

# A Comparative-Advantage Approach to Government Debt Maturity\*

Robin Greenwood  
Harvard University and NBER

Samuel G. Hanson  
Harvard University

Jeremy C. Stein  
Harvard University and NBER

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## Abstract

We study optimal government debt maturity in a model where investors derive monetary services from holding riskless short-term securities. In a simple setting where the government is the only issuer of such riskless paper, it trades off the monetary premium associated with short-term debt against the refinancing risk implied by the need to roll over its debt more often. We then extend the model to allow private financial intermediaries to compete with the government in the provision of money-like claims. We argue that if there are negative externalities associated with private money creation, the government should tilt its issuance more towards short maturities. The idea is that the government may have a comparative advantage relative to the private sector in bearing refinancing risk, and hence should aim to partially crowd out the private sector's use of short-term debt.

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## I. Introduction

In this paper, we study the question of how the government should optimally determine the maturity structure of its debt. We focus on situations where there is no question about the government's ability to service its obligations, so the analysis should be thought of as applying to countries like the U.S. that are seen to be of high credit quality.<sup>1</sup> The primary novelty of our approach is that we emphasize the monetary benefits that investors derive from holding riskless securities, such as short-term Treasury bills. These benefits lead T-bills to embed a convenience premium, i.e. to have a lower yield than would be expected from a conventional asset-pricing model.

We begin with the case where the government is the *only entity* able to create riskless money-like securities. In this case, optimal debt maturity turns on a simple tradeoff. On the one hand, as the government tilts its issuance to shorter maturities, it generates more in the way of monetary services, which are socially valuable; this is reflected in a lower expected financing cost. On the other hand, a strategy of short-term financing also exposes the government to rollover risk, given that future interest rates are unpredictable. As a number of previous papers have observed, such rollover risk leads to real costs insofar as it makes future taxes more volatile.<sup>2</sup>

This tradeoff yields a well-defined interior optimum for government debt maturity, unlike traditional tax-smoothing models which imply that government debt should be very long term. It also implies a number of comparative statics that appear to be borne out in the data. Most notably, it predicts that government debt maturity will be positively correlated with the ratio of government debt to GDP, a pattern which emerges strongly in U.S. data. The intuition is that as the aggregate

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<sup>1</sup> This is by contrast to a literature that argues that countries with significant default or inflation risk may have a signaling motive for favoring short-term debt, or at the extreme, may have little choice but to issue short-term securities. See, e.g., Blanchard and Missale (1994).

<sup>2</sup> See, e.g. Barro (1979), Lucas and Stokey (1983), Bohn (1990), Angeletos (2002), Aiyagari, Marcet, Sargent, and Seppälä (2002), and Nosbusch (2008).

debt burden grows, the costs associated with rollover risk—and hence with failing to smooth taxes—loom larger.

The simple tradeoff model also captures the way in which Treasury and Federal Reserve practitioners have traditionally framed the debt-maturity problem. According to former Treasury Secretary Lawrence Summers:<sup>3</sup>

*“I think the right theory is that one tries to [borrow] short to save money but not [so much as] to be imprudent with respect to rollover risk. Hence there is certain tolerance for [short term] debt but marginal debt once [total] debt goes up has to be more long term.”*

Our focus on the monetary services associated with short-term T-bills is crucial for understanding Summers’ premise that the government should borrow short to “save money”.<sup>4</sup> As we demonstrate formally below, if short-term T-bills have a lower expected return than longer-term Treasury bonds simply because they are less risky in a standard asset-pricing sense (i.e., because they have a lower beta with respect to a rationally priced risk factor) this *does not* amount to a logically-coherent rationale for the government to tilt to the short end of the curve, any more so than it would make sense for the government to take a long position in highly-leveraged S&P 500 call options because of the positive expected returns associated with bearing this market risk.

In our baseline formulation of the tradeoff model, it is optimal for the government to issue T-bills because the social planner internalizes the monetary benefits that bills create for households. However, one can also ask under what circumstances the government would still want to issue short-term securities when the planner does not directly internalize the monetary benefits they generate.

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<sup>3</sup> Private email correspondence April 28, 2008, also cited in Greenwood, Hanson and Stein (2010).

<sup>4</sup> In a similar spirit, Bennett, Garbade and Kambhu (2000) explain the appeal of short-term financing by saying: “Minimizing the cost of funding the federal debt is a leading objective of Treasury debt management...liquidity is an important determinant of borrowing costs...Longer-maturity debt is inherently less liquid than short-term debt...” Thus they are careful to link the lower cost of short-term financing with a non-risk-related attribute, in this case its greater liquidity. This can be seen as quite close to our emphasis on monetary services. Assistant Treasury Secretary Lee Sachs highlights similar themes in his 1999 address to Congress.

Suppose for example that money demand comes entirely from foreign investors, and that the planner is parochial in the sense of not caring about the utility of these foreign investors. We show that the government may still want to issue short-term securities to satisfy the foreign money demand, motivated in this case by a desire to minimize its interest costs, and hence the taxes that its own citizens must pay.

After fleshing out the simple tradeoff model, we go on to examine the case where the government is not the only entity that can create riskless money-like claims, but instead competes with the private sector in doing so. Following Gorton and Metrick (2010), Gorton (2010), and Stein (2010), we argue that financial intermediaries engage in private money creation, thereby capturing the same monetary convenience premium, when they issue certain forms of collateralized short-term debt—e.g., overnight repo, or asset-backed commercial paper. As Stein (2010) observes, the incentives for such private money creation can be excessive from a social point of view, as individual intermediaries do not fully take into account the social costs of the fire sales that can arise from a heavy reliance on short-term financing.

In the presence of these fire-sale externalities, there is an additional motivation for the government to shift its own issuance towards short-term bills. By doing so, it reduces the equilibrium money premium, thereby partially crowding out the private sector's socially excessive issuance of short-term claims. This is desirable as long as the marginal social costs associated with government money creation—which take the form of greater intertemporal volatility in taxes—remain lower than the marginal social costs associated with private money creation, which stem from fire sales. In other words, the government should keep issuing short-term bills as long as it has a *comparative advantage* over the private sector in the production of riskless money-like securities.

This line of reasoning adds what is effectively a regulatory dimension to the government's debt-maturity choice. An alternative way to address the fire-sales externalities associated with private money creation would be to try to control the volume of such money creation directly, e.g., with either a regulatory limit or a Pigouvian tax on short-term debt use by financial intermediaries. However, to the extent that such caps and taxes are not perfectly enforceable—say because some of the money creation can migrate to the unregulated “shadow banking” sector—there will also be a complementary role for a policy that reduces the incentive for private intermediaries to engage in money creation in the first place, by lowering the convenience premium that money commands. Our point is that this can be done by shortening the maturity of government debt.

To be clear, we intend for this comparative-advantage argument to be taken in a normative, rather than positive spirit. That is, unlike with the simple government-only model, we don't mean to suggest that the comparative-advantage aspect of the theory provides further testable predictions regarding how government debt maturity is actually chosen in the real world. Rather, we offer it as a framework for thinking about policy going forward—albeit one grounded in an empirically-relevant set of premises. In this sense, it is like much other recent work on financial regulatory reform.

The ideas here build on four strands of research. First there is the prior theoretical work on government debt maturity, especially that which has examined a tax-smoothing motive for long-term finance (Barro (1979), Lucas and Stokey (1983), Bohn (1990), Angeletos (2002), and Nosbusch (2008))<sup>5</sup>. Second, there is a literature that documents significant deviations from the predictions of standard asset-pricing models—patterns which can be thought of as reflecting money-like convenience services—in the pricing of Treasury securities generally, and in the pricing of short-

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<sup>5</sup> See also Calvo and Guidotti (1990), Barro (2002), Benigno and Woodford (2003), and Lustig, Sleet, and Yeltekin (2006). More closely related to our work is Guibaud, Nosbusch, and Vayanos (2007), who propose a clientele-based theory of the optimal maturity structure of government debt. However, in their model, government maturity policy is motivated by imperfect intergenerational risk sharing.

term T-bills more specifically (Krishnamurthy and Vissing-Jorgensen (2010), Greenwood and Vayanos (2010), Duffee (1996), Gurkaynak, Sack and Wright (2006)). Third, there is the set of recent papers alluded to above, which emphasize how private intermediaries try to capture the money premium by relying heavily on short-term debt, even when this creates systemic instabilities (Gorton and Metrick (2010), Gorton (2010), and Stein (2010)). And finally, there is evidence that changes in government debt maturity influence private-sector debt-maturity choices, consistent with a crowding-out view: when the government issues more short-term debt, private firms issue less, and substitute towards long-term debt instead (Greenwood, Hanson and Stein (2010)).

In Section II, we further motivate our theory by laying out a set of key stylized facts, drawing on the papers cited just above, as well as on some new empirical work of our own. In Section III, we develop the simple tradeoff model of optimal debt maturity when the government is the only entity that can create riskless money-like securities. In Section IV, we add financial intermediaries and private money creation to the mix, and pose the comparative-advantage question: to what extent should the government actively try to crowd out private money creation? Section V discusses some further implications of our framework, and Section VI concludes.

## **II. Stylized Facts**

### *A. Convenience Premia in Treasury Securities*

Krishnamurthy and Vissing-Jorgensen (2010) argue that Treasury securities have some of the same features as money, namely liquidity and “absolute security of nominal return”, and that these monetary attributes lead Treasuries to have significantly lower yields than they otherwise would in a standard asset-pricing framework—their estimate of the monetary premium on Treasuries over the period 1926-2008 is 72 basis points. Their identification is based on measuring the impact of

changes in Treasury supply on a variety of spreads. For example, they show that an increase in the supply of Treasuries reduces the spread between Treasuries and AAA-rated corporate bonds—most likely because as the quantity of money-like securities available to investors goes up, the equilibrium money premium goes down.

Krishnamurthy and Vissing-Jorgensen (2010) treat all Treasury securities as having similar money-like properties, and do not distinguish between Treasuries of different maturities. However, other work (e.g. Duffee (1996), Gurkaynak, Sack and Wright (2006)) has documented that the yields on short-term T-bills are on average strikingly low relative to those on longer-term notes and bonds. Gurkaynak et al write: “...bill rates are often disconnected from the rest of the Treasury yield curve, perhaps owing to segmented demand from money market funds and other short-term investors.” Panel A of Figure 1 provides an illustration. We plot the average spread, over the period 1980-2009, between actual T-bill yields (on bills with maturities from 1 to 26 weeks) and fitted yields, where the fitted yields are based on a flexible extrapolation of the Treasury yield curve from Gurkaynak, Sack, and Wright (2006) that is calibrated using only Treasuries with maturities greater than three months.<sup>6</sup> In other words, the spreads in Figure 1 represent the extent to which the shortest-term bills have yields that differ from what one would expect based on an extrapolation of the rest of the yield curve. As can be seen, the differences are large: four-week bills have yields that are roughly 50 basis points below their fitted values; and for one-week bills, the spread is over 60 basis points.

Our preferred interpretation of these spreads is that they reflect the extra “moneyness” of short-term T-bills, above and beyond whatever money-like attributes longer-term Treasuries may already have. For example, short-term bills not only offer absolute security of ultimate nominal

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<sup>6</sup> Gurkaynak, Sack, and Wright (2006) estimate a parametric model of the instantaneous forward rate curve that is characterized by five parameters. Zero coupon yields are then derived by integrating along the estimated forward curve. The parameters for each day are estimated by minimizing a weighted sum of pricing errors. The set of sample securities each day includes almost all “off-the-run” Treasury notes and bonds with a remaining maturity of more than 3 months.

return, as Krishnamurthy and Vissing-Jorgensen (2010) stress for Treasuries in general, but also have no interest-rate exposure—so they are completely riskless at short horizons. This is presumably what makes them so attractive to money-market mutual funds and so desirable as collateral in backing repurchase agreements and other financial contracts.<sup>7</sup> It may also explain the apparent steepness of the spread at the very shortest horizons (e.g., 1- and 2-week bills).

This interpretation is supported by the work of Greenwood and Vayanos (2010). They find that the relative returns on short-maturity Treasuries go up (as compared to those on longer-maturity Treasuries) when the government does a greater proportion of its issuance at the short end of the yield curve. In other words, when there are more of the most money-like short-term securities in the system, the convenience premium on these securities shrinks. Panel B of Figure 1 shows that this logic applies especially to short-term T-bills. We plot the spread between the actual 4-week T-bill yield and the fitted yield alongside the ratio of short-term debt to GDP, over the period from 1980-2009, where short-term debt includes all bills, notes and bonds with remaining maturities of less than one year. As can be seen, there is a strong positive relation between the two series ( $\rho=0.60$ ). The corresponding annual regression of spreads on the short-term debt-to-GDP ratio shows that a one-percent decrease in the ratio leads to an 11.5 basis-point decline in T-bill yields relative to the fitted Treasury curve ( $t = 2.26$ ).

#### *B. The Correlation Between Debt Maturity and the Debt-to-GDP Ratio*

Figure 2 plots the weighted average maturity of outstanding U.S. government debt against the debt-to-GDP ratio over the period 1952-2009. As can be seen, the two series are strongly

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<sup>7</sup> Several authors, starting with Van Horne and Bowers (1968), have pointed out that shorter-term Treasury debt instruments are more liquid, which they define as “the ratio of exchange between the asset and money.” For an early empirical estimate of the moneyness of T-bills, see Poterba and Rotemberg (1987). Bansal and Coleman (1996) develop a model that highlights the special liquidity role of short-term Treasury bills and explores the resulting implications for the equity premium and the term premium.

positively correlated—the correlation coefficient is 0.71 over the full sample period.<sup>8</sup> This relationship between debt maturity and the debt-to-GDP ratio, also noted in Greenwood and Vayanos (2010), Greenwood, Hanson and Stein (2010), and Krishnamurthy and Vissing-Jorgensen (2010), is one of the most direct implications of the simple tradeoff model of government debt maturity that we develop in the next section.

