.64	-0.41	0.24	1.86	-0.16	0.04	0.24	0.47
η_{ya}		0.24	0.39	0.83	1.91	0.56	0.69	0.83	0.98
η_{ha}		0.32	0.21	-0.12	-0.93	0.75	0.65	0.55	0.43

Notes: ϕ is the persistence parameter on the market technology process; IES is the intertemporal elasticity of substitution in consumption. The table reports the elasticities of market consumption, c, next period's market capital, k, market labor supply, n, market output, y, and home output, h, respectively. The first panel reports η_{ck}, η_{kk}, η_{nk}, η_{yk}, η_{hk}; the elasticities c, k, n, y, and h, respectively, with respect to this period's market capital which do not vary with ϕ. The last three panels give η_{ca}, η_{ka}, η_{na}, η_{ya}, η_{ha}; the elasticities with respect to market technology for selected values of ϕ. Table 1 gives a summary of the parameter values for each case.

Table 3

Elasticities With Respect to Capital and Technology For Cases 3 and 4

		IES				IES			
		0	0.2	1	5	0	0.2	1	5
Elasticities		Case 3: Home capital model				Case 4: Symmetric home production model			
η_{cf}	All ϕ	0.11	0.31	0.59	1.21	0.11	0.31	0.59	1.21
η_{ff}		1.00	0.98	0.96	0.90	1.00	0.98	0.96	0.90
η_{kf}		1.44	1.06	0.49	-0.75	1.44	1.06	0.49	-0.75
η_{df}		0.78	0.97	1.26	1.87	0.78	0.97	1.26	1.87
η_{nf}		0.44	0.06	-0.51	-1.75	0.44	0.06	-0.51	-1.75
η_{yf}		0.78	0.40	-0.18	-1.14	0.78	0.40	-0.18	-1.14
η_{hf}		0.11	0.31	0.59	1.21	0.11	0.31	0.59	1.21
η_{ca}	$\phi=0$	0.00	0.01	0.05	0.35	0.01	0.02	0.05	0.10
η_{fa}		0.03	0.04	0.08	0.26	0.08	0.08	0.08	0.07
η_{ka}		-0.01	0.24	1.24	5.97	1.32	1.29	1.24	1.14
η_{da}		0.00	-0.12	-0.62	-2.98	-0.66	-0.64	-0.62	-0.57
η_{na}		-0.01	0.24	1.24	5.97	1.32	1.29	1.24	1.14
η_{ya}		0.66	0.91	1.91	6.64	1.98	1.95	1.91	1.81
η_{ha}		0.00	-0.12	-0.62	-2.98	0.01	0.02	0.05	0.10
η_{ca}	$\phi=0.95$	0.05	0.08	0.23	1.57	0.15	0.25	0.23	-0.11
η_{fa}		0.02	0.03	0.06	0.15	0.07	0.06	0.06	0.09
η_{ka}		-0.10	0.10	0.88	3.54	1.04	0.83	0.88	1.55
η_{da}		0.05	-0.05	-0.44	-1.77	-0.52	-0.42	-0.44	-0.77
η_{na}		-0.10	0.10	0.88	3.54	1.04	0.83	0.88	1.55
η_{ya}		0.57	0.77	1.54	4.20	1.70	1.50	1.54	2.21
η_{ha}		0.05	-0.05	-0.44	-1.77	0.15	0.25	0.23	-0.18
η_{ca}	$\phi=1$	0.30	0.22	0.41	2.16	0.89	0.70	0.41	-0.21
η_{fa}		0.00	0.02	0.04	0.10	0.00	0.02	0.04	0.10
η_{ka}		-0.59	-0.18	0.51	2.34	-0.44	-0.06	0.51	1.75
η_{da}		0.30	0.09	-0.26	-1.17	-0.22	0.03	-0.26	-0.87
η_{na}		-0.59	-0.18	0.51	2.34	-0.44	-0.06	0.51	1.75
η_{ya}		0.07	0.49	1.18	3.01	0.22	0.60	1.18	2.41
η_{ha}		0.30	0.09	-0.26	-1.17	0.89	0.70	0.41	-0.21

Notes: ϕ is the persistence parameter on the market technology process; IES is the intertemporal elasticity of substitution in consumption. The table reports the elasticities of market consumption, *c*, next period's aggregate capital, *f*, market capital, *k*, home capital, *d*, market labor supply, *n*, market output, *y*, and home output, *h*, respectively. The first panel reports η_{cf}, η_{ff}, η_{kf}, η_{df}, η_{nf}, η_{yf}, η_{hf}; the elasticities *c, f, k, d, n, y,* and *h*, respectively, with respect to this period's aggregate capital which do not vary with ϕ. The last three panels give η_{cf}, η_{ff}, η_{kf}, η_{df}, η_{nf}, η_{yf}, η_{hf}; the elasticities with respect to market technology for selected values of ϕ. Table 1 gives a summary of the parameter values for each case.

