

Harvard University
Department of Economics

**Economics 2010b: Final Examination and
General Examination in Microeconomic Theory**

Spring Term 2006

1. You have **FOUR** hours.
2. Answer all questions

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE
QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.
PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**

Question (60 Points)

1. Consider a city where land area differs only by distance to the city center and everyone commutes into the city center. As a reminder, you will receive fewer points if you work with specific functional forms or if you assume that there are only two levels of income, but you may still find it worthwhile to make those sort of assumptions.

a. If people are homogenous and lot sizes are fixed, what condition will characterize the relationship between distance from the city center and housing costs? (5 points)

b. If people are homogeneous and lot sizes vary, what condition will characterize the relationship between distance from the city center and housing costs? What condition determines the relationship between lot size and distance from the city center (5 points)

c. If people differ by income and lot sizes are fixed, under what condition will richer people live further from the city? What will the presence of income heterogeneity do to the relationship between housing costs and distance to the center? (5 points)

d. If people differ by income and lot sizes are flexible, what condition will ensure that income rises monotonically with distance from the city center? What condition will ensure that income will fall monotonically from the city center? (10 points).

e. Now allow there to be two modes of transportation. One of which is time-intensive, and the other of which is cash intensive (cars vs. buses, perhaps). If people are homogeneous and lot sizes are flexible, how will this change the relationship between prices and distance and between lot size and distance? (5 points)

f. If people are heterogeneous in income, and there are two transport modes, then under what conditions will the relationship between income and distance from the city be non-monotonic? (10 points)

g. Now assume that everyone is homogeneous, but assume that income rises, what will happen to the distribution of prices within the city? Under what condition will the population become more or less compact as income rises? (10 points)

h. What will be the impact of rising income inequality on prices and land area? Under what circumstance are the poor hurt by rising incomes of the rich? Here you may well need to assume that there are only two levels of income (10 points).

Consider a two-player, first-price, sealed-bid auction: Each bidder submits a sealed bid b_i ; the highest bidder gets the good and pays his price. (If there is a tie, each player wins with probability 1/2)

Suppose that each bidder i has value θ_i for the object, and that the θ_i are i.i.d., with a continuous, strictly increasing cumulative distribution function F on an interval $[0, \bar{\theta}]$.

Suppose there is a symmetric equilibrium: all players use the same function β to determine their bid as a function of their type.

Suppose also that this bid function is continuously differentiable and strictly increasing.

- a) In this equilibrium, given a type profile $\bar{\theta} = (\theta_1, \theta_2)$, which player wins the object?
- b) What is the expected payoff of a type θ_i as function of his bid?
- b) Write down the first-order condition for type θ_i 's maximization problem.
- c) Use this to derive a differential equation that relates β' and β .
- d) What is the boundary condition for this equation?
- e) What is the solution?

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1. (i) Explain how the concept of a production set can be extended to uncertainty using the idea of a state-contingent commodity. (Give examples.) Is profit maximization well defined under your formulation?

(ii) Consider a two-person exchange economy under uncertainty. There are two dates, 0 and 1, one good, and three states of the world. Consumption occurs only at date 1. Each consumer has a Bernoulli utility function given by $\log x$ and the probability of each state is $1/3$. The endowments of the consumers are $(4, 1, 4)$, $(2, 5, 2)$, respectively.

(a) Compute the Arrow-Debreu (complete markets) equilibrium for this economy.

(b) Suppose now that markets are incomplete and only two securities can be traded at date 0: $(1, 0, 0)$ and $(0, 1, 0)$. What is the new competitive equilibrium? [Short sales are permitted.]

(c) How would your answer to (ii) change if the only two securities available at date 0 were (a) $(1, 1, 0)$, $(1, 2, 0)$; (b) $(1, 1, 0)$, $(\frac{1}{2}, \frac{1}{2}, 0)$?

Microeconomics General Exam
Spring 2006
Part D

D1) *Core and Shapley Value (10 minutes)*

Give an example of a three-person cooperative game in which the core is non-empty and the Shapley value is not in the core.

