

*Harvard University*  
*Department of Economics*

General Examination in Macroeconomic Theory

Fall 2008

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PLEASE USE A SEPARATE BLUE BOOK FOR EACH PART AND WRITE THE QUESTION NUMBER ON THE FRONT OF THE BLUE BOOK.

PLEASE PUT YOUR EXAM NUMBER ON EACH BOOK.

PLEASE DO NOT WRITE YOUR NAME ON YOUR BLUE BOOKS.

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For those taking the **GENERAL EXAM** in macroeconomic theory:

1. You have **FOUR** hours.
  2. Answer **ALL QUESTIONS** in Parts I, II, III, IV, and V.
  3. Time allotted for each part:
    - I. 48 minutes
    - II. 48 minutes
    - III. 48 minutes
    - IV. 48 minutes
    - V. 48 minutes
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This part of the exam has two questions designed to take 48 minutes in total: 23+25. Good luck!

**Problem 1 (Equity model: 23 minutes):** Assume that a consumer with only equity wealth must choose period by period consumption in a discrete-time dynamic optimization problem. Specifically, consider the sequence problem:

$$v(x_0) = \sup_{\{c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \delta^t u(c_t)$$

subject to the constraints:

- $x_{t+1} = \exp(r + \sigma u_t - \sigma^2/2)(x_t - c_t)$
- $u_t$  iid
- $u_t \sim N(0, 1)$
- $c_t \in [0, x_t]$ ,  $x_0 > 0$ .

Here  $x_t$  represents wealth at period  $t$  and  $c_t$  represents consumption at period  $t$ . The consumer has discount factor  $\delta = \exp(-\rho)$  and the consumer can only invest in a risky asset with expected return  $\exp(r) = E \exp(r + \sigma u - \sigma^2/2)$ . Finally, assume that the consumer has log utility:

$$u(c) = \ln(c).$$

a. Explain why the associated Bellman equation is given by

$$v(x) = \sup_{y \in [0, x]} u(x - y) + E \delta v(\exp(r + \sigma u - \sigma^2/2)y).$$

Explain all of the terms in the Bellman equation.

b. Guess that the value function takes the special form

$$v(x) = \psi + \phi \ln x.$$

Assuming that the value function guess is correct, derive the consumption function:

$$c = \phi^{-1} x.$$

Now solve for  $\phi$ .

c. Show that

$$E \Delta \ln c_{t+1} = (r - \rho) - \frac{1}{2} \sigma^2.$$

d. Explain why consumption growth *falls* as the variance of returns rises. Provide intuition for your answer. Explain why this is the opposite sign of the comparative static on volatility in labor income (in models that have stochastic labor income instead of stochastic asset returns).

**Problem 2 (True, False, Ambiguous: 25 minutes):**

1. If a function is everywhere continuous, then it must be differentiable at some set of points.
2. A Bellman Operator has a fixed point if and only if the operator is a contraction mapping.
3. In a model with stochastic labor income, an increase in the coefficient of relative risk aversion will increase average consumption growth.
4. If there is no fixed cost of adjusting the capital stock, then the capital stock will respond to all shocks that affect the profitability of capital.
5. Investment should rise when firms become eligible to use an investment tax credit.

**Barro question (48 minutes)**

Consider the neoclassical growth model, discussed, for example, in Barro & Sala-i-Martin, *Economic Growth*, Ch. 2. Households maximize utility,  $U$ , over an infinite horizon, using a constant rate of time preference,  $\rho$ . Each period's utility,  $u(c)$ , is iso-elastic with curvature parameter  $\theta$  ( $c$  is consumption per person). The rate of growth of population and the labor force is  $n$ . **Assume that the rate of technological progress is zero.** Suppose that the economy starts with capital per worker,  $k$ , equal to  $k(0)$ , which is less than the steady-state value,  $k^*$ . The production function is  $y=A \cdot f(k)$ , where  $y$  is output per worker,  $A$  is a positive constant, and  $f(\cdot)$  satisfies the usual neoclassical properties.

1. Write down the two dynamic equations for  $c$  and  $k$ . Briefly sketch where these two equations come from.

2. Show the phase diagram—involving pairs of  $(k, c)$  that generate  $\dot{c} = 0$  and  $\dot{k} = 0$ . Sketch briefly where these curves come from. Use the phase diagram to discuss the steady-state values,  $k^*$  and  $c^*$ . Use the diagram to discuss the dynamic path of  $(k, c)$ . Why do  $k$  and  $c$  approach  $k^*$  and  $c^*$ , respectively, over time? That is, how do you rule out dynamic paths that lead asymptotically away from the steady-state position? What is the growth rate of  $k$ ,  $c$ , and  $y$  in the steady state?

