Harvard University
Department of Economics

Economics 2010b: Final Examination and
General Examination in Microeconomic Theory

Spring Term 2003

1. You have **FOUR** hours.
2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Question 1

A large number of identical consumers are born at time 0 and have initial wealth w. They do not consume in period 0. They will all live and consume in period 1. Some will die before reaching period 2, others will live and consume in period 2 and then die.

In period 0 all consumers think that their probability of living through period 2 is π. In period 1, before they have to make any consumption or investment decisions (which will be described shortly), they receive one of two "signals" about their future longevity. A fraction, α get good health news which implies that the survival probability is $\pi_H > \pi$, the remaining $1-\alpha$ get bad health news which implies a survival probability of $\pi_L < \pi$. (note that $\pi = \alpha\pi_H + (1-\alpha)\pi_L$.

Assume that the consumers have no bequest motive and that they are von Neumann-Morgenstern utility maximizers. Their objective function is $u(c_1) + u(c_2^a)$, if they remain alive through period 2, and $u(c_1)$ if they do not.

This question considers several different structures for markets for annuities. Annuities are contracts that payoff one unit of consumption in the event that the consumer in alive in period 2. If the consumer dies between period 1 and period 2, any annuities he or she has purchased are valueless.

a) Assume that the consumers can save at both time 0 and time 1, but cannot buy any annuities at either date. The real rate of interest is zero; any wealth saved is simply available again in the next period. Wealth saved in time 1 for use in time 2 will be wasted if the consumer dies. Since there is no consumption at time 0, and no active annuities market at that date, they will all save their entire wealth w into time 1. Write the first order condition that determines how much of their wealth will be consumed in time 1. Discuss the comparative statics of savings at time 1 with respect to changes in π_H and π_L, holding α and π constant.

b) Now assume that there is an active market for annuities at time 1. Annuities are offered competively by companies that must break even in equilibrium. These companies cannot see the information that the consumers have. The consumers cannot credibly disclose this information to the companies. Moreover, companies issuing annuities cannot observe the total purchases of annuities an individual, and therefore they cannot vary the price at which any given annuity contract is sold as a function of the total annuities that the person holds.

Write the first-order conditions to determine how much is saved at time 1 and how much is invested in annuities, by consumers with each of the two pieces of information. Show that the equilibrium price of annuities is greater than π and less than π_H.

c) Now suppose that the annuity market is open at time 0, but not at time 1. Since everyone has the same (null) information at time 0, there is no adverse selection in the annuity market at this date. What is the equilibrium price of annuities at time 0 given this structure of active markets. Write the first-order condition for optimal savings and annuity purchase at time 0. Show that all consumers are better off in the equilibrium with annuity markets open at time 0 only than they are in the equilibrium with markets open at time 1 only. Explain.

d) Finally, suppose that markets for annuities are open at time 0 and at time 1. As in part b), assume that the quantity of annuities purchased by any one consumer is not observable. However, the companies can observe whether the consumer wants to buy more annuities or to cancel some already purchased. Thus the company can quote different prices for these transactions at time 1. The price is q_b for those who want to buy more annuities than they originally bought at time 0, and a different price, q_c is received by those who want to cancel some of their originally purchased annuities and receive a cash refund which they can consume at time 1. What is the equilibrium price of an annuity purchased at time 0? What are the prices q_b and q_c?

e) Show that in any equilibrium of the type found in part d), no annuities are actually traded at time 1.

Question 2

2. Consider the following two player, infinite-horizon game of perfect information. The first period is period 1. Player 1 moves in odd-numbered periods, player 2 moves in even ones. Each period, the player moving chooses whether to "Stop" or "Continue"; once a player chooses "Stop" the game ends. If the game ends in period t, t odd, player 1's payoff is $\delta^{t-1}a$ and player 2's payoff is $\delta^{t-1}b$; if t is even, then player 1's payoff is $\delta^{t-1}b$ and player 2's payoff is $\delta^{t-1}a$, and if no player ever chooses to stop, the payoffs are (0,0). Assume that $0 < \delta < 1$.

a) Prove that this game is continuous at infinity.

