Harvard University
Department of Economics

General Examination in Microeconomic Theory

Spring 2010

1. You have **FOUR** hours.
   Part A: 55 minutes
   Part B: 55 minutes
   Part C: 60 minutes
   Part D: 70 minutes

2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Part A: Verbal Problem (Glaeser)

For credit you must give a mathematical answer or justification.

(1) A firm can invest in producing a new product. What is the socially optimal amount of investment? (5 points)

(2) How much investment will be produced by giving the firm a temporary monopoly? How does this compare to the first-best? What is the socially optimal length of the monopoly? (15 points)

(3) Compare the temporary-monopoly strategy with a more explicit reward-for-innovation policy, keeping in mind that the government must finance this policy. How could such a policy be designed? What would be the natural shortcomings of such a policy? Remember to provide a formal treatment. (10 points)

(4) Now assume that there are multiple firms all in the same industry. How would your results from (1)-(3) differ with the degree of competition in the industry? (25 points)

Consider the following game with observed actions and incomplete information. There are three players: the Red Sox, the Yankees, and a baseball player we'll call "M." Each player has initial private information, which corresponds to their "type." . For the Red Sox and Yankees the private information is their willingness to hire M, denoted by v_R and v_Y; these values are independent and both have the uniform distribution on $[80, 120]$. M's private information is the payoff r that he will get from declining the offer (and staying in Japan.) Assume that r is uniformly distributed on $[40, 60]$. The extensive form is as follows:

1. In the first stage, the Red Sox and Yankees simultaneously choose bids b_R and b_Y.

2. In the second stage, the team that made the higher bid in the first stage chooses a wage w to offer M. The team that made the lower bid does not get to make an offer.

3. In the third stage, M says yes or no.

If M says no then the Red Sox and Yankees both get payoffs of zero and M gets r.

If M says yes, then the winning bidder i gets $v_i - b_i - w$, Matsuzaka gets w, and the losing bidder gets zero.

(Note that the winning bid b_i from stage 2 is not paid to M.)

(a) In a perfect Bayesian equilibrium, each player has beliefs about the types of the others, and the beliefs need to satisfy certain consistency conditions. What do those conditions require here?

(b) Consider the final stage of the game in which M must say yes or no. What will M's stategy be in a perfect Bayesian equilibrium?

(c) Suppose that team i has value v_i and wins the first stage with a bid of b_i. What will it bid in the second stage in a perfect Bayesian equilibrium? What expected payoff does this bid yield?

(d) Now consider the full game. Using the result of parts (a) and (b) write down the conditions that the first stage equilibrium bids $b_R^*(v_R)$ and $b_Y^*(v_Y)$ must satisfy. Assuming that the equilibrium is symmetric and that the optimal bid is always the solution to the appropriate first-order condition, derive a differential equation that you could solve to find $b^*(v)$.

e) Without actually solving the equation, what comments can you make about the properties of the solution?

Micro Generals – Spring
Oliver Hart

1.

(a) A standard assumption of general equilibrium theory is that firms maximize profits under perfect competition. Justify this assumption. Do you think that this assumption is reasonable under imperfect competition? Do you think that it is reasonable when there is uncertainty about the state of the world?

(b) Consider a two good exchange economy with three consumers. The aggregate endowment of each good is 1. Consumers 1 and 2 each have utility function $x_1^2 + x_2^2$ (and will always be treated equally in what follows). Consumer 3 has utility function $x_1^{\frac{1}{2}} x_2^{\frac{1}{2}}$. For a given utility level $\overline{U}$ for consumers 1 and 2, what is the utility-maximizing allocation for consumer 3? (You can derive this graphically or mathematically.) Use this to determine the set of Pareto optima for this economy (when consumers 1 and 2 are treated equally). What is the Walrasian equilibrium (or equilibria) if consumer 1 and 2's initial endowment is $\left(\frac{1}{4}, \frac{1}{4}\right)$ and consumer 3's is $\left(\frac{1}{2}, \frac{1}{2}\right)$?

2.

(a) A result in the property rights theory of the firm is that joint ownership of an asset is suboptimal. Explain this result. Can you think of cases where it will no longer hold?

