



HARVARD UNIVERSITY
DEPARTMENT OF ECONOMICS

General Examination in Microeconomic Theory

Fall 2010

1. You have **FOUR** hours.
2. Answer all questions

PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.
PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

Verbal Problem

Reminder: For credit you must give an algebraic answer or justification.

Individuals either commit one or zero crimes and there is heterogeneity in the population with regards to propensity to commit a crime.

- (1) Derive comparative statics on the determinants of the amount of crime in society with respect to arrest probability, punishment level, and opportunity cost of time.
- (2) Make assumptions about the social cost of crime and punishment and derive conditions for the optimal punishment holding the arrest probability fixed. Explain your result intuitively.
- (3) Now assume that there are two periods, and that criminals caught in the first period can be incarcerated during the second period (put in jail instead of being punished with a fine or other penalty). Assume that during the period of incarceration, the criminal can neither commit a crime nor be economically productive. Under what conditions is it optimal to incarcerate if an individual's benefit/cost of committing a crime is constant over the two periods? How does your answer change if the individual's propensity to commit a crime changes over time? Explain your result intuitively.
- (4) Can your answer to part three help explain the move from corporal punishment (physical punishment and public humiliation such as putting people in stocks, beating them, etc.) to incapacitation during the 19th century that is discussed by Foucault and others? Provide an interpretation in the context of your model.

(1 hour)

Consider the following signalling game. The sender has two types, $\theta = H, L$. The sender sees her type and choose an action $a_1 \in [0, \infty)$. The buyer sees a_1 but not the seller's type, and chooses either "accept" or "reject." If the buyer accepts, the seller's payoff is a_1 and the buyer's payoff is $v_\theta - a_1$. If the buyer rejects, the seller's payoff is w_θ and the buyer's payoff is 0. There is a common prior probability p that the sender's type is H .

Suppose that $v_H > v_L > w_H > w_L > 0$.

- a) Is there a pooling PBE (perfect Bayesian equilibrium)?
- b) Is there a separating PBE?
- c) What is the set of PBE outcomes?
- d) Is there a Bayesian Nash equilibrium whose outcome is not the outcome of a PBE?

Now suppose that $v_H > w_H > v_L > w_L > 0$.

- e) Is there a pooling PBE?
- f) Is there a separating PBE?

1. Consider a closed economy/country A with m consumers, n firms, and l goods. Preferences and production sets are “well-behaved”. Denote a Walrasian equilibrium for this economy by (p^*, x^*, y^*) .

The economy now opens itself up to free trade with a second economy/country B, consisting of M consumers and N firms (and the same l goods). This second economy is also “well-behaved”. There is a new Walrasian equilibrium (p', x', y') after trade. Show that at least one consumer in country A is better off in the post-trade equilibrium.

You are now asked to show that there exist lump sum transfers within the country A so that every consumer in A is better off in the post-trade equilibrium.

To do this you might want to proceed as follows. For each price vector p , let

$$T_i = px_i^* - (pw_i + \sum_{j=1}^n \theta_{ij} \pi_j)$$

where $\pi_j = \text{Max}_{y_j \in Y_j} py_j$, and the w_i and θ_{ij} denote individual endowments and shareholdings in A. Show that

$$\sum_{i=1}^m T_i \leq 0.$$

Now consider a regime in which consumer i in A receives the lump sum transfer

$$T_i - \frac{1}{m}(T_1 + \dots + T_m).$$

Argue informally that a Walrasian equilibrium for the world economy in the presence of these lump sum transfers will exist. Confirm that the lump sum transfers sum to zero and that each consumer i in A can afford to purchase x_i^* at any prices p (where consumer i receives x_i^* in the pre-trade Walrasian

equilibrium). Conclude that every consumer in A is better off in the post-trade Walrasian equilibrium.

2. Consider a moral hazard situation with a risk neutral principal and a risk averse agent. The agent's utility function is $-\exp[-r(w - \frac{1}{2}ca^2)]$, where w is his wage and a his effort (r and c are positive constants). Profit is given by

$$q = a + \varepsilon$$

where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$. The principal does not see a , but does observe q and another signal z . Here (ε, z) are joint normal; the variance of z is σ_z^2 and the covariance of (ε, z) is $\sigma_{\varepsilon z}$.

The principal offers the agent a linear incentive scheme

$$w = \alpha + \beta q + \gamma z$$

Show how to compute the optimal values of β, γ . Discuss and interpret the conditions under which $\gamma \neq 0$.

Throughout this problem assume that utilities are all quasi-linear in money and all valuations and willingness to pay are expressed in the same monetary units.

Three travelers each have to make a trip from Boston to San Francisco by plane. Their willingnesses to pay for the trip are, \$3500, \$5000, \$6000, respectively. There are two possible ways to travel. They can rent a plane that seats up to two people for \$3000. In that case the third person will not travel. Alternatively they can rent plane with capacity three for \$5000.

- a) What is the core of this game? Is it empty or non-empty?
- b) What is the Shapley value of this game?

Now suppose that they each have to make *two* trips from Boston to San Francisco within a one-year period, but that the dates of their trips are flexible. Per-trip valuations and costs of renting planes are exactly as above. The three players can plan their travel in advance, negotiating with each other about payments for planes, monetary transfers among themselves, and travel arrangements (i.e. who travels with whom).

- c) What is the core of this game? Is it empty or non-empty?
- d) What is the Shapley value of this game?
- e) Comment on the characteristic function of the cooperative game you are using to analyze this problem. How does the value of the coalition of all players relate to the payoff these players can achieve if they all travel together?