ON THE DESIGN OF HIERARCHIES:

COORDINATION VERSUS SPECIALIZATION*

by

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ABSTRACT

We develop a model of hierarchies based on the allocation of authority. A firm's owners have ultimate authority over a firm's decisions, but they have limited time or capacity to exercise this authority. Hence owners must delegate authority to subordinates. However, these subordinates also have limited time or capacity and so further delegation must occur. We analyze the optimal chain of command given that different agents have different tasks: some agents are engaged in coordination and others in specialization. Our theory throws light on the nature of hierarchy, the optimal degree of decentralization, and the boundaries of the firm.

1. Introduction

The purpose of this paper is to develop a model of hierarchies based on the allocation of authority. We take the view that a firm's owners have ultimate authority over a firm's decisions, but that they have limited time or capacity to exercise this authority. Hence the owners must delegate some authority to subordinates, i.e., they must grant the subordinates the right to make decisions that they themselves are unable or unwilling to make. However, these subordinates also have limited time or capacity to exercise authority and so further delegation must occur to other subordinates. Thus we view a firm as a chain of command over decisions. We use our model to analyze the optimal chain of command given that different agents have different tasks; in particular, some agents are engaged in coordination and others in specialization. Our theory throws light on the nature of hierarchy and the optimal degree of decentralization inside a firm, as well as on the boundaries of the firm.

There is a vast literature on many of the issues we consider, and this is not the place to provide a review. Economists have studied hierarchical structure from the point of view of supervision and task assignment (see, e.g., Williamson (1967) and Rosen (1982)); from the point of view of incentive theory (see, e.g., Calvo and Wellisz (1979)); and from the point of view of information processing and team theory (see, e.g., Keren and Levhari (19791, Radner (1992), Bolton and Dewatripont (19941, and Segal (1998)). By and large, however, the existing literature does not analyze hierarchy in terms of authority; that is, in contrast to our model, it is not the case that if i is above j in the hierarchy, then i necessarily has authority over j. Rather, in much of the literature, ii i is above j in the hierarchy, then j provides information to i. Also the literature does not distinguish between what happens inside a firm and what happens between firms. In other words, the optimal hierarchies derived could apply just as well to the organization of production in the U.S.A. as to the organization of production in Microsoft. In contrast, our approach does distinguish between the firm and the economy. In our model, one firm has one person or group with ultimate authority over all decisions (one owner or group of owners), whereas the economy has many people with ultimate authority over different subsets of decisions (many owners or groups of owners).[1]

Although our approach differs from much of the literature, it has parallels with the paper by Aghion and Tirole (19971. In our model, a boss (e.g., an owner) has formal authority in the Aghion-Tirole sense, while a subordinate has real authority if his boss cannot exercise authority but he can. We discuss the relationship further in Section 5.

The basic elements of our model are as follows. We consider an economy consisting of a set of assets and a set of identical individuals. Each asset represents a (residual1 decision; that is, a decision must be taken with respect to that asset. We assume that these decisions are noncontractible, both ex ante and ex post. In addition to these basic decisions, there are also "higher-level" decisions, which correspond to the coordination of assets or to synergies among assets.[2] To be precise, we assume that, for each subset of assets A, there is a task t, which consists of trying to come up with an idea about what to do with the assets in A. A task does not

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[1] In future work, it would obviously be desirable to combine the informational approach in the literature and the authority approach in this paper.

[2] Decisions rather than assets are the key feature of the model. We introduce assets because they are a convenient way to think about decisions, particularly higher-level decisions (synergies).

necessarily reach fruition, that is, become an idea ex post. If an individual's task becomes an idea, we say that the individual is active. In this case he will carry out his idea if he can. If the individual does not have an idea, he is inactive: in this case not only can the individual not implement his own idea, but also he cannot implement anyone else's.

Because any particular individual may not have an idea, it is important for efficiency that each asset has a hierarchy of bosses, that is, a chain of command. The way a hierarchy works is as follows. If the first person in the hierarchy (the ultimate boss) has an idea, she implements it. If she does not have an idea, control passes to her subordinate, who implements his idea if he has one. If the subordinate does not have an idea, the subordinate's subordinate has a chance to implement his idea, and so on.

A key assumption that we make is that ideas are mutually exclusive in the following sense. If one individual implements an idea involving an asset, then someone else cannot implement an idea involving that asset (whether or not the idea involves other assets too). This assumption has a significant implication. The benefit of putting someone high up in a hierarchy is that, if the person has a good idea, he is likely to be able to implement it. The cost is that the person may block others from implementing better ideas.

A (stylized) example may help. Imagine that there are two assets, a hotel and an airplane. Then there are three tasks: coming up with an idea about the hotel; coming up with an idea about the plane; and coming up with an idea about how to coordinate the hotel and the plane (synergy). The model supposes that these ideas conflict. For example, the "synergy" idea to offer hotel discounts to airplane passengers is inconsistent with the "hotel" idea to refurbish the hotel in the next three months.

Assume that there are three individuals, one carrying out each task.

One hierarchical structure would make the coordinator--that is, the person working on the hotel-plane synergy--senior on both assets. In this case, if the coordinator is active, i.e., has an idea, she can implement her idea, whether or not the specialists have ideas, since she has authority. In contrast, each specialist can implement his idea only if (a) he has one; (b) the coordinator does not.

Another hierarchical structure would reverse the roles: the coordinator would be junior to the specialists on both assets. In this case, each specialist can implement an idea whenever he has one, while the coordinator can implement her idea oniy if (a) she has one; (b) neither specialist does. (If either specialist implements his idea, this preempts the use of one of the assets, which means that the coordinator cannot implement her idea.1

A priori it is not clear which of these two hierarchical structures is better, although the second one seems unconventional. It is in fact an implication of one of our main results that the second hierarchical structure is suboptimal (given some additional assumptions).

