

*Harvard University*  
*Department of Economics*

**General Examination in Microeconomic Theory**

*Spring 2008*

1. You have **FOUR** hours.
2. Answer all questions

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.**  
**PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**

Part I - Glaeser

- (1) There are two locations productoland and wasteland. Assume that the two areas have exogenous productivity levels and that productivity in productoland is higher than wasteland.
  - (a) Characterize the spatial equilibrium where a fixed population divides between the two areas. Given conditions for housing prices, wages and population in the two areas.
  - (b) Assume that the national government has to raise a fixed sum of taxes for national defense and it must use a fixed rate proportional tax system. When is it preferable to make housing costs deductible?
  - (c) Now assume that there are only local governments and that they raise tax revenues, again using a fixed-rate proportional tax system, to finance local amenities. Provide the conditions for socially optimal local amenity provision.
  - (d) Consider three different objective functions for local government maximizing total population, maximizing land values and maximizing average income. Write the conditions for local amenity provision that maximize these objective functions. Which local objective function comes closest to providing the socially optimal level of amenities?

Question B1 (15 points)

Consider the following variant of a second-price auction. There is a single indivisible good to be sold. The valuations of the  $n$  bidders are given by independent random variables  $v_i$  distributed on the interval  $[V, 0]$ . The number of bidders,  $n$ , is more than 3. The utilities of the bidders are

$v_i + t_i$  if player  $i$  gets the good  
 $t_i$  if player  $i$  does not get the good  
where  $t_i$  is the monetary transfer given to player  $i$  by the auctioneer, and  $t_i < 0$  means that player  $i$  has to pay money to the auctioneer

Any money collected by the auctioneer is kept by the auctioneer, not returned to the bidders.

The rules are as follows: All bids must be non-negative numbers. The good is given to the bidder with the highest bid. The monetary payments are:

The highest bidder pays the second highest bid to the auctioneer.  
Both the highest bidder and the second highest bidder receive a rebate from the auctioneer equal to  $1/n^{th}$  of the *third* highest bid. Thus the highest bidder ends up paying the difference between the second and third highest bids.  
The third highest bidder and all those who bid lower than that receive a rebate from the auctioneer equal to  $1/n^{th}$  of the *second* highest bid.

- i) Show that each bidder has a dominant strategy in this auction. (5)
- ii) Show that the auctioneer will never run a deficit (2)
- iii) Show that any bidder will willingly participate in this auction (where the alternative is non-participation, no chance of winning the good, no payment, and no rebate. (3)
- iv) Why do you think that so much attention has been given to the second-price auction instead of this auction design which seems to have all its good properties and returns more of the surplus to the bidders? (5)

Question B2 (35 points)

Two players are going to play a simultaneous move game in which each will make one of two moves. The payoffs are given by

|     | $L$      | $R$      |
|-----|----------|----------|
| $T$ | $(x, 9)$ | $(3, 6)$ |
| $B$ | $(6, 0)$ | $(6, 9)$ |

Player 1, the row player, is informed about the value of  $x$  before any decision must be made. Player 2, the column player, does not know  $x$ , but does know the distribution from which  $x$  is drawn, which is common knowledge, and knows that Player 1 will have seen  $x$  before making his choice of row.

i) Let  $x$  have the distribution  $x = 12$  with probability  $2/3$  and  $x = 0$  with probability  $1/3$ . Show that this game has a unique Nash equilibrium. (5)

ii) Let  $x$  have the distribution  $x = 12$  with probability  $p$  and  $x = 0$  with probability  $1 - p$ . For which values of  $p > 2/3$  are there multiple equilibria? (7)

iii) Which of these equilibria does player 1 prefer? Are any of them Pareto ranked? (2)

iv) Assume  $p = 5/6$ . Let us further modify the game as follows:

Suppose that player 1 has a new move,  $E$  (meaning "End the Game"), after seeing the value of  $x$ , but prior to any the choice among any other moves. Move  $E$ , if made, results in a payoff of 10 for player 1, and the game ends. If player 1 does not chose  $E$  then the game is played with both players choosing simultaneous moves ( $T$  or  $B$  and  $L$  or  $R$ ) as above. Player 2 does not observe the value of  $x$ , but does observe whether player 1 has chosen this new move and whether the game has been thereby ended. What are all the Nash equilibria of this game? (12)

v) Are there any Nash equilibria that are not subgame perfect? Are there any that are not Weak Perfect Bayesian Equilibria? (5)

vi) How would you predict that this game is played after Player 1 observes  $x = 12$ . That is, how would you select among the equilibria found above? (4)

C1. Consider the following (private ownership) perfectly competitive economy. There are two consumers with utility functions  $x_1^\alpha x_2^{1-\alpha}$  and  $x_1^\beta x_2^{1-\beta}$ , respectively, where  $x_1$  represents leisure and  $x_2$  consumption and  $0 < \alpha < 1$ ,  $0 < \beta < 1$ . There is also a single firm that has a production function  $q = z^{1/2}$ , where  $z$  is labor input and  $q$  is output of good 2. The aggregate endowment of leisure equals  $A > 0$ . Each consumer owns a fraction of the aggregate endowment and the firm. Consumer 1's share of each is  $\theta$ , and consumer 2's share of each is  $1-\theta$ , where  $0 < \theta < 1$ . (The economy's initial endowment of good 2 is zero; this good has to be produced.)