Characterize the set of *pure* subgame-perfect equilibria (SPE) for the following regions of the parameter space. (Some partial credit will be given for finding one SPE for each region, and substantial partial credit will be given for correctly stating the SPE sets without offering a proof.)

b) $a > 0, a > \delta b$

c) $a < 0, a < \delta b$

d) $a > 0, a < \delta b$

e) $a < 0, a > \delta b$

Question 3

Answer all parts of the question, showing your work clearly. If you need to make any additional assumptions, state them clearly. Be concise.

1. Consider an exchange economy in which there is a single physical commodity, two states of the world, θ_1 and θ_2, and two agents, $i = 1, 2$. The agents are von Neumann-Morgenstern expected utility maximizers, and agent i has utility function $u_i : R \rightarrow R$. We denote a state contingent allocation in this economy by (x^1, x^2), where $x^i = (x_1^i, x_2^i)$ denotes agent i's consumption of the commodity in states θ_1 and θ_2 respectively. We also denote the aggregate initial endowment of the commodity in state i by w_i, $i = 1, 2$.

a. Suppose that the aggregate quantity of the commodity is the same in state 1 and in state 2, and that the probability of state θ_1 is p. What is the competitive equilibrium price?

b. Suppose u_i is strictly concave for $i = 1, 2$ and that the aggregate quantity of the commodity is greater in state 1 than in state 2, that is $w_1 > w_2$, and that the probability of state θ_1 is p. Show that for any Pareto efficient allocation (x^1, x^2), $x_1^i > x_2^i$.

c. Suppose as in the previous question that the aggregate quantity of the commodity is greater in state 1 than in state 2, and that the probability of state θ_1 is p. Suppose further that the agents' initial endowments are $w^1 = (1, 0)$ and $w^2 = (0, 2)$. Lastly, suppose that $u_1(x) = x$ and that $u'' < 0$ and that the agents trade from their initial endowments to a competitive equilibrium allocation. Explain concisely which of the two agents will benefit more from this trade and why.

d. Suppose that the aggregate quantity of the commodity is the same in state 1 and in state 2, but that the agents disagree about the probability of state 1. Agent i believes the probability is p_i, and that $p_1 > p_2$. Suppose we consider the Pareto efficient allocations for the preferences based on these different probabilities. What can you say about the the quantities of the commodity that agent 1 consumes in the two states at a Pareto efficient allocation?

e. Suppose that the two states are equally likely and that $u_i(x) = \ln x$, $i = 1, 2$ Suppose also that the agents' initial endowments are $w^1 = (1, 0)$ and $w^2 = (0, 2)$. What is the core of the economy?

Question 4

Micro General

1. Consider a hidden action model with a risk neutral principle and a risk averse manager. The manager selects non-negative effort level e ($e \in R_+$). Conditional on effort level e the realization of profit π is normally distributed with mean e and variance σ^2 that is $\pi\tilde{}N(e, \sigma^2)$. (To make your life easier we do not specify manager's utility as a function of effort and wage.) Suppose the manager's preferences are defined over mean and variance of his income w and his effort level e as follows: Expected utility=$E[w]$-$k \cdot Var[W]$ -$g(e)$. where $g(0) = g'(0) = 0$, $\lim g'(e)_{e \to \infty} = \infty$ and for $e > 0$ all derivatives of $g(.)$ are positive. The payoff of the principle is equal to his profit minus the wage he pays to the manager. The manager has an opportunity to refuse a principle job offer and receive a reservation utility of zero.

(a) Restrict attention to linear compensation schemes $w(\pi) = \alpha + \beta\pi$. Show that the manager's expected utility given $w(\pi)$, e, and σ^2 is given by $\alpha + \beta e - k\beta^2\sigma^2 - g(e)$.

(b) Derive the optimal contract when e is observable.

(c) Derive the optimal linear compensation scheme when e is not observable. What effects do changes in k and σ^2 have.