Table 4

Elasticities With Respect to Capital and Technology For Cases 5 and 6

		IES				IES			
		0	0.2	1	5	0	0.2	1	5
Elasticities		Case 5: Labor-intensive home capital model				Case 6: Labor-intensive home production model			
η_{cf}	All ϕ	0.05	0.23	0.55	1.42	0.05	0.23	0.55	1.42
η_{ff}		1.00	0.98	0.95	0.91	1.00	0.98	0.95	0.91
η_{kf}		1.02	0.94	0.88	0.98	1.02	0.94	0.88	0.98
η_{df}		0.95	1.18	1.34	1.06	0.95	1.18	1.34	1.06
η_{nf}		0.05	-0.16	-0.30	-0.05	0.05	-0.16	-0.30	-0.05
η_{yf}		0.37	0.20	0.09	0.29	0.37	0.20	0.09	0.29
η_{hf}		0.05	0.17	0.25	0.11	0.05	0.17	0.25	0.11
η_{ca}	ϕ =0	0.00	0.02	0.07	0.31	0.01	0.04	0.07	0.05
η_{fa}		0.06	0.07	0.12	0.20	0.18	0.16	0.12	0.03
η_{ka}		0.00	0.08	0.33	0.88	0.64	0.56	0.33	-0.12
η_{da}		0.01	-0.22	-0.95	-2.54	-1.85	-1.60	-0.95	0.34
η_{na}		-0.01	0.20	0.85	2.28	1.66	1.44	0.85	-0.30
η_{ya}		0.66	0.83	1.34	2.48	1.99	1.81	1.34	0.43
η_{ha}		0.00	-0.11	-0.47	-1.25	0.01	0.13	0.45	1.09
η_{ca}	ϕ =0.95	0.05	0.14	0.28	0.30	0.16	0.33	0.28	-0.25
η_{fa}		0.05	0.05	0.08	0.20	0.15	0.11	0.08	0.07
η_{ka}		-0.04	0.00	0.21	0.89	0.54	0.36	0.21	-0.03
η_{da}		0.11	0.00	-0.63	-2.55	-1.55	-1.04	-0.62	0.08
η_{na}		-0.10	0.00	0.55	2.29	1.39	0.93	0.55	-0.07
η_{ya}		0.59	0.66	1.11	2.49	1.77	1.41	1.11	0.61
η_{ha}		0.05	0.00	-0.30	-1.26	0.16	0.41	0.62	0.96
η_{ca}	ϕ =1	0.32	0.32	0.45	0.29	0.95	0.78	0.45	-0.42
η_{fa}		0.00	0.02	0.05	0.20	0.00	0.03	0.05	0.09
η_{ka}		-0.22	-0.12	0.12	0.89	-0.02	0.06	0.12	0.02
η_{da}		0.64	0.35	-0.34	-2.55	0.05	-0.18	-0.34	-0.06
η_{na}		-0.58	-0.31	0.30	2.29	-0.05	0.16	0.30	0.05
η_{ya}		0.21	0.42	0.91	2.49	0.63	0.80	0.91	0.71
η_{ha}		0.32	0.17	-0.17	-1.26	0.95	0.83	0.75	0.89

Notes: ϕ is the persistence parameter on the market technology process; IES is the intertemporal elasticity of substitution in consumption. The table reports the elasticities of market consumption, *c*, next period's aggregate capital, *f*, market capital, *k*, home capital, *d*, market labor supply, *n*, market output, *y*, and home output, *h*, respectively. The first panel reports η_{cf} , η_{ff} , η_{kf} , η_{df} , η_{nf} , η_{yf} , η_{hf} ; the elasticities *c, f, k, d, n, y,* and *h*, respectively, with respect to this period's aggregate capital which do not vary with ϕ . The last three panels give η_{cf} , η_{ff} , η_{kf} , η_{df} , η_{nf} , η_{yf} , η_{hf} ; the elasticities with respect to market technology for selected values of ϕ . Table 1 gives a summary of the parameter values for each case.