- a) Show that consumer demand in this economy is equivalent to what would occur if there were a single consumer with "aggregated preferences." Write down this consumer's preferences.
- b) Using this fact or otherwise, compute the Walrasian equilibrium for this economy.

C2. Suppose a buyer and seller can trade a widget at date 1. The value to the buyer is  $i$ , and the seller's cost is zero. Here,  $i$  represents an (unverifiable) investment made by the buyer at date  $\frac{1}{2}$ . The cost of this investment, which is borne entirely by the buyer, is  $\frac{1}{2}i^2$ . No long-term

contracts can be written, and the buyer and seller divide the gains from trade 50:50 at date 1.

- a) What is the first-best or socially optimal value of  $i$ ?
- b) What is the value of  $i$  that the buyer will actually choose?
- c) Suppose that the buyer can obtain value  $\lambda i$  by trading with an alternative seller at date 1, where  $\lambda$  lies between zero and one. How does this change your answer to (b)?
- d) Suppose that there is no alternative seller but you can design the bargaining game at date 1. What game would achieve the first-best?

Part IV has three questions: IV.1, IV.2, IV.3. Answer all three. Question IV.1 is worth 40 points, question IV.2 is worth 20 points, and question IV.3 is worth 10 points.

IV.1: Consider a bilateral trading problem where the seller of a unique object has private information about its quality, which may be described as type H (high) or L (low). The buyer believes that each type has probability  $1/2$ , while the seller knows the actual type. The object would be worth \$300 to the seller or \$450 to the buyer if the seller's type is L, but it would be worth \$700 to the seller or \$1050 to the buyer if the seller's type is H.

(a) Find the trading plan that maximizes the buyer's expected gains from trade, subject to the constraints that each individual should have an incentive to participate honestly.

(b) If the value to the buyer when the seller's type is H were some other  $\pi_H$  different from 1050 (all else as above), what would be the lowest (infimum) value of  $\pi_H$  such that the answer to part (a) would involve a positive probability of trading with type H?

(c) Find the trading plan that is best for the seller among all incentive-compatible plans that are safe for the buyer (in the sense that the buyer has nonnegative expected gains from trade with each type).

(d) If the probability of type H were some other  $p_H$  different from  $1/2$  (with  $p_L = 1 - p_H$ , all else as above), what would be the smallest (infimum) value of  $p_H$  such that the best safe plan from part (c) could be interim Pareto-dominated by a pooling plan?

IV.2: Consider an auction with 3 bidders to buy an object for which they have independent private values drawn from a Uniform distribution on the interval from 2 to 7. Under the rules of this auction, each bidder independently submits a written bid  $\beta$  which must be accompanied by a nonrefundable cash deposit of half of the bid amount,  $0.5\beta$ . The high bidder will win the object, and then the winner will pay the other half of his bid amount, that is, another  $0.5\beta$ . The seller keeps the deposits from all three bidders. Suppose that the bidders will bid according to a symmetric equilibrium, in which each bidder uses the same bidding strategy, which is a strictly increasing function of his private value for the object.

- (a) As a function of bidder  $i$ 's private value  $t_i$  in  $[2, 7]$ , show a formula for computing bidder  $i$ 's conditional expected profit  $U_i(t_i)$  in this auction, given his private value  $t_i$ , when all bidders are expected to apply their equilibrium strategy.
- (b) As a function of  $i$ 's private value  $t_i$  in  $[2, 7]$ , show a formula for his equilibrium bid  $\beta(t_i)$ . (You do not need to simplify this formula. Just make sure that the formula is well defined so that it could be used to compute the bid  $\beta(t_i)$  for any  $t_i$  between 2 and 7.)

IV.3: Suppose that the alternating-offer bargaining game is played when the set of feasible utility allocations is  $\{(x_1, x_2) \mid x_1 + x_2 \leq 1, x_1 \geq 0, x_2 \geq 0\}$ , the threat point is  $(v_1, v_2) = (0, 0)$ , and the probability of going to the threat point after the rejection of any offer is 0.1 (regardless of which player made the offer).

In a stationary subgame-perfect equilibrium of this bargaining game, what offer  $(x_1, x_2)$  would be made by player 1, and what offer  $(y_1, y_2)$  would be made by player 2?