To be clear, such a positive correlation could potentially arise even if the government adopted a mechanical issuance strategy of always issuing new debt with the same average maturity: when debt-to-GDP rises, the ratio of newly-issued debt to outstanding debt rises, so even holding fixed the average maturity of new issues, the average maturity of outstanding debt can increase. However, as noted by Garbade and Rutherford (2007), the government actively manages its issuance and repurchases to achieve a target maturity of outstanding debt. To test whether issuance behavior actively adjusts in response to a target debt maturity that is itself a function of the debt-to-GDP ratio, we estimate a regression of the maturity of new debt *issues*  $M_t^{Iss}$  on the lagged maturity of debt issues  $M_{t-1}^{Iss}$ , the lagged maturity of outstanding debt  $M_{t-1}^{Out}$ , and the debt-to-GDP ratio. This yields

$$M_t^{Iss} = 0.98 + 2.86 \cdot (D/GDP)_t + 0.76 \cdot M_{t-1}^{Iss} - 0.34 \cdot M_{t-1}^{Out}, \quad (1)$$

[ $t=2.52$ ]
[ $t=3.40$ ]
[ $t=7.43$ ]
[ $t=-3.60$ ]

with an  $R^2$  of 0.55. One can interpret this regression in terms of a partial adjustment model of the maturity of government debt issues: when the debt-to-GDP ratio is high, the government gradually adjusts the maturity of its new issuance towards longer-term debt.<sup>9</sup>

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<sup>8</sup> Treasury faced statutory restrictions on coupon rates and note maturities between 1965 and 1982. Garbade (2007) argues that this kept the Treasury out of the long-term bond market for part of this time. However, even if we omit this interval, we still find correlations of 0.38 post-1981, and 0.64 pre-1965.

<sup>9</sup> To see the rationale, suppose the target maturity for all outstanding debt is  $M_t^{Out*} = \alpha + \beta \cdot (D_t/GDP_t) + v_t$ . This implies that the target maturity of new issues is approximately  $M_t^{Iss*} = k(1 + \alpha) + k\beta \cdot (D_t/GDP_t) - k \cdot M_{t-1}^{Out} + k \cdot v_t$ , where  $k$  is average value of  $D_t/ISS_t$  (this is because  $M_t^{OUT} = (D_{t-1}/D_t) \cdot (M_{t-1}^{OUT} - 1) + (ISS_t/D_t) \cdot M_t^{ISS}$ ). If we assume a partial adjustment model for the maturity of new issues of the form  $\Delta M_t^{Iss} = \lambda \cdot (M_t^{Iss*} - M_{t-1}^{Iss}) + u_t$ , this

### C. Private-Sector Responses to Government Debt-Maturity Choices

The comparative-advantage version of our model rests on the premise that the government can, by issuing more short-term bills, crowd out the issuance of short-term money-like claims by financial intermediaries. Greenwood, Hanson and Stein (2010) investigate a similar crowding-out phenomenon, looking at how the maturity choices of private debt issuers respond to changes in government debt maturity over the period 1963-2005. Figure 3 reproduces one of the main findings of that paper, extending the time series to 2009: it shows that as government debt maturity contracts, the debt maturity of non-financial firms rises significantly.

While this result provides general support for a debt-maturity crowding-out hypothesis, here we introduce another piece of evidence that is more precisely targeted to understanding the money-creation behavior of financial intermediaries. To do so, we build on Krishnamurthy and Vissing-Jorgensen (2010), who document a negative relationship between: i) the ratio of (M2 – M1) to GDP; and ii) the ratio of government debt to GDP. Recall that they are interested in making the case that *all government debt is to some degree money-like*. So they interpret their results as saying that when government debt is higher, financial intermediaries have less incentive to create private money, as proxied for by (M2 – M1).<sup>10</sup>

By contrast, we want to emphasize the notion that *short-term government debt is more money-like than long-term government debt*, and hence should be expected to have a more powerful crowding-out effect on private money creation. So we run regressions of the form:

$$\left( \text{PrivateMoney} / \text{GDP} \right)_t = a + b \cdot \left( D / \text{GDP} \right)_t + c \cdot \left( D_S / (D_S + D_L) \right)_t + u_t. \quad (2)$$

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implies that  $M_t^{Iss} = \lambda k(1 + \alpha) + \lambda k \beta \cdot (D_t / \text{GDP}_t) - \lambda k \cdot M_{t-1}^{Out} + (1 - \lambda) \cdot M_{t-1}^{Iss} + e_t$ . The partial adjustment feature captures Treasury's "regular and predictable" issuance philosophy, according to which abrupt changes in the maturity mix of new issues are seen as undesirable, because they might adversely affect market liquidity (Garbade (2007)).

<sup>10</sup> Bansal, Coleman, and Lundblad (2010) also find that an increase in the supply of government debt leads to a decline in the supply of short-term private sector debt.

Here the left-hand side variable is a proxy for private money creation,  $D_S$  denotes government debt with a remaining maturity of less than one year, and  $D_L$  denotes government debt with a maturity of one year or more. We sample the data annually from 1952 through 2009.<sup>11</sup>

In estimating (2), we use three measures of private money creation. The first follows Krishnamurthy and Vissing-Jorgensen (2010) exactly, and is the difference between M2 and M1.<sup>12</sup> The second is the difference between M3 and M1, minus retail and institutional money-market funds, which we remove to avoid double-counting, given that money-market funds are simply a vehicle by which investors hold money-like securities. The last measure, taken from the Federal Reserve *Flow of Funds*, is Open Market Paper issued by Commercial Banks, scaled by GDP.<sup>13</sup>

We start in Table 1 by verifying the key result from Table V of Krishnamurthy and Vissing-Jorgensen—that when the public sector issues more debt, the financial sector creates less money. As can be seen, the correlation between  $(M2-M1)/GDP$  and  $D/GDP$  in the first column is negative. We then add a variable capturing the maturity structure of government debt,  $D_S/(D_S + D_L)$ . In this specification, maturity structure attracts a significantly negative coefficient. In other words, holding fixed the total level of government debt, private money creation is more strongly crowded out when the government debt is of shorter maturity. Panels B and C of Table 1 show that the results are not

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<sup>11</sup> Krishnamurthy and Vissing-Jorgensen use the 1934-2008 period, but debt maturity data from CRSP is not available before 1952. We follow Krishnamurthy and Vissing-Jorgensen and calculate the debt-to-GDP ratio using total debt; however, this means that  $D_S+D_L$  is not exactly equal to  $D$ , because total debt also includes non marketable savings bonds, and inflation protected securities.

<sup>12</sup> Non-M1 M2 consists of savings deposits and small-time deposits at banks and thrifts as well as retail money market funds. Savings deposits and small-time deposits are typically non-reservable liabilities and hence private banks can expand and contract the supply of these forms of money. By contrast, checking deposits and other reservable liabilities in M1 are pinned down by the quantity of central-bank controlled reserves. Non-M2 M3 consists of large time deposits at banks and thrifts, Eurodollar deposits, repurchase agreements, and institutional money market funds.

<sup>13</sup> Commercial banks include US-chartered banks, foreign bank offices in the US, bank holding companies, and banks in US-affiliated areas. We have also studied issuance of open market paper from the broader financial sector, with similar results. This includes all commercial banks, issuers of asset-backed securities, finance companies, real estate investment trusts, and funding corporations. Open-market paper includes commercial paper and bankers acceptances.

sensitive to how we define private money creation. They are qualitatively similar when using both the broader measure of money based on M3, as well as that corresponding to short-term paper issued by commercial banks. We have also experimented with alternate measures of government debt maturity structure—such as the weighted average maturity of debt—again with similar results.

### III. A Tradeoff Model of Government Debt Maturity

The full model features three sets of actors: households, the government, and financial intermediaries. In this section we begin with a stripped-down version that leaves out the intermediaries, thus focusing on the optimal maturity structure of debt when the government is the sole creator of money. This setup generates a simple tradeoff between the monetary services provided by issuing more short-term debt, and the increased rollover risk that comes as a result. In the next section, we allow banks to compete with the government in money creation.

#### A. Households

There are three dates, 0, 1, and 2. Households receive a fixed exogenous endowment of one unit in each period. After paying taxes in each period, households can consume the remainder of their endowment, or invest some of it in financial assets.<sup>14</sup> Households have linear preferences over consumption at these three dates.

Households can transfer wealth between periods by purchasing government bonds. At date 0, households can purchase short-term bonds  $B_{0,1}$ , which pay one unit at date 1, or long-term zero-coupon bonds  $B_{0,2}$ , which pay one unit at date 2. Households can also purchase short-term debt at date 1,  $B_{1,2}$ , which pays one unit at date 2. The discount factor between date 1 and date 2 is random

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<sup>14</sup> Although the assumption that households receive an endowment of one unit is without loss of generality, we do require endowments to be sufficiently large to finance the required government expenditure as well as the quantity of private-sector projects that are financed by banks.

from the point of view of date 0, and is not realized until date 1, so refinancing maturing short-term debt at date 1 introduces uncertainty over date-2 taxation and hence over consumption.

In addition to direct consumption, households derive utility from monetary services. For starkness, we assume that these services only come from short-term debt issued at date 0, although the crucial assumption is just that short-term bonds provide more in the way of monetary services than long-term bonds.<sup>15</sup> The utility of a representative household is thus given by

$$U = C_0 + E[C_1 + \beta C_2] + v(M_0), \quad (3)$$

where  $\beta$  is the random discount factor which is realized at date 1, with  $E[\beta] = 1$ , and where  $M_0 = B_{0,1}$ , the amount of short-term government bonds held by households at date 0.<sup>16</sup> For now, we assume that  $v'(M_0) > 0$  and  $v''(M_0) \leq 0$ . However, in Section IV when we analyze whether the government should try to crowd out private money creation, we must assume that there are strictly diminishing returns to holding money, i.e., that  $v''(M_0) < 0$ .

Equation (3) can be used to pin down real interest rates. Long-term bonds issued at date 0 have price  $P_{0,2} = 1$ . Short-term bonds issued at date 0 have price  $P_{0,1} = 1 + v'(M_0)$ , thereby embedding an additional money premium. Short-term bonds issued at date 1 have a price that is uncertain from the perspective of date 0,  $P_{1,2} = \beta$ .

### *B. Government*

The government finances a one-time expenditure  $G$  at date 0, using a combination of short- and long-term borrowing from households, and taxes which it can levy in each period. The government budget constraint is given by:

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<sup>15</sup> We follow a long tradition in economics, starting with Sidrauski (1967), of putting monetary services directly in the utility function. As discussed further below, our results are qualitatively unchanged if we allow long-term government bonds to also carry a convenience premium, provided that the premium is strictly less than that on short-term bonds.

<sup>16</sup> The assumption that  $E[\beta] = 1$  is without loss of generality, but does help to simplify some of the resulting expressions.

$$\begin{aligned}
t=0: G &= \tau_0 + B_{0,1}P_{0,1} + B_{0,2}P_{0,2} \\
t=1: B_{0,1} &= \tau_1 + B_{1,2}P_{1,2} \\
t=2: B_{1,2} + B_{0,2} &= \tau_2
\end{aligned} \tag{4}$$

where  $P_{0,1}$  and  $P_{0,2}$  denote the prices of short- and long-term bonds issued at date 0, and  $P_{1,2}$  denotes the (uncertain) price of short-term bonds issued at date 1. At date 0, for example, the government may levy taxes of  $\tau_0$  on households, and sell short- and long-term bonds. If the government borrows short-term, then at date 1, it must levy taxes to pay off the maturing debt, or roll over the debt by reissuing short-term bonds  $B_{1,2}$ . At date 2, the government pays off maturing short- and long-term debt by levying taxes.<sup>17</sup>

We follow the standard assumption that taxes are distortionary (Barro (1979), Lucas and Stokey (1983), Bohn (1990)), and that the magnitude of these distortions is convex in the amount of revenue raised each period.<sup>18</sup> For simplicity, we use the quadratic function  $\tau^2 / 2$  to capture the resources that are wasted when taxes are  $\tau$ . Household consumption in each period is thus given by

$$\begin{aligned}
C_0 &= 1 - \tau_0 - (1/2)\tau_0^2 - B_{0,1}P_{0,1} - B_{0,2}P_{0,2} \\
C_1 &= 1 - \tau_1 - (1/2)\tau_1^2 + B_{0,1} - B_{1,2}P_{1,2} \\
C_2 &= 1 - \tau_2 - (1/2)\tau_2^2 + B_{1,2} + B_{0,2}.
\end{aligned} \tag{5}$$

Substituting in the government budget constraint from (4), consumption at the three dates can be rewritten as

$$\begin{aligned}
C_0 &= 1 - (1/2)\tau_0^2 - G \\
C_1 &= 1 - (1/2)\tau_1^2 \\
C_2 &= 1 - (1/2)\tau_2^2.
\end{aligned} \tag{6}$$

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<sup>17</sup> We restrict the government to issuing only non-contingent debt obligations. As we discuss below, in the absence of money demand, this restriction does not bind. However, when households derive utility from money services, there may be incentives for them to issue such securities, which we rule out by assumption.

<sup>18</sup> Bohn (1990) assumes that taxes are a linear function of endowments, and that the deadweight costs of taxation are a convex function of the *tax rate*. Given that we take endowments to be fixed, our approach amounts to the same thing.

Since we have assumed that endowments are fixed and that the government finances a known one-time expenditure of  $G$ , there is no endowment or fiscal risk in our model. As discussed further below, this implies that tax smoothing does not give rise to the sort of hedging motive that often makes state-contingent debt optimal in models of government debt maturity. The only source of risk in our model is the random discount factor,  $\beta$ , which one can think of as being driven by shocks to household preferences unrelated to endowments. This setup helps to simplify the analysis and to highlight the novel forces at work in our model.