Returning to the general case, we assume that the organizational form--characterized by a chain of command over each asset and an assignment of tasks to each individual--is chosen ex ante to maximize expected total surplus. From the design point of view, the key questions are, what tasks should each individual be assigned to carry out and what is the optimal chain of command for each asset? One of our principal results is that, given the assumption that the probability of an idea is decreasing in the set of assets being looked after, individuals with a broad remit, i.e., whose tasks cover a large subset of assets, should appear higher in the chain of command than those with a narrow remit. In other words, big thinkers or coordinators should be senior to small thinkers or specialists. We also establish that "criss-cross" hierarchies are never optimal; that is, if individual i appears above individuai j on one asset, j will not appear above i on another asset. Finally under an additional assumption, we show that the optimal hierarchy is a pyramid, in the sense that each individual has at most one boss.

We use these results to analyze the trade-off between centralization and decentralization. We define an organization to be centralized if most individuals in it are coordinators, and an organization to be decentralized if most individuals in it except for the top people are specialists. We show that if the gains to coordination are large enough, it is optimal for the organization to be centralized; if the gains to coordination are significant but not too large, it is optimal for the organization to be decentralized; and finally if the gains to coordination are small, then it is optimal for the organization to split up into several independent firms.

The paper is organized as follows. We set out the model in Section 2. In Section 3 we establish our main result, that individuals with a broad remit should be senior to those with a narrow remit. In Section 4 we provide a detailed analysis of the symmetric two-asset case. Section 5 is devoted to some foundational issues. Finally, Section 6 contains extensions and further discussion.

2. The Model

We consider an economy consisting of m assets, $a_1,...,a_m$, and n (risk neutral) individuals 1,...,n. The economy begins at date 0, and at this point organizationai form is chosen. Each asset represents a decision that has to be made in the future at date 1. These decisions are noncontractible both ex ante and ex post. However, authority over decisions can be allocated at date 0, as can the tasks that people are engaged in.

For simplicity we assume that all individuals are identical. At date 0 each individual is assigned a task. (More than one person can be assigned the same task.) A task consists of trying to come up with an idea about what to do with a subset of the m assets, i.e.. what decisions to make with respect to these assets at date 1. For each set $A \subset \bar{A} = \{a_1,...,a_m\}$, there is a corresponding task $t(A)$. Not all tasks reach fruition, that is, become ideas. We write the probability that task $t(A)$ becomes an idea as $p(A)$, where $0 < p(A) < 1$.

An individual who has an idea about the subset of assets A, and is able to implement it, generates value $v(A) \geq 0$ (measured in money). We put few restrictions on the function v, other than to suppose that $v(\{a_k\}) > 0$ for all k. In particular, v may depend on the identity of the assets in A as well as on their number. Also v may not be superadditive or even nondecreasing in A.

This last point deserves discussion. We have in mind a situation where thinking about how to use two assets is a very different activity from thinking about how to use one of them. The first activity involves coordination while the second does not. If coordination possibilities are limited, then the value of having an idea about how to coordinate two assets may be very low. Thus $v(\{a_1, a_2\})$ could be smaller than $v(\{a_1\}) + v(\{a_2\})$, or even than Min $\left[v(\{a_1\}), v(\{a_2\})\right]$ if synergies between the assets are sufficiently small.[3]

We will make the following (quite strong) assumptions about the generation of value:

(A1) To realize v(A), an individual carrying out task t(A) needs access to all the assets in A. If he has an idea but has access only to a non-empty, strict subset of A, he obtains a positive, but insignificant, value.

(A2) (No externalities.) All the value from an idea accrues to the individual whose idea it is (think of a pet project). In particular, ideas cannot be transferred: i cannot carry out j's idea.

(A3) Having an idea is an independent event across individuals.

(A4) There is no ex post renegotiation (e.g., because of shortage of time). That is, authority cannot be bought and sold at date 1.

We suspect that not all of these assumptions (except possibly for (A4)) are essential, but they greatly simplify the analysis. We discuss (A4) further in Section 5. Note that, as will be seen below, it is the absence of costless ex post renegotiation that provides a role for hierarchical

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[3] Take the hotel-plane example of the introduction. Consider the comparison between the profit from offering hotel discounts to airplane passengers and the profit from refurbishing the hotel and charging higher hotel prices. The first may be bigger than the second if the plane flies to an airport near the hotel; but smaller if it does not.

structure in our model. In fact, we view hierarchical structure as a substitute for ex post renegotiation.

(A1) - (A4) have a simple but useful implication:

(*) An individual who has an idea and can implement it (even if only partially, i.e., even if he obtains only an insignificant value) will always do so; he will never defer to someone junior, however productive the junior person is. Also a senior individual who is inactive will never wish to veto the idea of a subordinate.

The first part of (*) Coilows from the fact that, if a senior person with an idea defers to a junior person, he loses his private value (for which he cannot be compensated--given (A4)). The second part follows from the fact that, given (A2), a senior person without an idea neither gains nor loses from his subordinate's idea.

We now turn to the allocation of authority at date 0. We associate with each asset a hierarchy of bosses, that is, a chain of command. Formally, a chain of command is a list, i.e., a sequence of a subset of the numbers 1,...,n (the list may contain all the numbers 1,...,n, none of the numbers, or a strict subset of the numbers; no number is repeated). The first number in the list refers to the ultimate boss, the second number to his subordinate, the third number to the subordinate's subordinate, and so on. Given a chain of command, the most senior person on the list with an idea implements it. If no one in the chain has an idea, the asset yields zero value.