3. Suppose that the economy starts from its steady state. Assume now that productivity—that is, the parameter  $A$  in the production function—shifts upward once and for all. How does this change affect the growth rates of  $k$  and  $y$  in the short run and the long run?

4. How would you modify the model to generate growth in  $y$  in the long run?

## Part III

Consider each of the following claims. State (at the beginning of your answer) whether the claim is TRUE, FALSE, or PARTLY TRUE. Explain your answer.

1. In the IS-LM model, an exogenous increase in money demand is contractionary, unless consumption and investment are interest-inelastic.
2. In the Lucas imperfect information model, a 1 percent surprise increase in the money supply has a larger effect on both output and inflation if the central bank has a reputation for monetary stability than if it has a reputation for volatile policy.
3. In the Calvo model of price adjustment, an announced, gradual reduction in inflation can cause an economic boom.
4. In New Keynesian models, such as those explored by Ball and Romer, nominal rigidities such as menu costs are not necessary to explain monetary nonneutrality if the economy instead has substantial real rigidities.
5. In the Caplan-Spulber menu-cost model, the aggregate price level moves proportionately with fluctuations in the money supply.
6. The Golosov-Lucas calibrated menu cost model finds large, persistent effects of money on output because most price movements are driven by idiosyncratic shocks.

Question for fall 2008 macro theory generals

During the last year both America and Europe have faced a combination of negative economic shocks, including both supply shocks (most obviously the increase in oil prices) and demand shocks (most obviously the decline in house prices). Their respective central banks, however, have pursued sharply different policies. The U.S. Federal Reserve System, emphasizing concerns over a potential decline in output and further reductions in employment, has repeatedly cut its policy interest rate. The European Central Bank, emphasizing concerns over faster price increases, and even more so fears of potential increases in inflation expectations associated with expectations about the Bank's own future policy, has instead held its policy interest rate steady.

(a) Under what circumstances would concern for expectations, importantly including expectations about the central bank's own future policy actions, optimally lead policymakers to forego any attempt to reduce fluctuations in real economic activity and instead simply use monetary policy to keep inflation as steady as possible at some designated rate – even when, presumably, both policymakers and the public would prefer that fluctuations in output and employment be reduced?

(b) If those conditions are not satisfied – that is, if it is not optimal to forego altogether attempts to use monetary policy to stabilize output and employment – in what way might the effect of expectations about future policy still lead the optimal monetary policy to differ from what it would be if neither households nor firms thought in advance about what the central bank was likely to do?

Answer both (a) and (b), being as explicit as you can about your analytical reasoning and also any assumptions that underlie your answers.

## Part V 48 Minutes

Please answer all three equally-weighted questions in this section.

1. a. The United States is running an enormous trade balance deficit of over five percent of GDP. What are some explanations of why this equilibrium might be sustainable for an extended period?

b. Suppose we have a world of complete asset markets but some goods are not tradeable. Further suppose that the representative agent's utility function (in every country) is identical and separable in traded and nontraded goods. Thus the utility function takes the form

$$u(C_N) + \left( \frac{C_T^{1-\rho}}{1-\rho} \right)$$

where  $C_N$  denotes consumption and  $C_T$  is consumption of traded goods. Explain why there might be a home bias in equity holdings in this model even if there are no exogenous restrictions on asset markets.

2. Consider a stochastic two-country world, two-period endowment economy, in which agents in the home country have utility function

$$U = \left( \frac{C_1^{1-\rho}}{1-\rho} \right) + E \left( \frac{\beta C_2^{1-\rho}}{1-\rho} \right).$$

Agents in the foreign country have an identical utility function. Home income in the first period is given by  $Y$ , and in the second period by  $Y(s)$ , where  $s$  is the state of nature. Agents abroad receive income stream  $Y^*$  and  $Y^*(s)$ .

a. Assuming that there are complete state contingent markets in this global economy show, analytically, in what sense consumption risk is shared. It is sufficient to show your result in terms of relating consumption growth rates across countries.