The social planner maximizes household utility subject to the government budget constraint. Substituting household consumption (6) and money ( $M_0=B_{0,1}$ ) into the household utility function (3) and dropping exogenous additive terms, the planner's problem can be written as

$$\max_{\{B_{0,1}, B_{0,2}, B_{1,2}\}} \left[ v(B_{0,1}) - \frac{1}{2}(\tau_0^2 + E[\tau_1^2] + E[\beta\tau_2^2]) \right]. \quad (7)$$

The three right-hand terms in (7) capture the standard tax smoothing objective—the planner would like taxes to be low and constant over time. However, this objective must be balanced against the utility that households derive from holding short-term bonds.

### *C. Optimal Maturity Structure in the Absence of Money Demand*

We first solve the planner's maximization problem in the benchmark case where households derive no utility from monetary services (i.e.,  $v(B_{0,1}) = 0$ ). In this case, the prices of short- and long-term bonds issued at date 0 are the same and the planner solves

$$\min_{\{B_{0,1}, B_{0,2}, B_{1,2}\}} \left[ \frac{1}{2}(\tau_0^2 + E[\tau_1^2] + E[\beta\tau_2^2]) \right]. \quad (8)$$

The planner's problem can be solved by working backwards. At date 1, the discount factor between dates 1 and 2 is realized. From the government budget constraint, taxes at date 1 and date 2 are  $\tau_1 = B_{0,1} - B_{1,2}\beta$  and  $\tau_2 = B_{1,2} + B_{0,2}$ . Substituting into the planner's date-1 problem yields

$$\min_{B_{1,2}} \left[ \frac{1}{2} (\tau_1^2 + \beta \tau_2^2) \right] = \min_{B_{1,2}} \left[ \frac{1}{2} (B_{0,1} - B_{1,2} \beta)^2 + \frac{1}{2} \beta (B_{1,2} + B_{0,2})^2 \right] \quad (9)$$

Taking first order conditions with respect to  $B_{1,2}$  yields the solution

$$B_{1,2} = \frac{B_{0,1} - B_{0,2}}{1 + \beta}, \quad (10)$$

which implies that  $\tau_1 = \tau_2 = (B_{0,1} + B_{0,2} \beta) / (1 + \beta)$ . Intuitively, the planner chooses  $B_{1,2}$  to perfectly smooth taxes between dates 1 and 2 and the tax rate is such that the present value of taxes equals the present value of required debt payments. To get the quantity of short- and long-term debt issued at date 0, we substitute (10) into (8) and take first-order conditions with respect to  $B_{0,1}$  and  $B_{0,2}$ . The solution is given by Proposition 1.

**Proposition 1:** *In the absence of money demand, the government perfectly smoothes taxes by setting  $\tau_0 = \tau_1 = \tau_2 = G/3$ ,  $B_{0,1} = B_{0,2} = G/3$ , and  $B_{1,2} = 0$ .*

**Proof:** *See appendix.*

Proposition 1 captures the intuition that, absent money demand, the government can insulate the budget and taxes from uncertain future refinancing by never rolling over debt at date 1. With convex costs of taxation in each period, the planner sets the marginal social cost of taxation equal across dates. The government can accomplish this by issuing a long-term “consol” bond with face value of  $2G/3$  that makes the same payment at dates 1 and 2.

One might wonder whether, contrary to Proposition 1, total welfare could be increased if the government were able to issue risky state-contingent securities whose payoffs depend on the realization of the discount factor  $\beta$ . For example, suppose the government can issue risky debt with a payoff of  $X_R(\beta)$  at  $t = 2$  when the realization of the discount factor is  $\beta$ . However, as long as these securities are fairly priced, (i.e., as long as  $P_R = E[\beta X_R(\beta)]$ ), it is straightforward to show that the government cannot improve upon the simple tax-smoothing solution described in Proposition 1.

That is, Proposition 1 continues to hold even when we allow the government to issue all possible risky securities. This is an important result, because it implies that absent money demand, it does not make sense for the government to try to lower its expected financing costs by selectively selling those securities that have low betas with respect to priced risks.

To see the intuition for this result, note that from (8) the planner cares about minimizing  $E[\beta\tau_2^2] = Cov[\beta, \tau_2^2] + (E[\tau_2])^2 + Var[\tau_2]$ . Suppose that the government reduces its issuance of 2-period riskless bonds and instead issues state-contingent securities that deliver a high payout at date 2 when  $\beta$  is high. On the one hand, this would reduce expected financing costs and hence expected taxes, leading  $(E[\tau_2])^2$  to fall. This is because the risky securities command a higher price than riskless ones with the same expected payout, given that they have a high payoff in states where consumption is valued most. On the other hand, the issuance of these risky securities increases  $Cov[\beta, \tau_2^2]$ . That is, servicing the risky debt requires the government to impose higher taxes on households in states where consumption is highly valued and hence where taxes are most painful. Thus these two effects tend to offset one another, and we are left with the fact that issuing risky securities always increases  $Var[\tau_2]$ . Consequently, the effect on  $E[\beta\tau_2^2]$  of shifting from riskless to risky securities is always positive, the opposite of what the planner would like to accomplish.

In summary, absent a specific hedging motive, the government should not issue a security that has a low required return simply because it is less risky in the standard asset-pricing sense. This conclusion is similar to that of Froot and Stein (1998), who argue that a financial institution cannot create value for its shareholders simply by taking on priced risks that are traded in the marketplace.

The result that the government chooses not to issue state-contingent debt depends on our simplifying assumptions that endowments are deterministic and that there are no fiscal shocks. If instead endowments were stochastic or there were government spending shocks, the government

might have a motive to issue state-contingent debt that hedges these risks as in Bohn (1988) and Barro (1997). Fiscal uncertainty may also alter the optimal long-short mix if we restrict attention to non-contingent debt. Suppose, for example, that there are interim shocks to government spending that must be financed at date 1. If short-term government rates are positively correlated with these interim spending shocks, this would lead the government to favor a longer initial maturity structure. This hedging rationale for long-term debt is explored in greater detail in Angeletos (2002). Since these hedging motives are well understood, we dispense with them in what follows in order to highlight the new forces at work in our model.

#### *D. Optimal Maturity Structure with Money Demand*

We now turn to the case in which households derive utility from their holdings of short-term bonds. Before doing so, we introduce a notational simplification. We denote the total scale of government borrowing at date 0 as  $D = B_{0,1} + B_{0,2}$ , and the short-term debt share as  $S = B_{0,1} / D$ . Applying this notation, the benchmark optimal debt structure in the absence of money demand is given by  $S = 1/2$  and  $D = (2/3)G$ .

We solve the planner's problem from (7) subject to the government budget constraint shown in (4). As before, the long-term bond has price  $P_{0,2}=1$ , and the short-term bond issued at date 1 has uncertain price  $\beta$ . However, the short-term bond issued at date 0 now embeds a money premium, i.e.,  $P_{0,1} = 1 + v'(M_0)$ .

As shown in the Appendix, we can rewrite the planner's problem as

$$\min_{S,D} \left[ \frac{1}{2} (G - D - DSv'(DS))^2 + \frac{D^2}{2} \left( b \left( S - \frac{1}{2} \right)^2 + \frac{1}{2} \right) - v(DS) \right], \quad (11)$$

where  $b \equiv E[(\beta - 1)^2 / (1 + \beta)] \approx \text{Var}[\beta] / 2$  is a measure of the magnitude of date-1 refinancing risk.

The first-order condition for the short-term debt share  $S$  can be written as

$$\overbrace{Db(S - 1/2)}^{\text{Marginal tax-smoothing cost}} = \overbrace{v'(SD)}^{\text{Marginal benefit of money services}} + \overbrace{\tau_0 \cdot [v'(SD) + SDv''(SD)]}^{\text{Marginal tax lowering benefit}}. \quad (12)$$

Each of the three terms in (12) has a natural interpretation. The left-hand side represents the marginal tax-smoothing cost of shifting government financing towards short-term debt. Note that this cost depends on the difference between  $S$  and  $1/2$ , i.e., on the extent of the departure from perfect tax smoothing. It also depends on the magnitude of date-1 refinancing risk  $b$ , as well as on the raw scale of government debt  $D$ . The first term on the right-hand side of (12) reflects the direct money benefit of short-term bills—the marginal convenience services enjoyed by households. The second term on the right-hand side of (12) captures the net benefit from the *lower level of taxes* that arises when the government finances itself at a lower average interest rate. The government can raise revenue either by taxing, or by creating more money, with the marginal revenue from creating money given by  $v'(SD) + SDv''(SD)$ . If the latter method of revenue-raising is non-distortionary, it pushes the social planner towards further issuance of short-term bills.

Nevertheless, for much of the remainder of the paper, we ignore this latter *tax-lowering* benefit, in which case (12) reduces to:<sup>19</sup>

$$Db(S - 1/2) = v'(SD). \quad (12')$$

The argument in favor of focusing on (12') rather than (12) is as follows. Given that our formulation of the deadweight costs of taxation is ad hoc and lacking in microfoundations, we don't have any basis for asserting that one form of taxation—seignorage from money creation—is less distortionary than some other form, such as income or capital taxation. Fortunately, as we demonstrate below, our qualitative results are not sensitive to whether we derive them from (12) or (12').

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<sup>19</sup> Equation (12) also reduces to (12') in the special case where the utility function over money is of the log form, i.e., if  $v(M) = \gamma \log(M)$ , since in this case the marginal revenue from creating more money is equal to zero.

The one scenario where it makes most obvious sense to include the tax-lowering benefits of short-term debt is when this debt is held by foreign investors. In this case, issuing more short-term debt corresponds to raising more seignorage revenue from parties whose utility a parochial planner may not internalize, while allowing for the reduction of other taxes on domestic households. We return to this case at the end of this section.

The solution to (12') leads directly to Proposition 2.<sup>20</sup>

**Proposition 2:** *Define  $S^*$  as the optimal short-term debt share which solves  $Db(S - 1/2) = v'(SD)$ . We have that  $S^* > 1/2$ , and  $S^*$  is decreasing in both uncertainty about date-1 short rates, as well as in  $G$ , i.e.,  $\partial S^* / \partial b < 0$ , and  $\partial S^* / \partial G < 0$ . Furthermore, suppose that  $v(M) = \gamma f(M)$ , where  $f(\cdot)$  is an increasing and weakly concave function and  $\gamma$  is a positive constant. Then  $\partial S^* / \partial \gamma > 0$ .*

**Proof:** *See appendix.*

Proposition 2 establishes that money demand increases the willingness of the government to issue short-term bills and thereby take on refinancing risk, with this short-term bias being more pronounced when either the intensity of money demand is stronger, or the variance of short rates at date 1 is lower. At the extreme, if the variance of short rates is low enough, or if the social costs of rollover risk are sufficiently small, the government may go so far as to finance its entire debt using short-term bills.<sup>21</sup>

A similar logic can be used to understand the relationship between the government's total debt burden and its choice of debt maturity. The greater is the size of the debt—as captured here by

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<sup>20</sup> In establishing Proposition 2, we restrict the government to issuing only simple, non-state-contingent securities. Unlike in the case with no money demand, since the government now incurs refinancing risk in choosing  $S^* > 1/2$ , it might want to hedge this risk by issuing some  $\beta$ -contingent security, which we rule out here by assumption.

<sup>21</sup> While there is nothing that prevents the government from choosing  $S^* > 1$ , so that it issues bills to make long-term loans to households, we find that  $1/2 < S^* < 1$  for almost all reasonable parameter values.

the parameter  $G$ —the larger is the refinancing risk in dollar terms, and thus the less willing is the planner to deviate from  $S = 1/2$ . Furthermore, when  $v''(\cdot) < 0$ , the premium on short-term debt will fall as  $G$  rises, further reducing the incentive to tilt towards short-term debt. As discussed earlier, these predictions capture the intuition used by practitioners to describe the government’s approach to debt-maturity policy. And, as can be seen in Figure 2, they are clearly borne out in the U.S. data, where the correlation between debt maturity and Debt/GDP has historically been strong.<sup>22</sup>

**Example:** Suppose that money demand is linear:  $v(M_0) = \gamma M_0$ . Then it is straightforward to show that the solution to (12') is given by  $S^* = 1/2 + 3\gamma / (2Gb + \gamma b)$  and  $D^* = 2G/3 + \gamma/3$ . For instance, suppose that  $G = 1$ ,  $\gamma = 0.02\%$ , and  $b = 0.20\%$ , implying that  $\sqrt{\text{Var}[\beta]} \approx 6.4\%$ . With these parameter values,  $S^* = 0.65$  and  $D^* = 0.67$ .

Although the total level of borrowing  $D$  is endogenous in our model, one insight that emerges from the Example is that the quantitatively interesting implications of our model are almost exclusively about  $S$ . Specifically, for most reasonable parameters and functional forms of  $v(\cdot)$  that we have explored,  $D^*$  is approximately  $2/3$ .

Proposition 2 has some potentially interesting implications for the shape of the yield curve. All else equal, factors that lower short-term debt issuance will raise  $v'(SD)$ , thereby raising the equilibrium price of short-term bonds relative to long-term bonds, i.e., increasing the slope of the

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<sup>22</sup> Proposition 2 also implies that government debt should be longer maturity if interest-rate volatility is high. While interest-rate volatility is in fact positively correlated with debt maturity in the time series, one should be cautious not to over-interpret this finding in light of our static model. In a more explicitly dynamic model, we would not expect government maturity structure to respond to short-lived shocks to volatility, which can be important in the data. For instance, when considering replacing a 5-year note with a strategy of rolling over a one-year bill five times, the government would be concerned about the total volatility of interest expense over the entire 5-year period, not simply the uncertainty about rates in the next year.

yield curve. Thus an increased concern with tax smoothing due to greater interest-rate volatility or higher costs of taxation will lower short-term issuance and thereby steepen the yield curve.