(d) Now consider a modification of the above problem. There are N risk neutral managers. Each manager chooses effort level $e_i \in R_+$ $i = 1..N$. All managers and the principle are risk neutral. The utility of $i - th$ manager is $E[w_i] - g(e_i)$. The profit of the firm is normally distributed with mean $\sum_{i=1}^{N} e_i$ and variance σ^2 thus $\pi \sim N(\sum_{i=1}^{N} e_i, \sigma^2)$. The utility of the principle is $\pi - \sum_{i=1}^{N} w_i$. The principle can only offer managers wage contracts that depend on profit of the firm. Is it true that the expected utility of the principle in this problem is more than N times greater than the expected utility of the principle in part (c) of this question? Explain.

2. Consider independent private value environment. There are two bidders and one object available for sale. The private values of both bidders are drawn independently from uniform distribution U[0,100]. Denote by V_i the private value of bidder i, $i = 1, 2$. The rules of the auction are somewhat unusual. We consider an auction that looks like a standard open cry English auction with a twist, no bids over 95 are allowed. The first bidder to bid 95 wins the auction (and pays 95 for the object). If nobody bids 95 the bidder with the highest bid under 95 wins the auction. (You can imagine that bidders look at the dial where the price starts at zero and increases to 95 in ten minutes. Each bidder has two buttons in front of him, one button says "in" the other says "95". All bidders start holding the "in" button, if a bidder releases the "in" button it means that he is no longer willing to pay the price on the dial, thus the first bidder to release the "in" button looses the object and pays zero, the other bidder pays the price that was on the dial when the loosing bidder dropped out. A bidder may press "95" button while holding the "in" button. The first bidder to press "95" button wins the object and pays 95 for it. In case of two bidders pressing "95" simultaneously the winner is determined by a coin toss. The loosing bidder pays nothing.)

a) Does the auction mechanism described above have a dominant strategy equilibrium. Explain.

b) Formulate the revenue equivalence theorem.

c) Assume that the above described mechanism has a symmetric equilibrium. Can you use the revenue equivalence theorem to compute the expected revenues in this auction? If yes compute the revenues in this auction, if your answer is no compare the revenue of this auction mechanism with an efficient second price auction, which auction gives greater revenues?

d) Suppose bidder one has value 100 for the object (of course this is his private information). In equilibrium of this auction what is the probability that this bidder wins the object? What is the expected price that this bidder pays for the object?

e) Find a symmetric equilibrium of the mechanism described above (or prove that it does not exist).

f) Consider a modification of the mechanism described above. The modified mechanism has two distinctions first, the maximum bid is now 10 (far lower than 95). In case both bidders jump to 10 simultaneously the winner is determined by a coin flip. The second distinction is that the winning bidder can re-sell the object to the loosing bidder if they can agree on price (of course, to fully describe the mechanism we must describe the rules of the bargaining round). Can you design the rules of bargaining round such that the object will end up in the hands of the bidder who values it the most. If your answer is no explain why not, if the answer is yes design such a bargaining game.

3. (This question has the same weight as other questions however it will probably take you less time).

a) Formulate Arrow's impossibility theorem.

b) Show that if assumptions of Arrow's impossibility theorem are satisfied than there exists an extremely "extremely pivotal" voter in a sense that for some profile he can move alternative x from the bottom to the top of SWF ordering. (If you would like to use Arrow's impossibility theorem to answer this question you must prove it here).

c) Consider the following SWF that aggregate preferences of N players. The SWF depends only on preferences orderings of agents n and n-1. If alternative ranked second by agent n is also ranked second by agent n-1 than the social preferences are the same as preference of agent n. Otherwise the social preferences are the same as preferences of agent n-1. What are the assumptions of Arrow's impossibility theorem violated by this preference aggregation rule?

d) Consider a world populated by an odd number of agents who have strict preferences over alternatives. Does a Condorset winner always exist? If yes prove it, if no give an example. (Alternative x is a Condorcet winner iff for any alternative y no more than 50% of agents prefer y to x.)

e) Suppose the preferences of agents in the society are such that alternative x is the Condorcet winner. Does this mean that alternative x is ranked first by Borda count. If your answer is yes prove it, if your answer is no provide an example where this is not the case. (Borda count is defined as following: Each of N alternatives alternative is ranked by each agent 1...N, the score of an alternative is the sum of its ranks. Alternatives are ordered according to scores. The alternative with the lowest score is the most preferred.)