Table 5

Simulation Results

	x=				
	c	*i*	*w*	*k*	*n*
A. U.S. Data: std(y) = 2.0					
$\frac{std(x)}{std(y)}$	.49	2.44	.65	.25	.76
$corr(x,y)$	.90	.97	.65	.38	.76
B. Standard RBC Model, IES = 1: std(y) = 1.0					
$\frac{std(x)}{std(y)}$	.35	3.08	.56	.33	.47
$corr(x,y)$	.90	.99	.98	.04	.98
C. Case 1, Deterministic home technology model, IES = 0.2: std(y) = 0.7					
$\frac{std(x)}{std(y)}$	.28	3.18	1.10	.33	.13
$corr(x,y)$	.98	.99	.99	.01	-.77
D. Case 2, Stochastic home technology model, IES = 0.2: std(y) = 1.0					
$\frac{std(x)}{std(y)}$	.28	3.21	.43	.35	.60
$corr(x,y)$	.98	.99	.96	.02	.98
E. Case 3, Home capital model, IES = 0.2: std(Y) = 0.7					
$\frac{std(x)}{std(y)}$	.11	1.3	.88	.19	.15
$corr(x,y)$	.94	.99	.99	.68	.85
F. Case 4, Symmetric home production model, IES = 0.2: std(Y) = 1.4					
$\frac{std(x)}{std(y)}$	.17	1.3	.45	.56	.55
$corr(x,y)$	.98	.99	.99	.98	.99
G. Case 5, Labor-intensive home capital model, IES = 0.2: std(Y) = 0.6					
$\frac{std(x)}{std(y)}$	.21	2.57	1.01	.26	.04
$corr(x,y)$	.97	.99	.99	0.00	-.12
H. Case 6, Labor-intensive home production model, IES = 0.2: std(Y) = 1.3					
$\frac{std(x)}{std(y)}$	.24	2.51	.35	.34	.67
$corr(x,y)$	.98	.99	.97	.74	.99

Notes: All Series are filtered using the Hodrick-Prescott technique. The following variables are in logs: *y* is output, *c* is market consumption, *i* is investment, *k* is market capital, *n* is market hours, and *w* is average productivity. The top of each panel is the percentage standard deviation of output; std(x)/std(y) gives the standard deviation of the series *x* relative to that of Y and corr(x,y) gives the correlation of *x* with *y*. The numbers are averages over 100 simulations of 150 periods each.

Table 6

	Cross-Correlations of Output with *x* =				
	y	*c*	*i*	*n*	*w*
A. U.S. Data					
x(-4)	0.32	0.31	0.30	-0.15	0.67
x(-3)	0.54	0.52	0.52	0.07	0.77
x(-2)	0.74	0.69	0.71	0.31	0.80
x(-1)	0.90	0.83	0.87	0.55	0.77
x	1.00	0.90	0.97	0.76	0.65
x{+1)	0.90	0.80	0.88	0.87	0.40
x(+2)	0.74	0.66	0.72	0.86	0.16
x(+3)	0.54	0.50	0.52	0.78	-0.05
x(+4)	0.32	0.29	0.30	0.66	-0.26
B. Case 6, Labor-intensive home production model					
x(-4)	0.07	-0.03	0.08	0.12	-0.04
x(-3)	0.22	0.13	0.24	0.27	0.12
x(-2)	0.42	0.35	0.43	0.46	0.33
x(-1)	0.68	0.63	0.68	0.70	0.62
x	1.00	0.98	0.99	0.99	0.97
x{+1)	0.68	0.72	0.67	0.64	0.73
x(+2)	0.42	0.50	0.41	0.36	0.52
x(+3)	0.22	0.32	0.20	0.15	0.34
x(+4)	0.07	0.18	0.05	0.00	0.20

Notes: Cross-correlations computed from simulated data of the home production model 6. All series are filtered using the Hodrick-Prescott technique. The following variables are in logs: *y* is output, *c* is market consumption, *i* is investment, *k* is market capital, *n* is market hours, and *w* is average productivity.