*E. Allowing for Monetary Services from Long-term Bonds*

To keep things simple, we have assumed that long-term bonds provide no monetary services whatsoever. However, all that we really need is for short-term bills to be more money-like than long-term bonds. Suppose instead that short-term bills offer one unit of monetary services and that long-term bonds offer  $0 < q < 1$  units of monetary services. In this case, while the government tilts less toward short-term debt than if  $q = 0$ , our basic results remain qualitatively the same.<sup>23</sup>

Digging deeper, one can also think about how the magnitude of  $q$  might be derived from first principles. Suppose that for a security to provide monetary services, it must be completely riskless between dates 0 and 1. Long-term government bonds are not inherently riskless, since their date-1 value depends on the realization of  $\beta$ . However, the private market may still be able to create some amount of riskless claims by using long-term bonds as collateral for short-term borrowing—as is done in the repo market. Following Geanakoplos (2010), the quantity of riskless collateralized claims that can be manufactured in this way is given by the *minimum* period-1 price of the long-term bond, i.e.,  $q = \beta^{\min} < 1$ .

A version of this argument is likely to hold even in a more elaborate long-horizon model where the interval between the periods becomes arbitrarily short, so long as  $v''(M_0) < 0$ . To see the intuition, think about the present value of the expected stream of future monetary services provided by a long-term bond that is originally issued at a market value of 100. Suppose that from one day to the next, the bond's price can rise or fall by at most one percent. Thus on the first day, it is possible

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<sup>23</sup> Suppose that  $M_0 = B_{0,1} + qB_{0,2} = DS + qD(1 - S)$ . The analog to (12') is  $Db(S - 1/2) = (1 - q)v'(DS + qD(1 - S))$ . Relative to the previous solution, the right-hand side is now scaled by a factor of  $1 - q < 1$  and the argument of  $v'(\cdot)$  also reflects the monetary services derived from long-term bonds.

to borrow 99 on a riskless overnight basis against the bond, i.e. to generate almost the same amount of monetary services as would come from 100 of short-term bills. However, over time, as the price of the bond fluctuates, the quantity of money that it can be used to collateralize will rise or fall. Given that  $v''(M_0) < 0$ , the value of such a risky stream of monetary services is less than the value of the sure stream of monetary services that would come from the 100 of short-term bills being rolled over repeatedly. The ratio of the value of the risky stream to that of the safe stream is equivalent to the concept of  $q$  in our simpler model.

*F. The Tax-Lowering Benefits of Short-Term Debt*

In the above analysis, we assumed that the social planner internalizes the monetary benefits enjoyed by households who invest in short-term debt, but does not put any weight on the tax savings that short-term debt generates—because these savings ultimately reflect a (potentially distortive) seignorage tax on its own citizens. Now we explore the opposite configuration, where the planner cares about the tax-lowering benefits of short-term debt, but not about the monetary services. As suggested above, this case is most naturally interpreted as corresponding to a situation where *all* the short-term debt is sold to foreign investors, and where a nationalistic planner looks out only for the interests of domestic households.

The nationalistic planner's date-1 problem is the same as before, since all monetary services are consumed at date 0. The planner's date-0 problem is similar to that in (7), except that we drop the direct utility of money services. Instead, as shown in the Appendix, there is a term which reflects the fact that the planner can lower date 0 taxes on domestic households by raising seignorage revenue from foreign investors:

$$\min_{\{B_{0,1}, B_{0,2}, B_{1,2}\}} \left[ \frac{1}{2} (\tau_0^2 + E[\tau_1^2] + E[\beta \tau_2^2]) - v'(M)M \right]. \quad (13)$$

The expression in (13) can be rewritten as

$$\min_{S,D} \left[ \frac{1}{2}(G-D-R(DS))^2 + \frac{D^2}{2} \left( b \left( S - \frac{1}{2} \right)^2 + \frac{1}{2} \right) - R(DS) \right] \quad (14)$$

where we make use of the substitutions  $D = B_{0,1} + B_{0,2}$ ,  $S = B_{0,1} / D$ ,  $M = DS$ , and where  $R(M) = v'(M)M$  denotes seignorage revenue—the interest savings from issuing more short-term bills to foreign investors. In the case of linear money utility (i.e.,  $v(M) = v'(M)M = \gamma M$ ), equation (14) is equivalent to equation (11) above. We restrict attention to money demand functions for which  $R'(M) > 0$  and  $R''(M) \leq 0$ .<sup>24</sup>

As shown in the Appendix, the solution takes the form

$$S^* = \frac{1}{2} + \frac{R'(D^*S^*)}{b} \frac{(G - R(D^*S^*) + 3)}{R'(D^*S^*)(G - R(D^*S^*) + 3) + 2(G - R(D^*S^*) - R'(D^*S^*))}, \quad (15)$$

and

$$D^* = \frac{(2 + R'(D^*S^*))(G - R(D^*S^*)) + R'(D^*S^*)}{3 + R'(D^*S^*)}. \quad (16)$$

Equations (15) and (16) yield the following proposition.

**Proposition 3:** *Let  $R(M) = v'(M)M$  and suppose that  $R'(M) > 0$  and  $R''(M) \leq 0$ . Then, with foreign investors holding all the short-term debt, and with a nationalistic planner, we have that, as in Proposition 2:  $S^* > 1/2$ ,  $\partial S^* / \partial b < 0$ , and  $\partial S^* / \partial G < 0$ .*

In summary, the case with foreign investors and a nationalistic planner works similarly to our baseline closed-economy case. The government still finds it optimal to issue more short-term debt than in the perfect tax-smoothing benchmark, and the comparative statics are directionally the same.

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<sup>24</sup> This is equivalent to  $v'(M) + Mv''(M) > 0$  and  $2v''(M) + Mv'''(M) \leq 0$ , which means that the money demand function cannot be too concave in the region of optimal  $S$ , and that  $v'''(M)$  cannot be too large. This rules out some utility functions, including  $v(M) = \gamma \log(M)$ , since in this case we have  $R(M) = Mv'(M) = \gamma$ .

#### IV. Adding Private-Sector Money Creation

We now extend the tradeoff model from Section III to allow private financial intermediaries to compete with the government in the provision of money-like securities. Our treatment of private-sector money creation follows Stein (2010). As in Stein's (2010) model, banks invest in real projects, and can choose whether to finance these projects by issuing short-term or long-term debt. However, given the structure of the risks on their projects, only short-term bank debt can ever be made riskless. Hence if they wish to capture the convenience premium associated with money-like claims, and thereby lower their financing costs, banks must issue short-term debt. While this has the same social benefits as government-created money, it can also lead to forced liquidations and fire sales. These fire sales in turn create social costs which the banks themselves do not fully internalize.

##### *A. Bank Investment and Financing Choices*

There are a continuum of banks in the economy with total measure one. Each bank invests a fixed amount  $I$  at date 0, financed entirely with borrowing from households—i.e., banks have no endowment of their own. With probability  $p$ , the good state occurs and the investment returns a certain amount  $F > I$  at date 2. With probability  $(1 - p)$ , the bad state occurs. In the bad state, expected output at date 2 is  $\lambda I < I$ , and there is some downside risk, with a positive probability of zero output. Importantly, the potential for zero output at date 2 in the bad state means that no amount of long-term bank debt can ever be made riskless, no matter how much seniority it is granted.

At date 1, a public signal reveals whether the good or bad state will prevail at date 2. Given the linearity of household preferences over consumption, the realization of the banks' investment risk has no direct impact on the price of new government bonds issued at date 1, which continues to be given by  $P_{1,2} = \beta$ . Therefore, to simplify the analysis we assume that this investment risk is independent of the realization of the discount factor  $\beta$ , which also happens at date 1. However, as

discussed further below, the analysis is virtually identical if we allow investment risk to be correlated with the realization of  $\beta$ .<sup>25</sup>

As demonstrated in Stein (2010), if the bad signal about investment output is observed at date 1, banks will be unable to roll over their short-term debt, and will be forced to sell assets to pay off departing short-term creditors. A bank that sells a fraction  $\Delta$  of its assets obtains date-1 proceeds of  $\Delta\beta k\lambda I$ , where  $k$  denotes the endogenous discount to fundamental value (i.e.,  $\beta\lambda I$ ) associated with the fire sale; we discuss the equilibrium determination of  $k$  momentarily. A bank that finances itself by issuing  $M_p$  dollars of short-term debt must therefore sell a fraction  $\Delta = M_p / (\beta k\lambda I)$  in the bad state when the discount factor is  $\beta$ .

Banks make an initial capital structure decision at date 0. They can finance their investment by issuing either short-term or long-term debt. The advantage of short-term debt is that as long as not too much is issued, it can be sufficiently well-collateralized as to be rendered riskless. This allows short-term bank debt to provide monetary services to households, and hence lowers its required rate of return. The disadvantage of short-term debt to banks is that in the bad state, it forces them to sell assets at discounted prices.

Since  $\Delta \leq 1$ , the upper bound on the quantity of private money  $M_p$  that banks can create is given by  $M_p \leq \beta^{\min} k\lambda I$ . In other words, banks can fully collateralize an amount of short-term debt equal to what they can obtain by selling off all of their assets at date 1 when the discount factor  $\beta$  is at its lowest possible value. Note that no bank will ever wish to issue an amount of short-term debt greater than this upper bound, since in this case the debt is no longer riskless, and hence does not sell at a premium, yet it still causes the bank to bear fire-sales costs.

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<sup>25</sup> This is because the date-0 price of money services,  $v'(M_0)$ , is the only factor connecting the debt maturity choices of the government and financial firms.

A bank that finances itself with an amount of private money  $M_P$  realizes total savings of  $M_P \cdot v'(M_0)$  relative to the case where it issues only more expensive long-term debt. Note that  $M_0$  is now the *total* amount of private plus government money, i.e.,  $M_0 = M_P + M_G = M_P + SD$ .

### B. Fire Sales

To pin down the fire-sale discount  $k$ , we follow Stein (2010) and assume that when a bank is forced to sell assets in the bad state at date 1, these assets are purchased by a set of “patient investors”.<sup>26</sup> Patient investors have a war chest of  $W$ , but cannot access capital markets at date 1 to raise more money in the event that the bad state occurs—in other words, their resources cannot be conditioned on the realization of the state. In addition to buying any assets sold by the banks, the patient investors can also allocate their war chest to investing in new physical projects at date 1. Given an investment of  $K$ , these new projects generate a discounted gross *social* return of  $\beta g(K)$ , where  $g' > 0$  and  $g'' < 0$ . However, these social returns are not fully pledgeable; only a fraction  $\phi < 1$  can be captured by the patient investors. Thus for an investment of  $K$ , the discounted gross *private* return available to the patient investors is  $\beta\phi g(K)$ . This imperfect pledgeability assumption is crucial in what follows, because it implies that the market fire-sale discount of  $k$  does not reflect the full social costs of underinvestment by patient investors in the bad state.

In the good state, banks do not sell any of their assets, so patient investors invest all of their capital in new projects, i.e.,  $K = W$ . In the bad state, banks are forced to sell assets to pay off the short-term debt that they have issued, and these assets sales are absorbed by the patient investors in equilibrium, so  $K = W - M_P$ . The fire-sale discount is determined by the condition that, in the bad

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<sup>26</sup> See also Shleifer and Vishny (2010) and Diamond and Rajan (2009) for similar formulations.

state, patient investors must be indifferent at date 1 between buying assets liquidated by the banks and investing in new projects. This implies that the fire-sale discount to fundamental value is

$$1/k = \beta\phi g'(K) = \beta\phi g'(W - M_p) \quad (17)$$

when the realization of the discount factor is  $\beta$ .

### C. Private Incentives for Money Creation

Individual banks maximize the expected present value of project cashflows, net of financing costs. Under the assumption that investment risk is independent of  $\beta$ , banks' objective is

$$\begin{aligned} \Pi &= (pF + (1-p)E[\beta\lambda I(1-\Delta) + (\Delta\beta\lambda kI)]) - I + M^P v'(M_0) \\ &= [pF + (1-p)\lambda I] - I + M_p[v'(M_0) - (1-p)(E[1/k] - 1)] \end{aligned} \quad (18)$$

Each bank treats total money  $M_0$  and hence  $v'(M_0)$  as given when choosing their capital structure, and similarly for the fire-sale discount  $k$ . Because investment  $I$  is independent of financing, we can focus just on the right-hand terms in (18), ignoring the first expression in brackets. These latter terms capture the tradeoff that banks face when creating more private money: doing so lowers their financing costs by an amount  $M_p \cdot v'(M_0)$ , but with probability  $(1-p)$  leads them to have to sell their assets at a discount to fundamental value. As noted above, the analysis here is virtually unchanged if we allow investment risk to be correlated with the realization of  $\beta$ .<sup>27</sup>

Substituting (17) into (18), equilibrium private money creation  $M_p^*$  is pinned down by

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<sup>27</sup> Let  $X \in \{G, B\}$  denote the realization of the investment risk and  $\beta_x = E[\beta | X = x]$ . Since  $1 = p\beta_G + (1-p)\beta_B$ , we have  $\beta_B = (1 - p\beta_G) / (1 - p)$  and  $Cov[\beta, \mathbf{1}_{\{X=G\}}] = p(\beta_G - 1)$ , so the parameter  $\beta_G$  governs the covariance between investment risk and  $\beta$ . We assume that  $\beta_G = \beta_B = 1$ . If we relax this assumption, banks' objective becomes  $\Pi = (p\beta_G F + (1-p)\beta_B \lambda I) - I + M_p (v'(M_0) - (1-p)(E[1/k | X=B] - 1))$  where  $E[1/k | X=B] = \beta_B \phi g'(W - M_p)$ . Thus, the expressions in this section would hold replacing  $g'(\cdot)$  with  $\beta_B g'(\cdot)$  and  $g''(\cdot)$  with  $\beta_B g''(\cdot)$ . Because the price of money services  $v'(M_0)$  is the only factor connecting government and private sector debt maturity choices, the covariance of investment risk and  $\beta$  is unimportant in our model, so we assume it is zero (i.e.,  $\beta_B = 1$ ) to avoid clutter.

$$v'(M_p^* + M_G) = (1-p)(\phi g'(W - M_p^*) - 1). \quad (19)$$

Note that this interior solution is only valid if  $M_p^*$  is below its technological upper bound, i.e., if  $M_p \leq \beta k \lambda I = (\lambda I) / (\phi g'(W - M_p))$ . As long as we do have an interior optimum,  $M_p^*$  is greater when the pledgeability parameter  $\phi$  is smaller. The intuition is that when  $\phi$  is small, banks do not internalize as much of the fire-sale costs associated with private money creation. The social costs of fire sales are given by the underinvestment in new date-1 projects that they ultimately displace; these projects have a marginal social value of  $g'(W - M_p^*)$ . But the private costs to the banks of fire sales are only felt to the extent that they result in a discount on the assets they sell, and this discount is related to  $\phi g'(W - M_p^*)$ .