(b) Two workers invest in a venture. Worker i can acquire a skill that is worth v_i at cost $\left(\frac{1}{2\theta_i}\right)v_i^2$, i=A,B. The ex post value of the venture is $v_A + v_B$ if both parties participate. However, each worker can quit ex post; in the event of a quit the worker who owns the asset (venture) gets his v and the other worker gets nothing . A threat to quit triggers renegotiation with a 50:50 division of surplus. There is symmetric information throughout but acquiring a skill is not verifiable and so cannot be contracted on. Compute the equilibrium for the case where worker A or worker B owns the venture (asset). Which arrangement is better in terms of generating net surplus? In this example would you describe a worker's investment as asset-specific? Person-specific?

Part D

Question D1 (24 points)

The relation in this table T is defined by xTy if x has a strict majority over y in a binary vote. xTy is denoted by a 1 in the (x, y) cell of the table.

$x \backslash y$	1	2	3	4	5	6
1	$-$	1	0	0	1	1
2	0	$-$	1	1	0	1
3	1	0	$-$	1	1	0
4	1	0	0	$-$	1	1
5	0	1	0	0	$-$	1
6	0	0	1	0	0	$-$

a) What is the top cycle of T? Define it and compute it for this example. (**6**)

b) Why might we be interested in the top cycle of the majority relation? (i.e. state any interesting theorem or property that applies to it) **(6)**

c) What is the uncovered set of T? Define it and compute it for this example. (**6**)

d) Why might we be interested in the uncovered set of the majority relation? (i.e. state any interesting theorem or property that applies to it) **(6)**

Question D2 (21 points)

There are two technologies for producing an output z that is used for consumption using inputs (x, y). Technology A produces output according to $z = 10x + 2y$. Technology B requires a fixed cost of **four million** units of y and has the output function $z = 5x + 10(y - 3 \times 10^6)$. There is a population of **30 million** people, each of whom desires only the output z.. Half of them have a unit of x in their endowment. The other half have a unit of y. Any group of people can choose between these these technologies, operating **only one of them**, so as to produce the largest amount of output they can from their collective endowment. The question is how should they divide the output that they produce.

They decide to use the Shapley value of the cooperative game where $v(S)$ is the highest output that S can produce.

a) Explain the formula for the Shapley value. (**6**)

b) Using a "large numbers" approximation in this model with 30 million players, compute the payoffs in units of output for the owners of the two inputs. **(15)**

Question D3 (25 points)

A central authority can produce a good that is privately consumed. This good can exist in any quality $q > 0$. Quality is perfectly observable by the consumers and the central authority sells to them by quoting a price schedule $p(q)$. Each consumer wants at most one indivisible unit of the good. Each consumer decides which quality, if any, to buy. Consumer's differ according to their "type" θ which is a parameter of their utility. Assume that $\theta \in [\theta_{\min}, \theta_{\max}]$. A consumer of type θ who buys one unit at quality q and pays $p(q)$ for it, as required by the pricing schedule $p(\cdot)$, gets the untilty

$$u(q; p(\cdot), \theta) = \theta q - p(q)$$

A consumer who chooses not to purchase at all gets $u = 0$ no matter what their value of θ is.

The parameter θ is not observable by the central authority.

a) The central authority can select any pricing schedule $p(\cdot)$. Let $W(\theta)$ be the realized utility level for type θ when they optimize $u(q; p(\cdot), \theta)$ – the best utility that they can achieve by a choice of q or by choosing not to purchase the good. Show that the realized utility levels for the consumers $W(\theta)$ must be an increasing convex function of θ. (**10**)

b) Write the formula for the $p(\cdot)$ that will generate a given increasing convex function W. Show that any increasing convex function W can be achieved by a suitable choice of $p(\cdot)$. (**5**)

c) Now we will complete the model by discussing the cost of producing the good and the central authority's objective function.

Cost of production: (**Increasing marginal cost of quality and constant returns to scale**). Unit cost of production for each quality q is an increasing convex function of q. Total production costs are the sum of these unit costs over all units produced.

Objective function #1 (**Total Surplus**): maximization of total utility of the consumers, **plus** net revenue collected as payments under the price schedule, **minus** the cost of production

Objective function #2 (**Central Authority's profit**): net revenue collected as payments under the price schedule, minus the cost of production.

This question concerns the relationship between the solution to the problem as posed above and the full information solution that would obtain if θ were observable by the central authority. Do not solve the problems. Use only basic economic reasoning and your knowledge of the structure of the solution to parts a) and b).

i) Assume objective function #1. Do these two problems have the same solution? (**5**)

ii) Assume objective function #2. Do these two problems have the same solution? (**5**)