We define an organizational form at date 0 to be a delineation of a chain of command for each asset and an assignment of tasks to each individual. We assume that both the chain of command and the tasks can be specified in an enforceable contract.[4]

We make a final assumption:

(A5) There is costless (Coasian) bargaining at date 0, and individuals are not wealth-constrained.

(A5) is in stark contrast to (A4). We have in mind that there is plenty of time for the parties to negotiate at date 0, but very limited time (no time) to negotiate at date 1. (A5) implies that organizational form will be chosen at date 0 to maximize expected total surplus, with the surplus being divided up using lump sum transfers.

Before we write down the formula for expected surplus, we can simplify matters a little. Suppose an individual's task consists of looking after assets in the subset A. Then it makes no sense to put the individual in the list (chain of command) involving an asset $a_k \notin A$, since the individual will never have an idea about a_k.

Similarly, suppose an individual is assigned the task t(A). Then it makes no sense <u>not</u> to put him in the list (chain of command) involving each asset $a_k \in A$, since he generates no significant value unless he has control over each of the assets in A.

Putting these two observations together, we can conclude the following. Once the lists for all the assets have been determined, we can figure out which task each person is doing by seeing which list he appears in: if the union of the lists he appears in corresponds to the set of assets A, then he will be doing task t(A).

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[4] In Section 6 we briefly discuss what happens if tasks are noncontractible.

An example might be useful at this point. Suppose there are two assets and four people ($m = 2$, $n = 4$). Figure 1 illustrates three possible organizational forms.

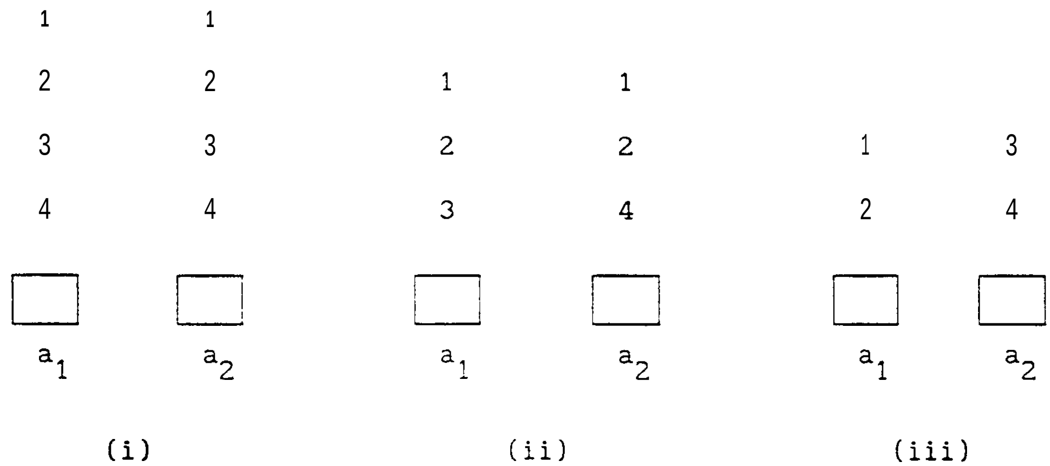

Figure 1

In the first form, 1 is the boss of al and a_2, and has 2, 3, and 4 as subordinates on both assets. The tasks correspond to this assignment of authority: all four individuals are engaged in looking after both assets. In the second form, 1 is the boss of both assets, 2 is l's subordinate on a_1 and a_2, 3 is 2's subordinate on a_1, and 4 is 2's subordinate on a_2. Again, the tasks correspond: 1 and 2 look after al and a_2, 3 looks after a_1 and 4 looks after a_2. In the third form, 1 is the boss of asset a_1, 2 is his subordinate on a_1, 3 is the boss of a_2, and 4 is his subordinate on a_2. 1 and 2 look after a_1 and 3 and 4 look after a_2.

The forms have a natural economic interpretation. The first two represent a single firm since both assets have the same ultimate boss, individual 1 (who can be interpreted as the owner of the assets). The second form can be thought of as corresponding to a more decentralized firm than the first because authority is more likely to be exercised by someone with a narrow remit--a specialist--and less likely to be exercised by a coordinator (in the second form, 3 or 4 gets to exercise authority if 1 and 2 don't have an idea). Finally, the third form represents two firms since assets a_1 and a_2 have different ultimate bosses: 1 is the boss of a_1, 3 is the boss of a_2.

We now write down the general formula for expected total surplus in the m asset, n individual case. Let L_k be the list associated with asset a_k. For individual i, define

$$A_i = \{\text{assets } a_k \mid i \text{ appears on list } L_k\}.$$

A_i is the set of assets over which i can exercise authority. From the above we know that individual i will be engaged in task $t(A_i)$. Also for individual i define

$$S_i = \{\text{individuals } j \mid \text{for some asset } a_k, i \text{ and } j \text{ both appear on list } L_k \text{ and } j \text{ appears above } i\}.$$

S_i is the set of individuals who are senior to i on some asset. Now we know that individual i receives value $v(A_i)$ if and only if i has an idea and nobody senior to i on any of the assets i looks after has an idea. Given (A3), we can therefore write the formula for total expected surplus as

$$V = \sum_{i=1}^{n} p(A_i) \left(\prod_{j \in S_i} (1 - p(A_j)) \right) v(A_i). \tag{2.1}$$

According to (A5), organizational form will be chosen to maximize (2.1).

3. An Example and the Main Theorem

In this section we establish some general results about optimal organizations. Part (a) of Theorem 1 provides a surprisingly powerful characterization of an optimal hierarchy. It says that an optimal organizational form has the property that an individual's place in the hierarchy is determined (entirely1 by his probability of having an idea: individuals with the lowest probability of having an idea are placed at the top of the hierarchy, individuals with the next lowest probability of an idea are placed next in the hierarchy, and so on. Part (b) (which pretty much follows from part (a)) says that criss-cross arrangements are never optimal. That is, if j is above i on one asset, i will never be above j on another asset.