D2) *Taking Turns (15 minutes)*

There are n individuals and m alternatives, with $m > n > 2$. The domain for preferences is the set of all possible n strict orders over the m alternatives. Consider the following method of constructing a social welfare function, called *Taking Turns*:

Player 1's favorite alternative is declared to be the socially best alternative. This alternative is then deleted from further consideration, leaving $m - 1$ others. Next player 2's favorite alternative from among these $m - 1$ is declared to be the socially second alternative. It is deleted, leaving $m - 2$. In each step the next player in numbered order determines the next alternative in the social order. When all n players have had a chance, and n alternatives have been ranked, the choice reverts to player 1 whose best alternative among those remaining becomes the $n + 1^{st}$ in the social order, etc.

a) Does the Taking Turns procedure satisfy the Paretian axiom of Arrow's Theorem? Explain.

b) Does the Taking Turns procedure satisfy the Pairwise Independence (or Independence of Irrelevant Alternative axiom of Arrow's Theorem? Explain, giving a proof if you think it does or a counterexample if you think it does not.

D3) *Allocating a set of indivisible objects (20 minutes)*

Two people have to allocate two indivisible objects. It is possible either to assign an object to a person or to allocate it randomly between them with any specified probability.

Preferences are as follows: There is no money or other transferable utility. For $i = 1, 2$ person 1's utility for object i is a_i ; person 2's utility for object i is b_i . If a randomized assignment is made and an object j is assigned to player 1 with probability π , the utilities are πa_j and $(1 - \pi)b_j$ respectively. The utility of receiving no objects is thus normalized to zero. If a player gets more than one object, the utilities are added. Thus, in general, if the objects are allocated so that player 1's probabilities of receiving them are (π_1, π_2) the resulting utilities are $(\pi_1 a_1 + \pi_2 a_2, (1 - \pi_1)b_1 + (1 - \pi_2)b_2)$.

This question is about Normative Bargaining Theory.

The status quo (or threat) point is that no one receives either of the objects and both utilities are zero. The set of utilities that can be achieved are those associated with all possible assignments of objects to people, including randomized assignments, as described above. There is perfect information, summarized by the four parameters (a_1, a_2, b_1, b_2) .

Below is a description of a normative bargaining solution called *Method P*. Method P is a way of coming up with an allocation of the objects based on (a_1, a_2, b_1, b_2) . The question concerns the normative properties that Method P may or may not have:

Description of Method P:

Method P proceeds by imagining an artificially constructed general equilibrium economy in which the two players behave as perfect competitors on the market where objects, and probability shares of objects, are traded. The allocation that would result in this equilibrium is taken as the normative bargaining solution.

In this artificial GE system, the objects are initially owned by a fictitious player 0 who has no utility for them and will supply them inelastically to the competitive market for any price. The two people buy objects on this competitive market, or probability shares of objects, at prices p_i . That is, a purchase of all of object i costs p_i , and a π probability share costs πp_i . In an equilibrium, each object must be demanded by only one person, or if both people demand it, the probability shares demanded must sum to one.

The two people are each given incomes in money of $y = \frac{1}{2}(\sqrt{a_1 b_1} + \sqrt{a_2 b_2})$. They have no utility for this money and will use all of it to buy the two objects.

Observe that following prices are equilibrium prices in this artificial GE economy: $p_i = \sqrt{a_i b_i}$. The reason is as follows: People will allocate their income in such a way as to maximize their utility. Player 1 spends her money initially on the object that has the higher a_i/b_i (since 1's utility per dollar spent on i is $\sqrt{\frac{a_i}{b_i}}$). If player 1's income, y exceeds p_i then player 1 will spend the

remaining income on the other object. Likewise, player 2, spends first on the object with the lower a_i/b_i (since 2's utility per dollar spent is $\sqrt{\frac{b_i}{a_i}}$). The system will be in a general equilibrium because each player will be maximizing utility at the given prices and incomes and all goods will be purchased by one or the other.