#### D. The Social Planner's Problem

The social planner now maximizes total household utility, plus the net present value of date-1 investment by the patient investors:

$$U_{SOCIAL} = E[g(K) - K] + v(M_0) - \frac{1}{2} [\tau_0^2 + E[\tau_1^2] + E[\beta \tau_2^2]] \quad (20)$$

We begin by considering a first-best case in which the planner is able to directly control private money creation  $M_p$ , in addition to total government debt  $D$ , and the short-term government share  $S$ .

Denoting the social planner's first-best values with two asterisks we have the following result.

**Proposition 4:** *In the first-best outcome, the marginal costs of both public and private money are set equal to the marginal social benefit of additional money services:*

$$\underbrace{(1-p)(g'(W - M_p^{**}) - 1)}_{\text{Fire sale cost of private money}} = \underbrace{v'(M_G^{**} + M_p^{**})}_{\text{Social benefit of money}} = \underbrace{b(M_G^{**} - D^{**} / 2)}_{\text{Tax-smoothing cost of government money}} \quad (21)$$

**Proof:** *See appendix.*

The latter equality in (21), that  $v'(M_G^{**} + M_p^{**}) = b(M_G^{**} - D^{**} / 2)$ , is just a restatement of (12') from the government-only case, generalized to allow for the existence of private money. The former

equality, that  $(1-p)(g'(W - M_p^{**}) - 1) = v'(M_G^{**} + M_p^{**})$ , differs from the private solution in (19) to the extent that the pledgeability parameter  $\phi$  is less than one. Simply put, for any value of government money creation  $M_G$ , the social planner always prefers a smaller value of private money creation than do the banks acting on their own, i.e.,  $M_p^{**} < M_p^*$ . Again, this is because with  $\phi < 1$ , the banks do not fully internalize the underinvestment costs that accompany their money-creation activities.

While the first-best outcome in Proposition 4 is a useful benchmark, it may be difficult to implement, because it requires the government to directly control all forms of private money creation. For example, the government can try to impose a cap or a tax on short-term debt issuance by regulated banks. However, as pointed out by Gorton (2010) and others, a significant fraction of private money creation in the modern economy takes place in the unregulated “shadow banking” sector, and so may be hard to police effectively at low cost. Indeed, more stringent regulation of traditional commercial banks may simply drive a greater share of private money creation into the unregulated sector.

With this limitation in mind, an alternative way to frame the government’s problem is as a second-best one in which it still seeks to maximize (20), but where it cannot directly constrain private money creation, and hence where its only choice variables are those pertaining to the government’s own debt structure, namely  $D$  and  $S$ . It is in this second-best setting that our crowding-out intuition emerges. Consider what happens if the government issues more short-term debt at the margin. The convenience premium  $v'(\cdot)$  falls, making it less attractive for the private sector to cater to money demand. In particular, for any given level of government money  $M_G$ , the corresponding level of private money creation is pinned down by (19). This implicitly defines a private-sector reaction function to government short-term debt issuance,  $M_p^*(M_G)$ . It is straightforward to show that:

$$-1 < \frac{\partial M_p^*}{\partial M_G} = -\frac{v''(M_p^* + M_G)}{v''(M_p^* + M_G) + (1-p)\phi g''(W - M_p^*)} < 0. \quad (22)$$

Using the private-sector reaction function, we can back out the amount of public money necessary to crowd out all socially excessive private money creation. However, because it is also socially costly for the government to issue more short-term debt, it will not be optimal for the government to issue so much short-term debt as to push private money creation all the way down to  $M_p^{**}$ . Using this logic, we can derive the government's first-order condition for the optimal short-term share in the second-best case, which we denote by  $S^{***}$ .

$$\underbrace{D^{***} b(S^{***} - 1/2)}_{\text{Tax-smoothing cost of government money}} = \underbrace{v'(M_p^* + S^{***} D^{***})}_{\text{Marginal benefit of money services}} + \underbrace{(1-p)(\phi-1)g'(W - M_p^*) \frac{\partial M_p^*}{\partial M_G}}_{\text{Crowding out benefit}}. \quad (23)$$

Relative to our previous condition in (12'), equation (23) shows that there is an additional crowding-out benefit of short-term government debt. (The last term in (23) is positive when  $\phi < 1$  since  $\partial M_p^* / \partial M_G < 0$ .) Thus we have the following result.

**Proposition 5:** *When there are externalities associated with private money creation (i.e.  $\phi < 1$ ), a government that recognizes the crowding out benefits of short-term debt issues more short-term debt than a government that ignores its impact on private money creation. Moreover, if  $v'''(\cdot)$  and  $g'''(\cdot)$  are not too large, the crowding-out motive grows monotonically stronger as banks' failure to internalize fire-sales costs becomes more extreme, that is,  $\partial S^{***} / \partial \phi < 0$ .*

**Proof:** *See appendix.*

As our second-best framing makes clear, the crowding-out approach is only attractive to the extent that direct regulatory control of private money creation is infeasible. This is because such an approach requires the government to take on more refinancing risk—thereby imposing more volatile taxes on households—as the price of mitigating forced liquidations by banks. In reality, it is likely that regulation can be somewhat useful in constraining private money creation, albeit imperfectly so.

In such a setting, as Stein (2010) discusses, it may be optimal to deploy multiple tools in combination, using both some form of direct regulation and shortened government maturities together in an effort to deter excessive private issuance short-term debt.

When is the potential role for activist government maturity policy the greatest? Under the plausible assumption that  $v'''(M) > 0$ , government maturity policy can have its greatest effect when the convenience premium on T-bills is high which occurs when the *level* of government debt  $D$  is low. Holding fixed maturity  $S$ , as  $D$  increases, money demand is satiated, and the role for activist government policy is diminished. (Formally, when  $v'''(M) > 0$ ,  $\partial M_p^* / \partial M_G$  becomes less negative as the level of total government debt and, hence, short-term government debt rises.) During periods of low government debt levels, however—such as experienced by the United States in the late 1990s through 2007—there may be greater scope for activist maturity policy to discourage excessive private sector liquidity creation, at the same time incurring little in the form of rollover risk.

## **V. Further Implications**

### *A. Accommodating Shocks to the Capacity for Private Money Creation*

Thus far, our primary focus has been on what might be thought of as the long-run steady-state determinants of government debt-maturity policy. However, the model can also be used to think about sharp changes in policy that occur at times when the private sector's ability to manufacture short-term riskless claims unexpectedly becomes compromised, such as in the midst of a financial crisis, or when there is a flight to quality. To take an extreme case, consider equation (23) and think about suddenly setting  $M_p^* = 0$ , leading to a spike in the marginal value of monetary services. Assuming that this does not have too strong an offsetting effect on the crowding-out motive, the government should respond to such a shock by expanding the supply of riskless short-

term bills. Such reasoning seems to have been borne out in Treasury policy during the height of the 2008 financial crisis, when Treasury issued \$350 billion of short-term bills within a week of Lehman Brothers' failure, as part of the "Supplementary Financing Program." The proceeds from this program were lent to the Federal Reserve, which in turn bought long-maturity assets. The dramatic shortening of Treasury maturity structure in late 2008 is readily apparent in Figure 3.<sup>28</sup>

*B. Crowding out Private Money Creation: Practical Considerations*

In our simple model, there are only two maturities of government debt: short-term and long-term. Of course, in reality, the government can issue at any maturity from a few days to 30 years or potentially even longer. Although we have not solved a model along these lines explicitly, informal reasoning suggests that the existence of multiple maturities may allow the government more flexibility in terms of pursuing the dual objectives of money creation and tax smoothing. As can be seen in Figure 1A, there is what appears to be a very significant money premium in the shortest-maturity T-bills—those with maturities of less than four weeks. And much short-term financing on the part of private financial intermediaries is of extremely short maturity, often overnight. Thus it would seem that the Treasury could both create valuable incremental monetary services, as well as have a potentially powerful crowding-out effect on the private sector, by issuing more in the way of, say, two and four-week bills.

To get a sense of the relevant magnitudes, consider the average maturity distribution of T-bills between 1995 and 2009, a period during which the spread between yields on 4-week bills and

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<sup>28</sup> Following the abrupt shortening in late 2008, Figure 3 shows that government debt maturity quickly reverts in 2009. In explaining this reversal, the Treasury Borrowing Advisory Committee says that "the potential for inflation, higher interest rates, and rollover risk should be of material concern." (Report to the Secretary of the Treasury from the Treasury Borrowing Advisory Committee of the Securities Industry and Financial Markets Association, November 4, 2009. Available at <http://www.treas.gov/press/releases/tg348.htm>.) Thus, when the financial sector's ability to produce money-like securities disappeared, Treasury quickly ramped up its issuance of money-like claims; however as the anticipated debt burden grew, concerns about rollover risk eventually trumped the desire to cater to money demand, and maturity structure was once again lengthened.

their fitted values averaged minus 33 basis points (Figure 1B). According to the Treasury Bulletin, T-bills averaged 24% of total marketable debt outstanding. Based on CRSP data, we estimate that only 23% of bills had a maturity of 4 weeks or less during this period (the weighted average maturity of bills was 90 days). This suggests that simply by reshuffling maturities *within the category of T-bills*, there is significant scope for the Treasury to cater to the demand for monetary services.

Moreover, such a move into shorter-maturity bills need not come at the cost of much if any loss of tax-smoothing benefits—at least not if one thinks that the right summary statistic for the degree of tax smoothing is, roughly speaking, the weighted average duration of the overall government debt. In particular, any reduction in duration brought about by an increase in short-maturity bills can be easily offset by, for example, shifting some 10-year bonds into 20-year bonds. For concreteness, suppose the government had cut the weighted average maturity of T-bills by half, to an average of 45 days during the period. To offset the change in duration, the Treasury would have had to do a 10-year for 20-year swap equal to only 0.6% of total debt outstanding.<sup>29</sup>

Although this back-of-the-envelope reasoning is suggestive, we should emphasize a few important qualifications. First, our simple two-maturities model does not completely explain why so much of the monetary premium is concentrated at the very shortest end of the yield curve—i.e., it does not really say why one-month bills should have yields that are so much lower on average than three-month bills. Absent a better understanding of this phenomenon, any recommendation to shorten maturities within the category of T-bills must be somewhat tempered.

Second, as emphasized above, government maturity policy is likely to have its greatest effect when the convenience premium on T-bills is high, which occurs when the *level* of government debt is low. Thus, following significant increases in the debt-to-GDP ratio—such as the US experienced

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<sup>29</sup> These estimates are based on estimated duration of 8.34 for the 10-year bond and 13.21 for the 20-year bond on December 31, 2009:  $(0.24 \times 45 / 365) / (13.21 - 8.34) = 0.6\%$  of total debt outstanding.

in 2008 and 2009—the direct money benefits of such a policy would be diminished. Furthermore, the “crowding out” benefit of short-term debt may also be smaller when debt-to-GDP is large. These points are underscored by Figure 1B, which shows that the spread between 4-week T-bill yields and their fitted values is currently near a 30-year low in absolute terms.

Third, we suspect that our formulation of the tax-smoothing motive may not fully capture all the concerns that Treasury debt managers have in mind when they talk about the “rollover risk” associated with short-term financing. In particular, one can imagine that even if the household discount factor  $\beta$  were a fixed constant, it might be imprudent for the Treasury to put itself in the extreme position of having to, say, roll over all of its outstanding T-bills every day. Such a strategy might increase its vulnerability to bank-run-type problems, whereby a sudden fear about the government’s ability to service its debts so sharply elevates its interest expenses that it becomes a self-fulfilling prophecy.<sup>30</sup> It seems plausible that, in contrast to tax smoothing, the magnitude of this bank-run problem is not fully summarized by the average duration of the government’s debt, but rather depends on the amount of debt with the very shortest maturities. If so, this would be another reason to take the above recommendations with a grain of salt.

At the same time, before one invokes run risk as a reason for the government not to issue more at the shortest maturities, it is important to remember the core message of the model: what matters is not the absolute cost imposed on the government by a given debt-maturity structure, but rather its *comparative advantage* in bearing this cost. And while it is one thing to argue that the government may face some amount of run risk when issuing a large quantity of short-maturity paper, it is quite another to argue that it is not better-suited than the private sector to bearing such run risk. In other words, we expect that the basic comparative-advantage insight of our model is likely to

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<sup>30</sup> See Bohn (2010) for a recent discussion of this mechanism.

survive even in a more elaborate setting where the government faces not just a tax-smoothing problem, but also some degree of run risk when it issues at very short maturities.