Before we state Theorem 1, and two corollaries, it is useful to get some intuition from a special case. Suppose there are two assets, al and a_2, and two individuals, 1 and 2 ($m = n = 2$). Given our assumption that individuals are identical, but assets may not be, there are nine distinct organizational forms. (In what follows, everything is unique up to the permutation of the individuais' names.) To see this, note that there are two organizational forms where both individuals look after both assets (2 can be senior to 1 on both, or senior on one and junior on the other); four forms where 2 looks after two assets and 1 looks after one (2 can be senior or junior on the asset 1 looks after; and 1 can look after al or a_2); and three forms where 1 **and** 2 both look after one asset (they can look after different assets or the same asset, which may be a_1 or a_2).

Some of these forms are illustrated in Figure 2. (We leave out the symmetric version of (iii), (iv) and (v), where assets al and a_2 are reversed.1

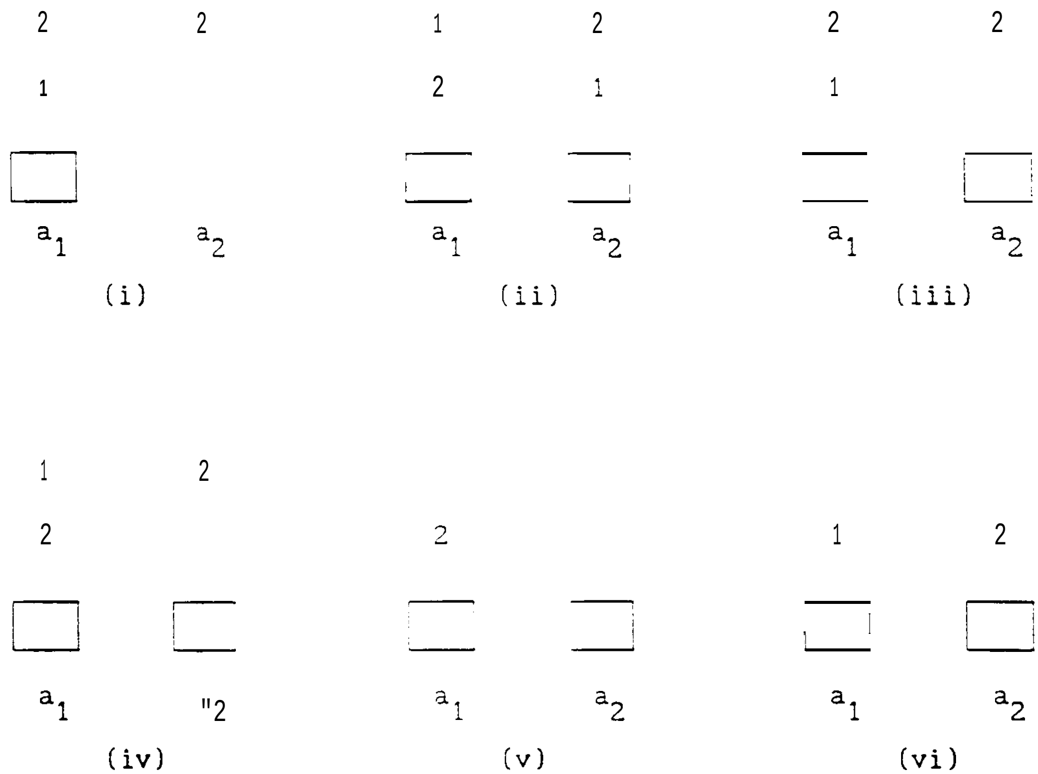

Figure 2

Forms (i), (iii), (v) and (vi) make good economic sense. We would expect form (i) to be desirable if coordination is very important; form (iii) to be desirable if some coordination is important but not too much; form (v) to be desirable if coordination is not valuable and a_1 is very productive; and form (vi) to be desirable if coordination is not valuable and both assets are comparably productive. (Each of these forms can be shown to be optimal for a suitable choice of the parameters.)

However, forms (ii) and (iv) seem strange. Does it make sense to have someone coordinate and yet be junior on some asset, given that this implies that he will rarely be able to implement his coordination idea?

Fortunately, (ii) is never optimal, and neither is (iv) (under an additional assumption). To see this, note that the expected surpluses (values) from forms (i) - (v) (represented by $V_1,\ldots,V_5$, respectively) are given by

$$V_1 = [1-(1-p_2)^2]v_2 = (2p_2 - p_2^2)v_2,$$
$$V_2 = 2p_2(1-p_2)v_2,$$
$$V_3 = p_2 v_2 + (1-p_2)p_1 v_1,$$
$$V_4 = p_1 v_1 + (1-p_1)p_2 v_2,$$
$$V_5 = [1-(1-p_1)^2] v_1 = (2p_1 - p_1^2)v_1,$$

where $v_1 = v(\{a_1\})$, $v_2 = v(\{a_1,a_2\})$, $p_1 = p(\{a_1\})$, $p_2 = p(\{a_1, a,\})$. (To understand these formulae, note that in (i) coordination occurs if either 1 or 2 (or both) is active; in (ii) coordination occurs if exactly 1 or 2 is active (but not both); in (iii) coordination occurs if 2 is active whether or not 1 is active; and in (iv) coordination occurs if 2 is active but 1 is not.)

It is immediate that $V_2 < V_1$ and so (ii) is not optimal. To see whether (iv) can be optimal, note that, if it is, we must have $V_4 \geq V_1$ and $V_4 \geq V_5$. The first implies $p_1v_1 \geq p_2 v_2 (1 + p_1 - p_2)$, while the second implies $p_2v_2 \geq p_1v_1$. These cannot both be true, as long as we are prepared to assume $P2 < p_1$.

