

Harvard University
Department of Economics

General Examination in Microeconomic Theory

Fall 2009

1. You have **FOUR** hours.
2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.
PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Glaeser
Fall General 2009

Question:

People live in place a and place b. There is an option to build a rail line between the two places.

- (1). If the rail line is built, what is the optimal price.
- (2) derive conditions for whether building the rail line is optimal if the total number of trips is unaffected by the new line
- (3). Derive the benefits from the line if the trips are not fixed.
- (4).How does your answer change if the populations of the two places are not fixed.
- (5). Under what conditions can property values help you to evaluate the project

Economics 2010a
Final Examination

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Question 2.

a) State the definition of subgame-perfect equilibrium, and give an example of a game with a Nash equilibrium whose outcome is not the outcome of any subgame-perfect equilibrium..

b) State the definition of perfect Bayesian equilibrium of a signalling game, and give an example of a game with a perfect Bayesian equilibrium whose outcome is not the outcome of any subgame-perfect equilibrium.

c) State the definition of sequential equilibrium, and give an example of a game of complete information that has no move by Nature and has a subgame-perfect equilibrium whose outcome is not the outcome of any sequential equilibrium.

d) In a game of perfect information, what is the relationship between the set of outcomes of sequential equilibria and the set of outcomes of subgame-perfect equilibria?

1.(a) Consider a two good exchange economy with a single consumer. The consumer's endowment is $(\omega_1, \omega_2) \gg 0$ and she has strictly convex preferences.

(i) Is there a Walrasian equilibrium? Is the Walrasian equilibrium allocation unique?

(ii) What extra conditions are required for Walrasian equilibrium relative prices to be unique?

(iii) What extra conditions are required for the relative price of good 2 to be decreasing in ω_2 ?

(b) Consider a two good, two consumer exchange economy. Consumer 1 cares only about good 1 and consumer 2 only about good 2. Consumer 1's endowment is (ω_1^1, ω_2^1) and consumer 2's is (ω_1^2, ω_2^2) .

(i) Compute the Walrasian equilibrium for this economy. Is it unique?

(ii) Is the relative price of good 2 decreasing in the aggregate endowment of good 2?

2. (a) Consider the following principal-agent problem. The agent can choose action a_H or a_L . There are three possible outcomes, $q_1 < q_2 < q_3$. Action a_H yields these outcomes with probabilities $0, 1/6, 5/6$, respectively. Action a_L yields these outcomes with probabilities $1/3, 1/3, 1/3$, respectively. The principal is risk neutral and the agent is risk averse with no wealth constraints. The principal cannot observe the agent's action. Show that the first-best can be achieved. What is the form of the optimal incentive scheme?

(b) Suppose now that the agent has utility function I-a (she is risk neutral), but has zero wealth. Her reservation wage is zero. The principal has all the bargaining power. Assume that the principal wants the agent to choose $a_H > a_L$. What is the optimal incentive scheme?

Question D1

Consider the following variant of a second-price auction. There is a single indivisible good to be sold. The valuations of the n bidders are given by independent random variables v_i distributed on an interval. The number of bidders, n , is more than 3. The utilities of the bidders are $v_i + t_i$ if player i gets the good t_i if player i does not get the good where t_i is the monetary transfer given to player i by the auctioneer, and $t_i < 0$ means that player i has to pay money to the auctioneer. Any money collected by the auctioneer is kept by the auctioneer, not returned to the bidders.

The rules are as follows: All bids must be non-negative numbers. The good is given to the bidder with the highest bid. The monetary payments are:

The highest bidder pays the second highest bid to the auctioneer.

Both the highest bidder and the second highest bidder receive a rebate from the auctioneer equal to $1/n$ th of the third highest bid. Thus the highest bidder ends up paying the difference between the second and third highest bids.

The third highest bidder and all those who bid lower than that receive a rebate from the auctioneer equal to $1/n$ th of the second highest bid.

i) Show that each bidder has a dominant strategy in this auction.

ii) Show that the auctioneer will never run a deficit.

iii) Show that any bidder will willingly participate in this auction (where the alternative is non-participation, no chance of winning the good, no payment, and no rebate).

iv) Why do you think that so much attention has been given to the second-price auction instead of this auction design which seems to have all its good properties and returns more of the surplus to the bidders?

Question D2

There is a seller ($i=0$) owning two identical indivisible goods that she can sell. There are two buyers ($i=1,2$). The buyers and the seller each have a willingness to pay $\theta_{\{i\}}$ for a unit of the good that is known only to them, and which everyone believes are independent random variables with a uniform distribution on $[0,1]$. No one is willing to pay anything for more than one unit of the good. (In particular the second unit of the good owned by the seller has a zero value to her.)

a) Describe the efficient allocations of the two units of the good as a function of $(\theta_0, \theta_1, \theta_2)$.

b) Compute the interim expected utilities that can be realized by the three agents by an incentive compatible mechanism that achieves this efficient allocation of the two available units (without regard to the existence of a source or sink for funds, or for participation constraints.). How much flexibility is there in designing these expected utilities as a function of the players types-- that is, what free parameters can still be chosen. Explain.

c) What are the interim participation constraints for the three agents?

d) Is there an incentive compatible way of allocating these two units efficiently while respecting these constraints and not using any outside source or sink for funds? Comment, relative to results discussed in MWG.