## **VI. Conclusions**

A growing body of empirical evidence suggests that low-risk short-term debt securities provide significant monetary services to investors. Moreover, while both the government and private-sector financial intermediaries have the capacity to produce such money-like claims, the private sector's incentives to engage in money creation may be excessive from a social point of view, because intermediaries do not fully internalize the fire-sale costs associated with their reliance on short-term funding.

We have argued that these two observations can be used as a basis for thinking about government debt maturity policy. Perhaps the most novel insight to emerge from our framework is that government debt maturity can be a useful complement to prudential financial regulation. Rather than addressing private-sector financial fragility solely by writing rules that attempt to constrain the use of short-term debt by intermediaries, the government can also reduce the incentives that lead to excessive private money creation by issuing more short-term debt of its own, thereby compressing the monetary premium and crowding out private issuance. Particularly in a world where it is so easy for financial activity to migrate out of the reach of regulators, and thereby frustrate the intentions of more traditional capital or liquidity regulations, this crowding-out approach may be a powerful tool.

## Appendix- Omitted Proofs

**Proof of Proposition 1:** The planner's date-1 problem is given by Eq. (9). Differentiating with respect to  $B_{1,2}$  yields the first order condition

$$-\beta(B_{0,1} - \beta B_{1,2}) + \beta(B_{1,2} + B_{0,2}) = 0. \quad (\text{A1})$$

The second order condition is  $\beta(1 + \beta) > 0$ . The solution to (A1) is then

$$B_{1,2} = (B_{0,1} - B_{0,2}) / (1 + \beta), \quad (\text{A2})$$

which implies that

$$\tau_1 = \tau_2 = (B_{0,1} + B_{0,2}\beta) / (1 + \beta). \quad (\text{A3})$$

Consider the problem at  $t = 0$  where  $\tau_0 = G - B_{0,1} - B_{0,2}$ . Substituting (A3) into Eq. (8), yields

$$\min_{\{B_{0,1}, B_{0,2}\}} \left[ \frac{1}{2} \left( (G - B_{0,1} - B_{0,2})^2 + E \left[ \left( \frac{B_{0,1} + \beta B_{0,2}}{1 + \beta} \right)^2 \right] \right) + E \left[ \beta \left( \frac{B_{0,1} + \beta B_{0,2}}{1 + \beta} \right)^2 \right] \right], \quad (\text{A4})$$

which is equivalent to

$$\min_{\{S, D\}} \left[ \frac{1}{2} (G - D)^2 + \frac{1}{2} E \left[ \frac{(SD + (1 - S)D\beta)^2}{1 + \beta} \right] \right] \quad (\text{A5})$$

where we have made the change of variables to  $D = B_{0,1} + B_{0,2}$ , and  $S = B_{0,1} / D$ .

We first differentiate (A5) with respect to  $S$ , yielding

$$D^2 E \left[ \frac{1 - \beta}{1 + \beta} (\beta + S(1 - \beta)) \right] = 0. \quad (\text{A6})$$

We note that  $S^* = 1/2$  is the solution to this the first order condition, since

$$E \left[ \frac{1 - \beta}{1 + \beta} \left( \beta + \frac{1}{2}(1 - \beta) \right) \right] = E \left[ \frac{1}{2}(1 - \beta) \right] = 0. \quad (\text{A7})$$

Noting that

$$E \left[ \frac{\beta(1 - \beta)}{1 + \beta} \right] + \frac{1}{2} E \left[ \frac{(1 - \beta)^2}{1 + \beta} \right] = 0, \quad (\text{A8})$$

and defining  $b = E[(1 - \beta)^2 / (1 + \beta)]$ , we can rewrite (A6) as

$$D^2 b (S - 1/2) = 0. \quad (\text{A9})$$

We now solve for  $D$ , the level of debt. Note that

$$E\left[\frac{(S^* + (1 - S^*)\beta)^2}{1 + \beta}\right] = E\left[\frac{(\frac{1}{2} + \frac{1}{2}\beta)^2}{1 + \beta}\right] = \frac{1}{4}E[1 + \beta] = \frac{1}{2}. \quad (\text{A10})$$

Optimal  $D$  thus satisfies

$$\min_D \left[ \frac{1}{2}(G - D)^2 + \frac{1}{4}D^2 \right], \quad (\text{A11})$$

which has first order condition

$$-(G - D) + D/2 = 0 \Rightarrow D^* = 2G/3. \quad (\text{A12})$$

Using these facts, we can show that

$$\frac{1}{2}E\left[\frac{(SD + (1 - S)D\beta)^2}{1 + \beta}\right] = \frac{D^2}{2}\left(b(S - 1/2)^2 + 1/2\right). \quad (\text{A13})$$

One can confirm that the second order conditions are satisfied at this solution. Moreover, as demonstrated below, the objective is globally convex in  $B_{0,1}$  and  $B_{0,2}$ , so the solution is unique.<sup>31</sup>

*Allowing the government to issue risky securities*

We now show that, in the absence of money demand, these results continue to hold if we allow for arbitrary risky securities whose payouts are possibly contingent on the realization of  $\beta$ . Specifically, we now allow the government to issue face value  $B_R$  of risky securities with payoff  $X_R(\beta)$  at  $t = 2$ . We assume that these securities are fairly priced by households with price  $P_R = E[\beta X_R(\beta)]$ . The government's budget constraint becomes

$$\begin{aligned} t = 0: G &= \tau_0 + B_{0,1} + B_{0,2} + B_R P_R \\ t = 1: B_{0,1} &= \tau_1 + B_{1,2} P_{1,2} \\ t = 2: B_{1,2} + B_{0,2} + B_R X_R(\beta) &= \tau_2 \end{aligned} \quad (\text{A14})$$

As above, we work backwards from  $t = 1$ . The planner's date-1 problem is

$$\min_{B_{1,2}} \left[ \frac{1}{2}(\tau_1^2 + \beta \tau_2^2) \right] = \min_{B_{1,2}} \left[ \frac{1}{2}(B_{0,1} - B_{1,2}\beta)^2 + \frac{1}{2}\beta(B_{1,2} + B_{0,2} + B_R X_R(\beta))^2 \right]. \quad (\text{A15})$$

Taking first order conditions with respect to  $B_{1,2}$  yields

$$B_{1,2} = (B_{0,1} - B_{0,2} - B_R X_R(\beta)) / (1 + \beta), \quad (\text{A16})$$

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<sup>31</sup> While the objective may not be globally convex in  $S$  and  $D$ , global convexity in  $B_{0,1}$  and  $B_{0,2}$  shows the solution is unique.

which implies  $\tau_1 = \tau_2 = (B_{0,1} + B_{0,2}\beta + B_R\beta X_R(\beta)) / (1 + \beta)$ .

Consider the problem at  $t = 0$  where  $\tau_0 = G - B_{0,1} - B_{0,2} - B_R P_R$ :

$$\min_{\{B_{0,1}, B_{0,2}, B_R\}} \left[ \frac{1}{2} (G - B_{0,1} - B_{0,2} - B_R P_R)^2 + \frac{1}{2} E \left[ \frac{(B_{0,1} + B_{0,2}\beta + B_R\beta X_R(\beta))^2}{1 + \beta} \right] \right]. \quad (\text{A17})$$

The first order conditions are

$$\begin{aligned} -(G - B_{0,1} - B_{0,2} - B_R P_R) + E \left[ \frac{1}{1 + \beta} (B_{0,1} + B_{0,2}\beta + B_R\beta X_R(\beta)) \right] &= 0, \\ -(G - B_{0,1} - B_{0,2} - B_R P_R) + E \left[ \frac{\beta}{1 + \beta} (B_{0,1} + B_{0,2}\beta + B_R\beta X_R(\beta)) \right] &= 0, \\ -P_R (G - B_{0,1} - B_{0,2} - B_R P_R) + E \left[ \frac{\beta X_R(\beta)}{1 + \beta} (B_{0,1} + B_{0,2}\beta + B_R\beta X_R(\beta)) \right] &= 0. \end{aligned} \quad (\text{A18})$$

Since  $E[\beta] = 1$  and  $P_R = E[\beta X_R(\beta)]$ , it is easy to see that  $B_{0,1} = B_{0,2} = G/3$  and  $B_R = 0$  satisfies these three conditions for an arbitrary risky security.

We now show that the objective function is globally convex in its three arguments, showing that this is the unique solution to the planner's problem. Specifically, the Hessian is

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & P_R \\ 1 & 1 & P_R \\ P_R & P_R & P_R^2 \end{bmatrix} + \begin{bmatrix} E[(1 + \beta)^{-1}] & E[(1 + \beta)^{-1}\beta] & E[(1 + \beta)^{-1}\beta X_R] \\ E[(1 + \beta)^{-1}\beta] & E[(1 + \beta)^{-1}\beta^2] & E[(1 + \beta)^{-1}\beta^2 X_R] \\ E[(1 + \beta)^{-1}\beta X_R] & E[(1 + \beta)^{-1}\beta^2 X_R] & E[(1 + \beta)^{-1}\beta^2 X_R^2] \end{bmatrix} \quad (\text{A19})$$

The first matrix is positive semi-definite with eigenvalues of  $2 + P_R^2 > 0$  and 0 (multiplicity 2). Let

$$E^*[Z] = \frac{E[(1 + \beta)^{-1}Z]}{E[(1 + \beta)^{-1}]} \quad (\text{A20})$$

denote the expectation with respect to the  $(1 + \beta)^{-1}$  twisted probability measure and note that the second term can be written as

$$E[(1 + \beta)^{-1}] \cdot E^* \left[ \begin{bmatrix} 1 \\ \beta \\ \beta X_R \end{bmatrix} \begin{bmatrix} 1 \\ \beta \\ \beta X_R \end{bmatrix}' \right] \quad (\text{A21})$$

which is positive definite, assuming that 1,  $\beta$ , and  $\beta X_R$  are linearly independent. This shows that the objective function is globally convex for an arbitrary  $X_R$  and, hence, that the unique optimum is  $B_{0,1}^* = B_{0,2}^* = G/3$  and  $B_R^* = 0$ .<sup>32</sup>

**Proof of Proposition 2:** The planner solves

$$\min_{S,D} \left[ \frac{1}{2}(G-D)^2 + \frac{D^2}{2} \left( b(S-1/2)^2 + 1/2 \right) - \gamma f(SD) \right]. \quad (\text{A22})$$

The first order conditions for  $S$  and  $D$  are

$$0 = D^2 b(S-1/2) - D\gamma f'(SD), \quad (\text{A23})$$

and

$$0 = -(G-D) + D \left( b(S-1/2)^2 + 1/2 \right) - S\gamma f'(SD). \quad (\text{A24})$$

The solution takes the form

$$\begin{aligned} S^* &= \frac{1}{2} + \frac{\gamma f'(S^*D^*)}{D^*b} \\ D^* &= \frac{2}{3}G + \frac{\gamma f'(S^*D^*)}{3}. \end{aligned} \quad (\text{A25})$$

Note that the Hessian evaluated at the solution in (A25) is

$$\mathbf{H} = \begin{bmatrix} D^2b - D^2\gamma f''(SD) & \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) \\ \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) & b(S-1/2)^2 + \frac{3}{2} - \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right)^2 \gamma f''(SD) \end{bmatrix}, \quad (\text{A26})$$

with determinant  $\det(\mathbf{H}) = 3bD^2/2 - \gamma f''(SD)(3/2 + b/4) > 0$ , so this is a minimum. Furthermore, so long as  $f''(\cdot) \leq 0$ , the objective is globally convex in  $B_{0,1}$  and  $B_{0,2}$  and the solution is unique.

We now derive the comparative statics. Consider the impact of  $\gamma$  on  $S^*$  and  $D^*$ :

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<sup>32</sup> The matrix is positive semi-definite if these three variables are linearly dependent. Specifically, if  $X_R = c$ , a constant, the security is equivalent to 2-period riskless bonds. In this case, all solutions with  $B_{0,2} + cB_R = G/3$  are equivalent, so while  $B_{0,2} + cB_R$  is determined, neither  $B_{0,2}$  nor  $B_R$  is determined. Similarly, if  $X_R = c/\beta$  so that  $\beta X_R = c$ , the security is equivalent to 1-period riskless from an ultimate tax-perspective. Of course, these are simply different ways of implementing perfect tax-smoothing, so these two indeterminate cases do not alter our substantive conclusion.

$$\begin{aligned}
\begin{bmatrix} \partial S^* / \partial \gamma \\ \partial D^* / \partial \gamma \end{bmatrix} &= \begin{bmatrix} D^2 b - D^2 \gamma f''(SD) & \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) \\ \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) & b(S-1/2)^2 + \frac{3}{2} - \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right)^2 \gamma f''(SD) \end{bmatrix}^{-1} \begin{bmatrix} Df'(SD) \\ Sf'(SD) \end{bmatrix} \\
&= \frac{2f'(SD)}{6b - \gamma(6-b)f''(SD)} \begin{bmatrix} \frac{3D - \gamma f'(SD)}{D^2} \\ b \end{bmatrix}.
\end{aligned} \tag{A27}$$

Since  $D^* = 2G/3 + \gamma f'(S^* D^*)/3$ , we have  $3D^* > \gamma f'(S^* D^*)$  since  $G > 0$ . Therefore, we have  $\partial S^* / \partial \gamma > 0$  and  $\partial D^* / \partial \gamma > 0$ .