This example illustrates the theorem (and corollaries) stated below: (a) If individual 2's task is such that he has a lower probability of an idea than individual 1, then it is not optimal to put 1 above 2 on any asset ((iv) is not optimal). (b) Criss-cross arrangements are not optimal, i.e., it cannot be the case that 2 is above 1 on one asset and 1 is above 2 on another ((ii) is not optimal).

Without the monotonicity assumption on probabilities, $p_2 < p_1$, we cannot rule out organizational forms like (iv). In the above example, let $v_1 = 10$, $v_2 = 8$, $p_1 = 1/4$, $p_2 \simeq 1$. Then direct calculation shows that (iv) is optimal.[5]

Let us return to the m asset. n individual case. Recall from Section 2 that A_i is the set of assets over which i can exercise authority and L_k is the list (chain of command) associated with asset a_k. We now state the main theorem.

<u>Theorem 1</u>. Consider an optimal organizational form.

(a) Suppose $p(A_i) > p(A_j)$. Then, for all assets $a_k \in A_i \cap A_j$, j appears above i on list L_k.

(b) Criss-cross arrangements are not optimal, i.e., if j appears above i on list L_k for asset a_k, then there does not exist k' such that i appears above j on list $L_{k'}$ for asset $a_{k'}$.

<u>Proof</u>: See appendix.

Notice that, when $p(A_i) \neq p(A_j)$, part (b) of the theorem follows immediately from part (a). The heart of part (b), therefore, lies in showing that criss-cross arrangements are also not optimal when $p(A_i) = p(A_j)$.

Of course, the leading case where $p(A_i) = p(A_j)$ is when $A_i = A_j$: part

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[5]Although (iv) may be optimal when $p_2 > p_1$, we can rule out (iii) in this case. It follows from direct calculation that $p_2 > p_1 \Rightarrow$ either $V_3 < V_1$ or $V_3 < V_5$. This finding provides another illustration of part (a) of Theorem 1: If a specialist has a lower probability of an idea than a coordinator ($p_2 > p_1$), then it is not optimal to put a coordinator above a specialist on any asset ((iii) is not optimal).

(b) of the theorem then says that, given two people with the same remit, one should be senior to the other on all the assets they work on. Below we state a corollary that deals with the more interesting case where $A_i \subset A_j$, $A_i \neq A_j$, i.e., j's remit is broader than i's.

First, we make an observation about Theorem 1. Part (a) at first looks a little suspicious. It would appear that the decision about who to put on top of a hierarchy is determined solely by the probability of the success of an idea and not at all by the value of an idea. For example, suppose $p(A_j)$ and $v(A_j)$ are very low. Then putting j at the top of the hierarchy is very inefficient, and yet the theorem suggests that this is optimal. The reason there is no contradiction is that the theorem says nothing about which tasks should be carried out. Given that j has a low probability of success and is unproductive even when he has an idea, j is clearly doing the wrong task. That is, in an optimal organizational form no-one will be doing task $t(A_j)$.

This observation about unproductive individuals with low probabilities having the wrong tasks gives the clue to how part (a) of the Theorem is proved. Take an asset a_k, and suppose that agent i is senior to agent j on list L_k, but $p(A_i) > p(A_j)$. in broad terms, we show that expected surplus can be increased by making one of two changes to the organizational form. Either i is relatively unproductive ($v(A_i)$ is relatively low), in which case expected surplus can be increased by switching i to task $t(A_j)$ and placing him just under j in seniority on all assets in A_j--akin to changing from hierarchy (iv) to hierarchy (i) in Figure 2 (with $i \equiv 1$, $A_i \equiv \{a_1\}$, $j \equiv 2$, $A_j \equiv \{a_1,a_2\}$, and $a_k \equiv a_1$). Or j is relatively unproductive ($v(A_j)$ is relative low), in which case expected surplus can be increased by switching j to task $t(A_i)$ and placing her just under i in seniority on all assets in A_i--akin to changing from hierarchy (iv) to hierarchy (v). The merit of these two kinds of maneuver is that one can keep track of how overall expected surplus changes. By contrast, if we consider a third kind of change to the organizational form, the ostensibly more straightforward maneuver of simply switching the seniorities of i and j, then the people who lie between i and j (on list L_k) are affected in subtle ways and so the overall change to expected surplus is complicated. (In terms of our earlier two-person analysis, this third maneuver amounts to switching the seniorities of 1 and 2 on asset a_1; i.e., changing from hierarchy (iv) to hierarchy (iii). This method happens to work when there are just two people, but not if there are others in between them.)

Corollary 1 follows directly from Theorem 1, given a further assumption.

<u>Monotonicity</u> <u>(M)</u>. $p(A) > p(B)$ if $A \subset B$, $A \neq B$.

<u>Corollary</u> <u>1</u>. Assume (M). Consider an optimal organizational form. Suppose $A_i \subset A_j$, $A_i \neq A_j$, where $i, j \in \{1, \ldots, n\}$. Then, for each asset a_k such that $a_k \in A_i$, j will appear above i on list L_k.

Corollary 1 says that, under the assumption that the probability of an idea is decreasing in an individual's span of control, it is optimal for someone with a broad remit to be senior to someone with a narrow remit.

Assumption (M) warrants further discussion. We would argue that this assumption is plausible. Consider the two functions p(A) and v(A) as the set A increases. It would be surprising if p and v moved in the same direction. If p and v both increase, this would say that coordinators are supermen, while if p and v both decrease, it would say that specialists are supermen. It is more likely that p and v move in opposite directions, i.e., p falls and v rises, or p rises and v falls. The first of these seems more reasonable than the second: that is, it seems to accord with common sense that a coordinator can (on average) achieve a sizable efficiency gain if he has an idea, but that he is not that likely to have an idea. (Note, however, that (M) requires only that p is decreasing in A; it does not require that v is increasing in A.)

