

*Harvard University*  
*Department of Economics*

**General Examination in Microeconomic Theory**

*Fall 2006*

1. You have **FOUR** hours.
2. Answer all questions

**PLEASE USE A SEPARATE BLUE BOOK FOR EACH QUESTION AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.**

**PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.**  
**PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.**

Country Alpha imports a lot of oil. Country Omega exports oil. Country Alpha believes that when Country Omega gets richer, they use that money to fund terrorism against Country Alpha.

Write down a formal model of this problem and solve for: (1) The optimal tax on oil, (2) The optimal tariff on imported oil and (3) the optimal subsidy for alternative energy.

First, assume that you can only use one policy and solve explicitly for the optimal taxes and subsidies in your model. (15 Points)

Second, compare the three policies. What should you do if you can use all of them (5 Points).

Are there any other strategies that you can think of that could improve the welfare of country Alpha (5 points).

As always, no points are given for verbal answers without mathematical derivation.

1. a) State the definitions of subgame-perfect equilibrium, sequential equilibrium, and perfect Bayesian equilibrium (PBE). (For PBE you should specialize to two-player games with observed actions and independent types.)
  - b) Give an example of a game with a subgame-perfect equilibrium outcome that is not the outcome of any sequential equilibrium.
  - c) What is the relationship between PBE and sequential equilibrium in signalling games? Why?
  - d) What is the relationship between sequential equilibrium and subgame-perfect equilibrium in finite games of perfect information? Explain.

	$Q_{11}$	$Q_{12}$	$Q_{13}$
$F$	1,1	0,0	-2,0
$U$	1,0	0,1	-2,0
$T$	1,0	0,0	-2,1

	$g$	$b$	$r$
$F$	$\frac{1}{2}$	$\frac{1}{2}$	0
$U$	0	1	0
$T$	$\frac{1}{2}$	0	$\frac{1}{2}$

Figure 4

1. (i) Explain the main reasons why a competitive equilibrium may fail to exist.

Illustrate using an Edgeworth box.

- (ii) "In a one-consumer exchange economy a competitive equilibrium, if it exists, is unique." True or false.

- (iii) Consider the following one consumer-one producer (private ownership) economy.

The consumer has utility function  $x_1^{1/2} x_2^{1/2}$ , where  $x_1$  represents leisure and  $x_2$  consumption. The firm's production function is  $q = z^{1/2}$ , where  $z$  is labor input and  $q$  is output. The consumer's endowment of labor (i.e., leisure) = 2. Compute competitive equilibrium prices, profit, and consumption.

D1 –

A number of individuals live in cities within a region where they are connected to each other by a road network. The shape of the road network is very simple. There is a central city,  $x_0$ , and other cities,  $x_k$ , ( $k=1, \dots, K$ ) each of which is connected by a single road to the  $x_0$ . No pair of cities are directly connected to each other by road segments except for the pairs  $(x_0, x_k)$  – that is, the road network is a simple form of “star” as shown below:

Attached

The road connecting  $x_k$  to  $x_0$  is of length  $a_k$ . The population at  $x_k$  is  $n_k$ .

Each individual has to travel to a location where an essential service is provided to them, to be chosen somewhere on the road network. It can be at a city  $x_0, x_1, \dots, x_K$ , or between two cities  $x_0$  and  $x_k$  at any distance  $b$  ( $< a_k$ ) from  $x_0$  (and therefore at a distance  $a_k - b$  from  $x_k$ ).

This question involves the choice of this location.

The individuals vote on the location in a set of pairwise majority rule votes. For any two locations each individual votes for the location that is closest to where they live, measured in total traveling distance. For example, if the locations are  $x_2$  and the midpoint between  $x_1$  and  $x_0$ , then someone at  $x_3$  would compare  $a_3 + a_2$  to  $a_1/2 + a_3$ , because to get to  $x_2$  they first have to go to  $x_0$  and then to  $x_2$ , whereas to go to the midpoint of  $x_0$  and  $x_1$  the first have to go to  $x_0$  and then only half way to  $x_1$ .

- a) Show that there is always some location that is a Condorcet winner of this voting scheme, regardless of the parameters  $x_k$  and  $n_k$ .
- b) Under what conditions is the central city,  $x_0$ , the Condorcet winner?

- c) Here are some definitions and a theorem from MWG 21.E, a section that we did not cover in 2010b:

A social choice function is a single-valued mapping from a set of preference profiles (the “domain”) to the set of alternatives.

An alternative  $x$  maintains its position when the profile changes from  $P$  to  $P'$  if for all alternatives  $y$  and all individuals  $i$ ,  $x P_i y$  implies  $x P'_i y$ .

A social choice function  $f$  is monotonic if for any two profiles  $P$  and  $P'$  such that  $f(P)$  maintains its position when the profile changes from  $P$  to  $P'$ , we have  $f(P) = f(P')$ .

Theorem: If the domain of profiles on which a social choice function is defined is unrestricted, any social choice function that is Pareto efficient and monotonic is dictatorial.

c(i): Show that the social choice function defined by the Condorcet winner in the problem of parts a) and b) above is monotonic.

c(ii): Since this social choice function is obviously Pareto efficient and non-dictatorial, comment on the application of the theorem stated above to this example.

D2-

Two players have to design the rules under which they will make a transaction in which one of them (the seller) might transfer an indivisible good to the other (the buyer) in exchange for the receipt of some money. At the time that they agree to these rules they do not know their valuations for the good. All they know is that they both have quasi-linear utility for the good and money and that their valuations  $(s, b)$ , will be drawn independently from a pair of distributions  $F_s, F_b$ . They want to design a mechanism that they will play after their valuations have become known to them, but at a time that each one is still private information. They can enlist the help of a third-party who can enforce the agreement that they make at the ex ante stage.

- a) To evaluate different procedures you want to rely on the concepts of incentive theory as covered in 2010b. Explain the use of the revelation principle and the interim incentive constraints in this context?
  - b) What are the participation constraints in this set up, or do they not apply here?
  - c) Consider a “fixed-price contract”, where the buyer can obtain the good at his discretion by paying an amount  $p$  to the seller. Can this contract be optimal in some cases?
  - d) How about a contract a fixed-price contract with a right of refusal – that is, the buyer can signal to the third party that he is willing to buy at  $p$ , but the seller can refuse to sell at  $p$  and in that case the seller retains the good.
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