We next examine the impact of  $b$  on  $S^*$  and  $D^*$ :

$$\begin{aligned}
\begin{bmatrix} \partial S^* / \partial b \\ \partial D^* / \partial b \end{bmatrix} &= \begin{bmatrix} D^2 b - D^2 \gamma f''(SD) & \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) \\ \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) & b(S-1/2)^2 + \frac{3}{2} - \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right)^2 \gamma f''(SD) \end{bmatrix}^{-1} \begin{bmatrix} D^2(S-1/2) \\ D(S-1/2)^2 \end{bmatrix} \\
&= \frac{2f'(SD)}{6b - \gamma(6-b)f''(SD)} \begin{bmatrix} \frac{\gamma}{b^2 D^2} (f''(SD) \gamma^2 f'(SD) + f''(SD) b D \gamma / 2 - 3b D) \\ \frac{-\gamma^2 f''(SD)}{b} \end{bmatrix}.
\end{aligned} \tag{A28}$$

Thus,  $\partial S^* / \partial b < 0$  and  $\partial D^* / \partial b \geq 0$ .

Last, the impact of  $G$  is given by:

$$\begin{aligned}
\begin{bmatrix} \partial S^* / \partial G \\ \partial D^* / \partial G \end{bmatrix} &= \begin{bmatrix} D^2 b - D^2 \gamma f''(SD) & \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) \\ \gamma f'(SD) - D \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right) \gamma f''(SD) & b(S-1/2)^2 + \frac{3}{2} - \left( \frac{1}{2} + \frac{\gamma f'(SD)}{Db} \right)^2 \gamma f''(SD) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&= \frac{2}{6b - \gamma(6-b)f''(SD)} \begin{bmatrix} \frac{\gamma}{b^2 D^2} (f''(SD) b D - 2b f'(SD) + 2\gamma f'(SD) f''(SD)) \\ 2b - 2f''(SD) \gamma \end{bmatrix}.
\end{aligned} \tag{A29}$$

Thus,  $\partial S^* / \partial G < 0$  and  $\partial D^* / \partial G > 0$ .

**Proof of Proposition 3:** For simplicity, assume that all government debt is sold to foreign investors, including both short-term and long-term debt. Since all debt is sold to foreign investors, domestic household consumption is given by:

$$\begin{aligned}
C_0 &= 1 - \tau_0 - (1/2)\tau_0^2 \\
C_1 &= 1 - \tau_1 - (1/2)\tau_1^2 \\
C_2 &= 1 - \tau_2 - (1/2)\tau_2^2.
\end{aligned} \tag{A30}$$

(Compare (A30) with equation (5) in the text.) Substituting the government's budget constraint in (4) into (A30) we obtain

$$\begin{aligned}
C_0 &= 1 - (1/2)\tau_0^2 - G + (B_{0,1}P_{0,1} + B_{0,2}P_{0,2}) \\
C_1 &= 1 - (1/2)\tau_1^2 + (B_{1,2}P_{1,2} - B_{0,1}) \\
C_2 &= 1 - (1/2)\tau_2^2 - (B_{1,2} + B_{0,2}).
\end{aligned} \tag{A31}$$

As before, domestic consumption is impacted by distortionary tax costs, but relative to (6) there are additional terms which reflect *net* foreign borrowing in each period.

Assuming that  $P_{0,1} = 1 + v'(B_{0,1})$ ,  $P_{0,2} = 1$ , and  $P_{1,2} = \beta$  (i.e., foreign investors have the same preference shock as domestic households), it is easy to see that

$$U = C_0 + E[C_1 + \beta C_2] = 3 - G - \frac{1}{2}[\tau_0^2 + E[\tau_1^2] + E[\beta\tau_2^2]] + B_{0,1}v'(B_{0,1}). \tag{A32}$$

Thus, dropping constants, the nationalistic planner's problem can be rewritten as

$$\min_{S,D} \left[ \frac{1}{2}(G - D - R(DS))^2 + \frac{D^2}{2} \left( b \left( S - \frac{1}{2} \right)^2 + \frac{1}{2} \right) - R(DS) \right] \tag{A33}$$

where  $R(M) = v'(M)M$  denotes seignorage revenue and we assume  $R'(M) > 0$  and  $R''(M) \leq 0$ .

The first order conditions for  $S$  and  $D$  are

$$0 = -(1 + G - D - R(DS))DR'(SD) + D^2b(S - 1/2), \tag{A34}$$

and

$$0 = -(G - D - R(DS)) - (1 + G - D - R(DS))SR'(SD) + D(b(S - 1/2)^2 + 1/2). \tag{A35}$$

Solving (A34) and (A35) shows that the solution takes the form

$$\begin{aligned}
S^* &= \frac{1}{2} + \frac{R'(D^*S^*)}{b} \frac{(G - R(D^*S^*) + 3)}{R'(D^*S^*)(G - R(D^*S^*) + 3) + 2(G - R(D^*S^*) - R'(D^*S^*))} \\
D^* &= \frac{(2 + R'(D^*S^*))(G - R(D^*S^*)) + R'(D^*S^*)}{3 + R'(D^*S^*)}.
\end{aligned} \tag{A36}$$

We now derive the comparative statics. Consider the impact of  $b$  on  $S^*$  and  $D^*$ . We have:

$$\begin{aligned}
\begin{bmatrix} \partial S^* / \partial b \\ \partial D^* / \partial b \end{bmatrix} &= \begin{bmatrix} (DR')^2 - (1 + G - D - R)D^2R'' + D^2b & (1 + SR')DR' - (1 + G - D - R)DSR'' + Db(S - 1/2) \\ (1 + SR')DR' - (1 + G - D - R)DSR'' + Db(S - 1/2) & (1 + SR')^2 - (1 + G - D - R)S^2R'' + b(S - 1/2)^2 + 1/2 \end{bmatrix}^{-1} \begin{bmatrix} D^2(S - 1/2) \\ D(S - 1/2)^2 \end{bmatrix} \\
&= \frac{2(S - 1/2)}{6b + 4bR' + (2 + b)(R')^2 - R''(6 + b)(1 + G - D - R)} \begin{bmatrix} -(R' + 2SR' + S(R')^2 - SR''(1 + G - D - R) + 3) \\ D(2R' + (R')^2 - R''(1 + G - D - R)) \end{bmatrix}.
\end{aligned}$$

Since  $S^* > 1/2$ ,  $(1 + G - D - R) = (1 + \tau_0) > 0$ ,  $R' > 0$ , and  $R'' < 0$ , we have  $\partial S^* / \partial b < 0$  and  $\partial D^* / \partial b > 0$ .

The impact of  $G$  is given by:

$$\begin{aligned} \begin{bmatrix} \partial S^* / \partial G \\ \partial D^* / \partial G \end{bmatrix} &= \begin{bmatrix} (DR')^2 - (1+G-D-R)D^2R'' + D^2b & (1+SR')DR' - (1+G-D-R)DSR'' + Db(S-1/2) \\ (1+SR')DR' - (1+G-D-R)DSR'' + Db(S-1/2) & (1+SR')^2 - (1+G-D-R)S^2R'' + b(S-1/2)^2 + 1/2 \end{bmatrix}^{-1} \begin{bmatrix} DR' \\ (1+SR') \end{bmatrix} \\ &= \frac{1}{6b + 4bR' + (2+b)(R')^2 - R''(6+b)(1+G-D-R)} \begin{bmatrix} (4SR''(1+G-D-R) - 4(R'/D))/D \\ 2b + bR' - 2R''(1+G-D-R) \end{bmatrix}, \end{aligned}$$

where the second line follows by using (A36) to simplify the resulting expression. Thus,  $\partial S^* / \partial G < 0$  and  $\partial D^* / \partial G > 0$ .

**Proof of Propositions 4 and 5:** We solve the second-best problem. The first-best problem can be seen as a special case of the second-best problem which is obtained by setting  $\phi = 1$ . We start with the planner's objective function

$$U_{SOCIAL} = E[g(K) - K] + v(M_0) - \frac{1}{2} [\tau_0^2 + E[\tau_1^2] + E[\beta\tau_2^2]]. \quad (\text{A37})$$

We plug into this the reaction function implicitly defined by equation (19) in the text,  $M_p^*(M_G, \phi)$ :

$$\begin{aligned} U_{SOCIAL} &= p[g(W) - W] + (1-p)[g(W - M_p^*(SD, \phi)) - (W - M_p^*(SD, \phi))] \\ &\quad + v(M_p^*(SD, \phi) + SD) - \frac{1}{2}(G-D)^2 - \frac{D^2}{2} \left( b \left( S - \frac{1}{2} \right)^2 + \frac{1}{2} \right). \end{aligned} \quad (\text{A38})$$

The first order condition for  $S^{***}$  is

$$\begin{aligned} 0 &= -(1-p)(g'(W - M_p^*) - 1)D \frac{\partial M_p^*}{\partial M_G} + v'(SD + M_p^*)D \left( 1 + \frac{\partial M_p^*}{\partial M_G} \right) - D^2b(S-1/2) \\ &= (1-p)(\phi-1)g'(W - M_p^*) \frac{\partial M_p^*}{\partial M_G} D + v'(SD + M_p^*)D - D^2b(S-1/2). \end{aligned} \quad (\text{A39})$$

Where the second line follows from the fact that  $v'(M_p^* + SD) = (1-p)(\phi g'(W - M_p^*) - 1)$ .

Rearranging and dividing by  $D$ , we obtain

$$Db(S-1/2) = (1-p)(\phi-1)g'(W - M_p^*) \frac{\partial M_p^*}{\partial M_G} + v'(SD + M_p^*), \quad (\text{A40})$$

which is equation (23) in the text. Note that the first-best solution given in equation (21) obtains as a special case of (A40) by setting  $\phi = 1$ . The first order condition for  $D^{***}$  can be written as

$$0 = (1-p)(\phi-1)g'(W - M_p^*) \frac{\partial M_p^*}{\partial M_G} S + v'(SD + M_p^*)S + (G-D) - D \left( b(S-1/2)^2 + 1/2 \right). \quad (\text{A41})$$

We later use the fact that the first order conditions for  $S^{***}$  and  $D^{***}$  imply

$$G = D^{***} (3/2 - b(S^{***} - 1/2)/2). \quad (\text{A42})$$

Letting

$$\lambda_p = \frac{\partial M_p^*}{\partial M_G} = -\frac{v''(M_G + M_p^*(M_G, \phi))}{v''(M_G + M_p^*(M_G, \phi)) + (1-p)\phi g''(W - M_p^*(M_G, \phi))} < 0 \quad (\text{A43})$$

(recall that  $-1 < \lambda_p < 0$ ) denote the crowding out effect of short-term government issuance and

$$\Psi = (1-p)(1-\phi)g''(\cdot)\lambda_p^2 + (1-p)(\phi-1)g'(\cdot)\left(\frac{\partial \lambda_p}{\partial M_G} + \frac{\partial \lambda_p}{\partial M_p} \lambda_p\right) + v''(\cdot)(1 + \lambda_p). \quad (\text{A44})$$

The Hessian for this problem at the solution defined by (A39) and (A41) is

$$\mathbf{H} = \begin{bmatrix} \Psi D^2 - bD^2 & \Psi SD - Db(S - 1/2) \\ \Psi SD - Db(S - 1/2) & \Psi S^2 - b(S - 1/2)^2 - 3/2 \end{bmatrix} \quad (\text{A45})$$

We assume that  $\Psi < 0$  at  $S = S^{***}$  and  $D = D^{***}$ . This ensures that the second order conditions are satisfied since this implies  $\det(\mathbf{H}) = D^2[(3/2)b - (3/2)\Psi - (1/4)b\Psi] > 0$ . As above, if  $\Psi < 0$ , the objective will be globally concave in  $B_{0,1}$  and  $B_{0,2}$ , ensuring uniqueness.

We now examine the comparative statics with respect to  $\phi$ . Differentiate the first order condition for  $S$  with respect to  $\phi$  to obtain:

$$\left( (1-p)g'(\cdot)\lambda_p - (1-p)(\phi-1)g''(\cdot)\lambda_p \frac{\partial M_p^*}{\partial \phi} + (1-p)(\phi-1)g'(\cdot) \left[ \frac{\partial \lambda_p}{\partial \phi} + \frac{\partial \lambda_p}{\partial M_p} \frac{\partial M_p^*}{\partial \phi} \right] + v''(\cdot) \frac{\partial M_p^*}{\partial \phi} \right) D. \quad (\text{A46})$$

Noting that

$$\begin{aligned} \frac{\partial M_p^*}{\partial \phi} &= \frac{(1-p)g'(\cdot)}{v''(\cdot) + (1-p)\phi g''(\cdot)} < 0 \\ \frac{\partial \lambda_p}{\partial \phi} &= \frac{(1-p)v''(\cdot)g''(\cdot)}{[v''(\cdot) + (1-p)\phi g''(\cdot)]^2} > 0 \\ \frac{\partial \lambda_p}{\partial M_p} &= -(1-p)\phi \frac{v'''(\cdot)g''(\cdot) + v''(\cdot)g'''(\cdot)}{[v''(\cdot) + (1-p)\phi g''(\cdot)]^2}, \end{aligned} \quad (\text{A47})$$

which imply  $(1-p)g'(\cdot)\lambda_p + v''(\cdot)(\partial M_p^* / \partial \phi) = 0$ , the expression in (A46) simplifies to

$$\begin{aligned}
& -(1-p)(\phi-1)g''(\cdot)\lambda_p \frac{\partial M_p^*}{\partial \phi} + (1-p)(\phi-1)g'(\cdot) \left[ \frac{\partial \lambda_p}{\partial \phi} + \frac{\partial \lambda_p}{\partial M_p} \frac{\partial M_p^*}{\partial \phi} \right] \\
& = \frac{(\phi-1)g'(\cdot)(1-p)^2}{[v''(\cdot) + (1-p)\phi g''(\cdot)]^2} \left( 2v''(\cdot)g''(\cdot) - (1-p)\phi g'(\cdot) \frac{v'''(\cdot)g''(\cdot) + v''(\cdot)g'''(\cdot)}{v''(\cdot) + (1-p)\phi g''(\cdot)} \right) D^{***}.
\end{aligned} \tag{A48}$$