Corollary 1 covers only the case where the remits of individuals can be ranked. To this extent, the corollary leaves open the possibility that someone can have two bosses (a non-pyramidal hierarchical structure). That is, person i may be senior to person j on one asset; while person i' is senior to person j on another asset, whereas i is not. This can happen if the remits of persons i. j and i' cannot be ranked.

For example, consider the situation illustrated in Figure 3. There are three individuals and six assets, and synergies exist only between assets a_1, a_2, a_3, assets a_4, a_5, a_6, and assets a_3, a_4.

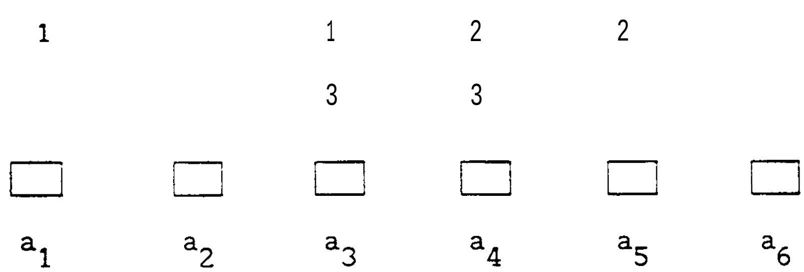

Figure 3

Assume $v(\{a_1,a_2,a_3\}) = v(\{a_4,a_5,a_6\}) = 14$, $v(\{a_3,a_4\}) = 8$, $p(\{a_1,a_2,a_3\}) = p(\{a_4,a_5,a_6\}) = 1/4$, $p(\{a_3,a_4\}) \simeq 1$. Then it is straightforward to show that it is optimal to put individual 1 in charge of a_1,a_2,a_3, individual 2 in charge of a_4,a_5,a_6, and to make individual 3 a subordinate on a_3,a_4. Individual 3 then has different bosses on a_3 and a_4.

In order to rule out this kind of situation, we need a further assumption.

Let us define a set of assets A to be synergistic if $v(A) > 0$.

<u>Nestedness</u> <u>(N)</u>. Synergies are <u>nested</u> if given two synergistic sets A, B, either $A \subset B$, or $B \subset A$, or $A \cap B = \emptyset$.

In other words, (N) says that if there is a synergy between a set of assets, then any synergy involving one of the set and a new asset requires the presence of the other assets in the set too. If synergies are nested, the situation in Figure 3 cannot arise, since if $\{a_3, a_4\}$ is a synergistic set (which is why 3 is working on these assets), then $\{a_1, a_2, a_3\}$ is not a synergistic set (and so 1 will not work on these assets).

(N) is quite strong. Note, however, that it is trivially satisfied in the two asset case.

<u>Corollary</u> <u>2</u>. Assume (M) and (N). Consider an optimal organizational form. If j appears above i on list L_k for some asset a_k, then j appears above i on every list on which i appears.

The proof of Corollary 2 is direct. Suppose i looks after the set of assets A_i and j looks after the set of assets A_j. A_i and A_j must be synergistic since otherwise one of the individuals creates zero value and expected surplus could be increased by assigning this individual to a single asset (any one) and making him the most junior person on this asset. It follows from (N) that $A_i \subset A_j$, $A_j \subset A_i$ or $A_i \cap A_j = \emptyset$. The last is impossible since j appears above i on some asset. If $A_i \subset A_j$, $A_i \neq A_j$, the conclusion of Corollary 2 follows from Corollary 1. If $A_i = A_j$, the conclusion follows from Theorem 1, part (a). Finally, $A_j \subset A_i$, $A_j \neq A_i$, is inconsistent with Corollary 1 since we know that i does not appear above j on list L_k.

At this point it is worth returning to the two asset-two individual example. Theorem 1 and Corollary 1 imply--and we have also observed this from direct calculation-- that the organizational forms (ii) and (iv) in Figure 2 cannot be optimal (assuming $p_2 < p_1$). In fact, if we are prepared to make the additional assumption of symmetry ($v(\{a_1\}) = v(\{a_2\})$, $p(\{a_1\}) = p(\{a_2\})$), (iii) and (v) can also be ruled out. To see this, note that the expected surplus from organizational form (vi) is given by

$$V_6 = 2p_1v_1,$$

which is obviously greater than V_5. (This is just an implication of diminishing returns.) In addition, it is easy to show that either $V_6 > V_3$ or $V_1 > V_3$ (so (iii) is not optimal).

So in the two asset-two individual example, the optimal arrangement is symmetric: either there should be two coordinators (as in (i)) or no coordinators (as in (vi)).

In the next section, we will show that symmetry always holds in the two asset case when the number of individuals (n) is even. This has an interesting implication for Figure 1. In Figure 1 we illustrated three possible organizational forms for the two asset-four individual case. It turns out that, under symmetry, these are the only candidates for optimality (as the next section will show). Moreover, the trade-off between them is as one would expect (at least if $p_1 > p_2$, p_1 not too close to p_2). Form (i) is optimal if the gains to coordination are large enough; (ii) is optimal if the gains to coordination are moderate; and (iii) is optimal if the gains to coordination are small.

4. The Symmetric Two Asset Case

In this section we analyze in detail the case of n individuals and two symmetric assets $(m = 2)$. We refer to an individual who looks after both assets as a cordinator and an individual who looks after one asset as a specialist. The values they generate if they have an idea and exercise authority over the appropriate assets are given by v_2, v_1, respectively (i.e., $v_2 = v(\{a_1, a_2\})$, $v_1 = v(\{a_1\}) = v(\{a_2\})$). The probabilities of having an idea are given by p_2, p_1, respectively (i.e., $p_2 = p(\{a_1, a_2\})$, $p_1 = p(\{a_1\}) = p(\{a_2\})$).