b. Now suppose that home utility is given by

$$U = \left( \frac{C_1^{1-\rho}}{1-\rho} \right) + E \left( \frac{\beta C_2^{1-\rho}}{1-\rho} \right)$$

and foreign utility by

$$U = \left( \frac{(C_1^*)^{1-\rho^*}}{1-\rho^*} \right) + E \left( \frac{\beta(C_2^*)^{1-\rho^*}}{1-\rho^*} \right)$$

where  $\rho \neq \rho^*$ . What relationship does this model predict between the  $\log(C_2/C_1)$  and  $\log(C_2^*/C_1^*)$ ?

c. Now, again assume  $\rho = \rho^*$ ; suppose that agents could trade only risk-free bonds paying fixed interest rate  $r$  (where the interest rate is determined endogenously in global equilibrium). Taking  $r$  as given, characterize how home first-period consumption  $C_1$  is related to expected second-period home consumption  $EC_2$ .

### 3. Morris and Shin Speculative Exchange Rate Attacks

Consider a model where a government is trying to maintain an overly high exchange rate. The economy is characterized by a state of fundamental,  $\theta$ :  $\theta \in [0, 1]$ . The exchange rate in the absence of government intervention (the “shadow” exchange rate) is given by  $f(\theta)$ , where  $f'(\theta) > 0$ .

Assume that, initially, the government has pegged the exchange rate at  $\bar{e} \geq f(\theta)$ ,  $\forall \theta$ . Facing the government is a continuum of speculators who may attack the currency by selling short one unit, or they may refrain from doing so. If an individual speculator sells the currency short, they must pay a fee,  $t > 0$ .

If a speculator chooses to “attack” and if the peg falls, the net gain to the speculator is  $\bar{e} - f(\theta) - t$ . If the peg does not fall, he loses  $-t$  from attacking. If he does not attack, his net gain is normalized at zero.

The government derives a value  $v > 0$  from defending the rate, which is known to all agents, but faces cost  $c(\alpha, \theta)$ ,  $c_1(\alpha, \theta) > 0$ ,  $c_2(\alpha, \theta) < 0$ , where  $\alpha$  is the percent of speculators that choose to attack. Assume that  $c(0, 0) > v$ ,  $c(1, 1) > v$ .

Suppose each individual speculator observes noisy signal  $x_i$  of the aggregate state of the economy where:

$$x_i \sim U[\theta - \varepsilon, \theta + \varepsilon].$$

That is,  $x_i$  is uniformly distributed between  $\theta - \varepsilon$  and  $\theta + \varepsilon$ . We assume that  $2\varepsilon < \min\{\theta_{\min}, 1 - \theta_{\max}\}$ , and that  $\theta_{\min} < k - \varepsilon$ ;  $\theta_{\max} > k + \varepsilon$ . Conditional on  $\theta$ , the signals are independent and identically distributed across agents.

The government observes the number of individuals who attack the currency  $\alpha$ , observes  $\theta$ , and decides whether or not to abandon the peg. Finally, assume each speculator follows an identical “cutoff” strategy, choosing to attack whenever  $x < k$ .

a. Derive an expression for the fraction of agents  $\alpha$  that decide to attack the currency, as a function of the fundamentals ( $\theta$ ), the cutoff strategy ( $k$ ) and other fixed parameters of the model. Plot this expression in a graph that has  $\theta$

on the horizontal axis and  $\alpha$  on the vertical axis. How does the slope depend on the "uncertainty" faced by traders (the noise)?

b. In the same space  $(\alpha, \theta)$ , draw the locus that characterizes the government indifference condition (that is, the combination of  $\alpha$ ,  $\theta$  that makes the government just indifferent between abandoning or sticking to the regime).

c. Combining your findings of letter (a) and (b), show graphically the threshold value  $\theta^*$ , such that the regime fails whenever  $\theta < \theta^*$ , and survives otherwise. Is this solution unique? Explain why, if  $\theta^*$  is the critical value below which the government abandons its peg, then in equilibrium

$$-t + \frac{1}{2\varepsilon} \int_{k-\varepsilon}^{\theta^*} [\bar{e} - f(\theta)] d\theta = 0.$$

d. The "cutoff" strategies of traders are not constants, but determined in equilibrium simultaneously with the threshold fundamental. Write the two conditions which allow to find  $k$  and  $\theta^*$  (you only need to express the equations, not solve for either variable).