To help you understand how this bargaining solution works, consider the following example: If $(a_1, a_2, b_1, b_2) = (1, 1, 1, 4)$, the incomes are $y = \frac{3}{2}$, and equilibrium prices are $(p_1, p_2) = (1, 2)$. Player 1 chooses to allocate her income to buy all of object 1 and buys a probability share of object 2 using the remaining income, which results in a probability of $(\frac{3}{2} - 1)/2 = \frac{1}{4}$. Player 2 choose to buy only object 2, but his income is only enough to buy a probability share of it: probability $\frac{3}{4} = (\frac{y}{p_2} = \frac{3}{2}/2)$. Notice that, by construction (due to Walras Law) the total of the probability shares in object 2 is 1. The resulting allocation is described by player 1's probability shares $(\pi_1, \pi_2) = (1, \frac{1}{4})$ which produces the utilities $(1\frac{1}{4}, 3)$ for the two players.

In summary, Method P is the normative bargaining solution that maps the utility possibility sets that are generated by the utilities for the two objects into the allocation resulting from this general equilibrium, and via the utility functions into the resulting point in the utility possibility set.

a) What does it mean for a normative bargaining solution to be invariant to a transformation of the scale of the individual utilities? Why is this thought to be desirable?

b) Over the domain of object allocation problems with two individuals and two objects, with preferences as described above, does Method P have this property? Prove it or give a counterexample.

c) What does it mean for a normative bargaining solution to satisfy Independence of Irrelevant Alternatives? Why is this thought to be desirable?

d) Does Method P have this property over the domain described above? Prove it or give a counterexample.

D4) *Allocating an innovation (30 minutes)*

Three firms have been engaged in a research joint venture that will produce a patentable innovation. Once it is made, one and only one of these firms can obtain the license for further development of the innovation, which will produce an increase in that firm's profits in the amount θ_i , for $i = 1, 2, 3$. The valuations θ_i are independently and identically distributed as uniform random variables on $[0, 1]$. The firms will observe only their own θ_i after the innovation has been made. They will have no information about it until then.

Before the innovation is achieved the firms agree that they want to obtain the maximum possible expected profit from their joint venture. To do that, they agree that they should find a way to allocate the license to the firm with the highest θ_i . They agree to hire an auctioneer who will run an auction for the license in which the three of them are the only bidders and the license is sure to be purchased by the firm with the highest θ_i . Because there are a large number of perfectly competitive risk neutral auctioneers available they know that they can charge the auctioneer a fee equal to the expected revenue that the auction will generate. (The auctioneer shares their belief that the θ_i are i.i.d. uniform and uses this information in calculating how much to pay for the right to run the auction.)

The total profit that each firm receives is thus the sum of (i) one third of the auctioneer's up-front payment, which is non-stochastic, and (ii) the random payoff that the firm receives as a result of participating in the auction itself.

a) What will be the conditional expected payoff from participating in the auction for a firm once it learns its value is θ_i but before participating in the auction?

b) How much will the auctioneer pay to run an auction whose outcome is that the license always goes to the firm with the highest θ_i ?

c) Define the *total conditional expected payoff* (including both the share in the auctioneer's fee and the value of participating in the auction) to be $\pi(\theta_i)$. Suppose that the firms contemplate this total conditional expected payoff function before the θ 's are determined. They decide, prospectively, that $\pi(0)$ is too low. They feel that it is unfair that a firm that has contributed to the research joint venture, and just happens to be unable to make use of the resulting innovation itself, should get an ex post payoff of only one-third of what the auctioneer will pay. In order to increase $\pi(0)$ they decide that they should ask the auctioneer to bid for the right to use an auction method that would not necessarily allocate the good to the highest evaluator (e.g. something other than a first-price or second-price auction) recognizing that $\pi(\theta)$ may well have to be lower for other values of θ . Can $\pi(0)$ be increased in this way? If so, give an example of a method that will do it. (You need only state what method the auctioneer should use and indicate why it would lead to a higher $\pi(0)$. You need not calculate exactly what the new $\pi(0)$ would be.)