We assume $p_1 > p_2$. Hence, from the Theorem, coordinators will be senior to specialists and, given any two coordinators, one will be senior to the other on each asset. Thus we can represent an optimal organization as follows:

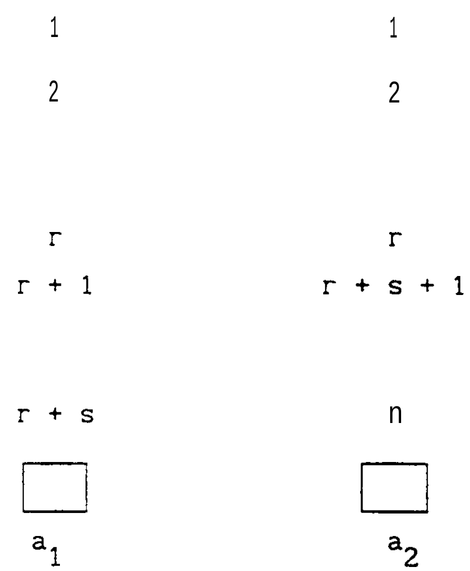

Figure 4

In Figure 4 there are r coordinators and $(n - r)$ specialists. Note that the optimal organization may not be symmetric, i.e., the number of specialists, s, on asset a_1 need not be the same as the number of specialists, $n - r - s$, on asset a_2.

In fact it turns out that the optimal organization <u>is</u> symmetric, i.e., $s = n - r - s$, except in the case where n is odd and there are no coordinators at all.

<u>Result 1</u>. Assume $m = 2$; the assets are symmetric, i.e., $v(\{a_1\}) = v(\{a_2\}) = v_1$, $p(\{a_1\}) = p(\{a_2\}) = p_1$; and $p_1 > p_2$. Consider an optimal organizational form in which individuals $1,\ldots,r$ look after both assets ($A_i = \{a_1,a_2\}$ for $i = 1,\ldots,r$), individuals $r + 1,\ldots,r + s$ look after asset a_1 ($A_i = \{a_1\}$ for $i = r + 1,\ldots,r + s$) and individuals $r + s + 1,\ldots,n$ look after asset a_2 ($A_i = \{a_2\}$ for $i = r + s + 1,\ldots,n$). Then, unless n is odd and $r = 0$, $s = n - r - s = (n-r)/2$.

<u>Proof</u>: See appendix.

The key step in proving Result 1 is to show that if there is some coordination ($r > 0$) and, say, one more specialist working on asset a_1 than on asset a_2 ($s = n - r - s + 1$), then it would be better either for the most junior coordinator to switch to specializing on a_2, or for the "extra" specialist on asset a_1 to switch to coordination. Either way, the number of specialists on a_1 and a_2 should be equalized: an asymmetric compromise is never optimal. And if there are no coordinators ($r = 0$), the number of specialists on each asset should be as equal as possible--i.e., the same when n is even.

From now on, to simplify matters, we will assume that n is even, in which case the optimal organization is symmetric even if there are no coordinators. As above, write the number of specialists on each asset as s. Since n is even, the number of coordinators must also be even: $r = n - 2s$.

We now consider the optimal choice of s. Before we get into the details, let us note a simple way of thinking about this. Imagine that everyone is a specialist, i.e., $2s = n$. Then the value of the organization is $2(1 - (1 - p_1)^{n/2})v_1$, which is a strictly concave function of n. In other words, not surprisingly, there are diminishing returns to having more specialists. As we will see, this has the following implication: in a class of cases (more precisely, when the surplus maximization problem is convex, and the solution to the problem is interior), then, after s has reached a certain value, it is better not to have further specialists, but rather to make any additional people in the organization (i.e., those at the top) coordinators. That is, for large enough n, the optimal value of s is independent of n.

However, this is not the only possibility. There is another class of cases (when the surplus maximization problem is nonconvex) where the optimum is a corner solution: $s = 0$ or $s = n$.

Now to the details. Denote by V_s the expected surplus (value) of an organizational form in which there are s specialists on each asset and $n - 2s$ coordinators. Suppose $s \leq n/2 - 1$. Let θ be the probability that at least one of the first $(n - 2s - 2)$ coordinators is active (i.e., has an idea). Then we can write

$$(4.1) \qquad V_s = \theta v_2 + (1 - \theta) \hat{V}_s,$$

where $\hat{V}_s$ is the value of the organizational form (call it "hat") consisting of everyone <u>but</u> the first $(n - 2s - 2)$ coordinators, i.e., the organizational form consisting of 2 coordinators and s specialists below them on each asset. The justification for (4.1) is that if one of the first (n - 2s - 2) coordinators is active, which happens with probability θ, v_2 is realized; while, if not, V_s is realized.

In turn we can write

$$(4.21 \qquad \hat{V}_s = (2p_2 - p_2^2)v_2 + (1 - p_2)^2(2W_s),$$

where W_s is the value of the organizational form consisting of s specialists working on asset a_1 (or a_2). The justification for (4.21 is that in the organizational form "hat," the probability that at least one of the coordinators is active is $(1 - (1 - p_2)^2)$, in which case v_2 is realized; otherwise $2W_s$ is realized.

Now increase s by 1. Since there are now 2 more specialists altogether, the number of coordinators falls to n - 2s - 2. Using the same logic as above, we can write

$$(4.3) \qquad V_{s+l} = \theta v_2 + (1 - \theta) \hat{V}_{s+1},$$

where $\hat{V}_{s+l}$ is the value of an organizational form consisting of (s + 1) specialists on each asset and no coordinators. In turn,

$$(4.41 \qquad \hat{V}_{s+1} = 2[p_1 v_1 + (1 - p_1)W_s].$$

Define $q_1 = 1 - p_1$, $q_2 = (1 - p_2)^2$. Combining (4.1) - (4.4), carrying out some manipulation, and using the fact that $W_s = (1-q_1^s)v_1$, weobtain

$$(4.51 \qquad V_{s+1} \geq V_s \Leftrightarrow \hat{V}_{s+1} \geq \hat{V}_s$$

$$\Leftrightarrow (q_2 - q_1)q_1^{\,s} \geq (1 - q_2)\left(\frac{v_2}{2v_1} - 1\right).$$

(4.5) provides us with important (marginal) information; it tells us when total value can be raised by increasing the number of specialists on each asset from s to s + 1 (and correspondingly reducing the number of coordinators by 2).

In what follows, it is helpful to consider separately the cases $q_1 \geq q_2$ and $q_1 < q_2$. (The first is likely to occur when $p_1 \simeq p_2$ and the second when $p_1 >> p_2$.)

<u>Case 1: $q_1 \geq q_2$</u>

From (4.51, the crucial inequality is:

$$(4.6) \qquad (q_2 - q_1)q_1^{\,s} \geq (1 - q_2)\left(\frac{v_2}{2v_1} - 1\right).$$

Now if $q_1 \geq q_2$ the left-hand side (LHS) is nonpositive and increasing in s. The right-hand side (RHS) nay be positive or negative, but it is constant. It follows that if (4.61 holds at a particular vaiue of s, it also holds at s + 1, i.e.,

$$(4.7) \qquad V_{s+1} \geq V_s \Rightarrow V_{s+2} \geq V_{s+1}.$$

(4.7) tells us that the problem of maximizing V_s in Case 1 is nonconvex. An implication is that an interior value of s is never (uniquely) optimal (if $0 < \hat{s} < n/2$ maximizes V_s, then $V_{\hat{s}} \geq V_{\hat{s}-1}$ and so, from (4.6), $V_{n/2} \geq V_{\hat{s}}$, which is a contradiction). Hence, in Case 1, we have a corner solution: s = 0 or s = n/2.

To see which corner is better, we compare

$$V_0 = [1 - (1 - p_2)^n]v_2 = (1 - q_2^{n/2})v_2$$

and

$$V_{n/2} = 2[1 - (1 - p_1)^{n/2}]v_1 = (1 - q_1^{n/2})2v_1.$$

It follows that, if $v_2 \geq 2v_1$, $s = 0$ is optimal (since $q_1 \geq q_2$); while, if $v_2 < 2v_1$, $s = n/2$ is optimal for large enough n (since $q_2^{n/2}$, $q_1^{n/2} \to 0$ as $n \to \infty$).

The results for Case 1 are summarized in Result 2.

Result 2. Assume $m = 2$; the assets are symmetric; n is even; $p_1 > p_2$; and $q_1 \geq q_2$, where $q_1 = 1 - p_1$ and $q_2 = (1 - p_2)^2$. Then if $v_2 \geq 2v_1$, it is optimal to have n coordinators on the two assets (a completely centralized firm). On the other hand, if $v_2 < 2v_1$, then, for large enough n, it is optimal to have $n/2$ specialists on each asset (two independent firms).

The intuition behind Result 2 is straightforward. If $v_2 \geq 2v_1$, coordination adds value. Since $q_1 \geq q_2$ (i.e., p_2 is not much smaller than p_1), the expected return from coordination is also quite large. It is therefore not surprising that all individuals will be assigned to the task of coordination. On the other hand, if $v_2 < 2v_1$, two specialists create more value than one coordinator, conditional on all of them having ideas.. Moreover, when n is large, the probability that at least one of n coordinators is active, or that at least one of $n/2$ specialists is active, is close to 1. So value, conditional on having an idea, is the only thing that matters. It follows that specialization is better when $v_2 < 2v_1$.

We turn next to the case $q_1 < q_2$.

Case 2: $q_1 < q_2$

In Case 2 the LHS of (4.6) is positive and decreasing in s, while the RHS is constant. It follows that, if (4.6) fails to hold for s = 0, it fails to hold for all s. Hence $(q_2 - q_1) \leq (1 - q_2)\left(\frac{v_2}{2v_1} - 1\right)$ is a sufficient condition for $V_{s+1} \leq V_s$ everywhere. In other words, if

$$v_2 \geq 2v_1 \left(\frac{1 - q_1}{1 - q_2}\right), \tag{4.8}$$

then V_s is maximized at s = 0.

At the other extreme, if $v_2 \leq 2v_1$, the RHS of (4.6) is nonpositive and so the LHS > RHS for all s. In this situation $V_{s+1} \leq V_s$ for all s, i.e., s = n/2 is optimal.

The interesting parameter range in Case 2 is where

$$2 < \frac{v_2}{v_1} < 2\left(\frac{1 - q_1}{1 - q_2}\right). \tag{4.9}$$

In this range, $V_{s+1} > V_s$ for small s, and $V_{s+1} < V_s$ for large s. To be more precise, when (4.9) holds, the problem of maximizing V is convex and there is an interior solution (for large enough n) characterized by the "first order condition": $V_{s-1} \leq V_s > V_{s+1}$. Write the optimal value as $\bar{s}$. From (4.51, $\bar{s}$ is given by the smallest integer greater than or equal to the solution x of

$$q_1^{x} = \frac{(1 - q_2)}{(q_2 - q_1)}\left(\frac{v_2}{2v_1} - 1\right). \tag{4.10}$$