The expression in (A48) will be negative when  $\phi < 1$  so long as  $g'''(\cdot)$  and  $v'''(\cdot)$  are not too large which we assume is the case.<sup>33, 34</sup>

Combining all of this we have

$$\begin{aligned}
\left[ \frac{\partial S^{***}}{\partial D} \ / \ \frac{\partial \phi}{\partial \phi} \right] & = \left( \frac{(1-\phi)g'(\cdot)(1-p)^2}{[v''(\cdot) + (1-p)\phi g''(\cdot)]^2} \left( 2v''(\cdot)g''(\cdot) - (1-p)\phi g'(\cdot) \frac{v'''(\cdot)g''(\cdot) + v''(\cdot)g'''(\cdot)}{v''(\cdot) + (1-p)\phi g''(\cdot)} \right) \right) \times \\
& \quad \begin{bmatrix} \Psi D^2 - bD^2 & \Psi SD - Db(S-1/2) \\ \Psi SD - Db(S-1/2) & \Psi S^2 - b(S-1/2)^2 - 3/2 \end{bmatrix}^{-1} \begin{bmatrix} D \\ S \end{bmatrix} \\
& = \frac{1}{\det(\mathbf{H})} \left( \frac{(1-\phi)g'(\cdot)(1-p)^2}{[v''(\cdot) + (1-p)\phi g''(\cdot)]^2} \left( 2v''(\cdot)g''(\cdot) - (1-p)\phi g'(\cdot) \frac{v'''(\cdot)g''(\cdot) + v''(\cdot)g'''(\cdot)}{v''(\cdot) + (1-p)\phi g''(\cdot)} \right) \right) \begin{bmatrix} -G \\ -bD^{***2} / 2 \end{bmatrix},
\end{aligned} \tag{A49}$$

where we have made use of the fact that  $G = D^{***} \left( 3/2 - b(S^{***} - 1/2) / 2 \right)$  from (A42). Thus,  $\partial S^{***} / \partial \phi < 0$  and  $\partial D^{***} / \partial \phi < 0$  so long as  $\phi < 1$  and  $g'''(\cdot)$  and  $v'''(\cdot)$  are not too large (this also implies that  $M_G^{***} = D^{***} S^{***}$  is decreasing in  $\phi$ ).

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<sup>33</sup> If  $g'''(\cdot)$  and  $v'''(\cdot)$  were too large, then  $\delta \lambda_p / \delta \phi = (\partial \lambda_p / \partial \phi) + (\partial \lambda_p / \partial M_p) (\partial M_p^* / \partial \phi)$ , the total derivative of  $\lambda_p$  with respect to  $\phi$ , would be a large negative number. In this case, as  $\phi$  declined,  $|\lambda_p|$  would decline significantly (since  $\lambda_p < 0$ ,  $\lambda_p$  would rise), greatly reducing the crowding-out benefit from issuing short-term government debt. If this force were strong enough, it could outweigh the direct effect,  $-(1-p)(\phi-1)g''(\cdot)\lambda_p (\partial M_p^* / \partial \phi) < 0$ , which reflects the fact that  $M_p^*$  rises as  $\phi$  falls, exacerbating the under-investment problem in the bad state. However, note that  $\partial \lambda_p / \partial \phi > 0$  which reflects the fact that, holding  $M_p$  and  $M_G$  fixed, private money creation becomes *more* not less sensitive to the money premium as  $\phi$  declines because firms more severely underweight its costs. Thus, if  $\partial \lambda_p / \partial M_p$  is not too large (i.e. the functions are well approximated locally by quadratics, so  $g'''(\cdot)$  and  $v'''(\cdot)$  are small), then we will have  $\delta \lambda_p / \delta \phi > 0$ , implying that  $\partial S^{***} / \partial \phi < 0$  and  $\partial D^{***} / \partial \phi < 0$ .

<sup>34</sup> The second order conditions for  $S^{***}$  and  $D^{***}$  also depend on  $g'''(\cdot)$  and  $v'''(\cdot)$  through  $\Psi$  which we assume is negative. Specifically, one can show that  $(\partial \lambda_p / \partial M_G) + (\partial \lambda_p / \partial M_p) \lambda_p$  is increasing in  $v'''(\cdot)$  and decreasing in  $g'''(\cdot)$ . Therefore,  $g'''(\cdot)$  cannot be too large if the second order conditions for  $S^{***}$  and  $D^{***}$  are to hold. Specifically, if  $g'''(\cdot)$  is too large, a rise in  $M_G$  would significantly raise  $|\lambda_p|$ , implying an increasing as opposed to diminishing crowding out benefit from issuing more short-term debt.

Finally, note that

$$\begin{aligned} \frac{\delta}{\delta\phi} [M_P^{***}(\phi, M_G^{***}(\phi))] &= \frac{\partial M_P^{***}}{\partial\phi} + \frac{\partial M_P^{***}}{\partial M_G} \frac{\partial M_G^{***}}{\partial\phi} > \frac{\partial M_P^{***}}{\partial\phi} \\ \frac{\delta}{\delta\phi} [M_P^{***}(\phi, M_G^{***}(\phi)) + M_G^{***}(\phi)] &= \frac{\partial M_P^{***}}{\partial\phi} + \frac{\partial M_G^{***}}{\partial\phi} \left(1 + \frac{\partial M_P^{***}}{\partial M_G}\right) < \frac{\partial M_P^{***}}{\partial\phi} < 0. \end{aligned} \quad (\text{A50})$$

Thus, the increase in private money following a decline in  $\phi$  is smaller when the government recognizes the “crowding out” benefit of short-term bills. However, the total increase in public plus private short-term debt is greater than in the absence of such a policy because each dollar of additional short-term government debt crowds out less than one dollar of short-term private debt. Finally,  $\delta[M_P^{***}(\phi, M_G^{***}(\phi))]/\delta\phi < 0$ , so long as  $g'''(\cdot)$  and  $v'''(\cdot)$  are not too large and  $\phi$  is not too small (e.g. if  $g'''(\cdot) = v'''(\cdot) = 0$  and  $\phi > 1/2$ ). Since the first best solution obtains when  $\phi = 1$ , the second-best solution involves a larger quantity of government bills and more private money creation.

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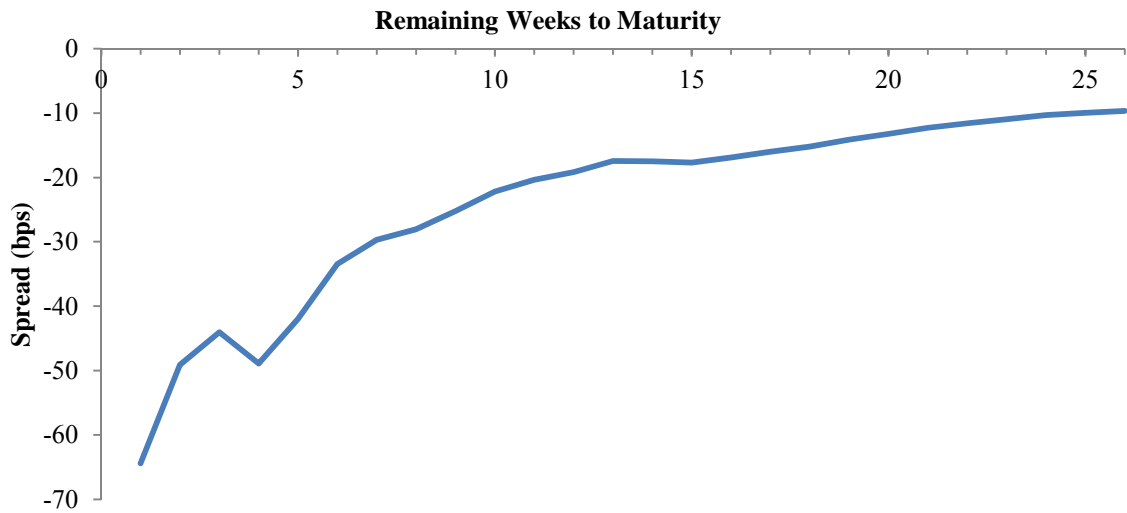
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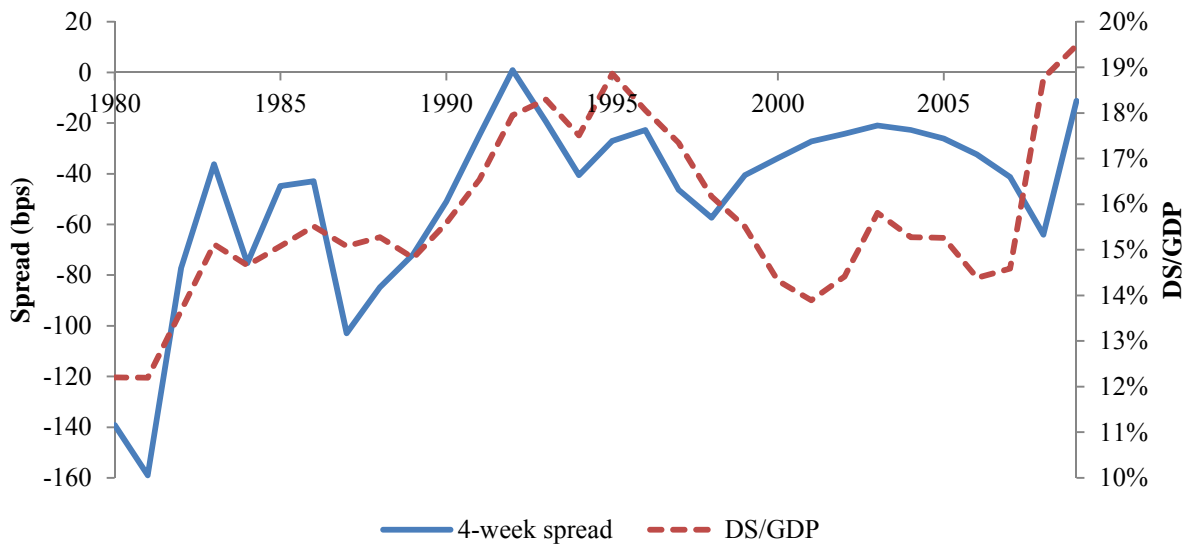
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**Figure 1. The money premium on short-term Treasury Bills.** Panel A plots the average spread, over the period 1980-2009, between actual Treasury-bill yields (“on-cycle” Treasury bills with maturities from 1 to 26 weeks) and fitted yields, where the fitted yields are based on a flexible extrapolation of the Treasury yield curve from Gurkaynak, Sack and Wright (2006), with updates available from the Federal Reserve. Gurkaynak et al (2006) estimate a parametric model of the instantaneous forward rate curve that is characterized by five parameters. The set of sample securities used to estimate the curve each day includes almost all “off-the-run” Treasury notes and bonds with a remaining maturity of more than 3 months. Panel B plots the 4-week T-bill premium (actual yield minus fitted yield) against the ratio of short-term debt to GDP. This ratio is the product of the debt with remaining maturity of one-year or less to total marketable debt from CRSP, times the debt-to-GDP ratio obtained from Henning Bohn’s website.

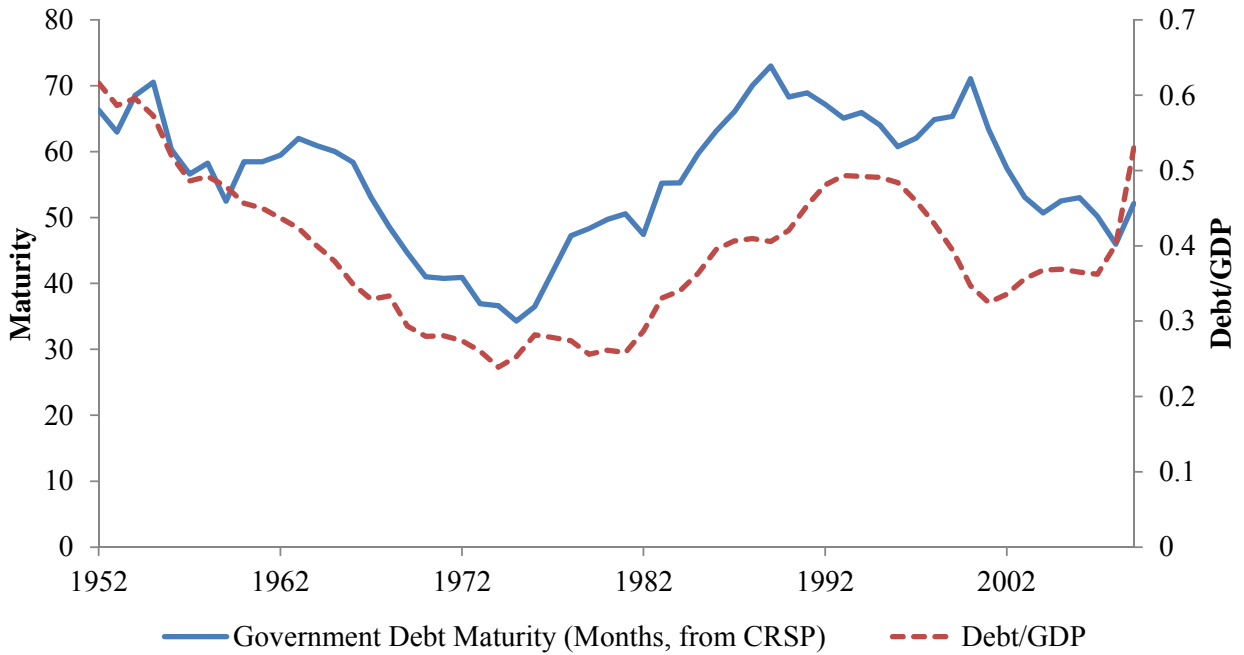
Panel A: Average premium (actual Treasury-bill yield minus fitted yield) by week to maturity



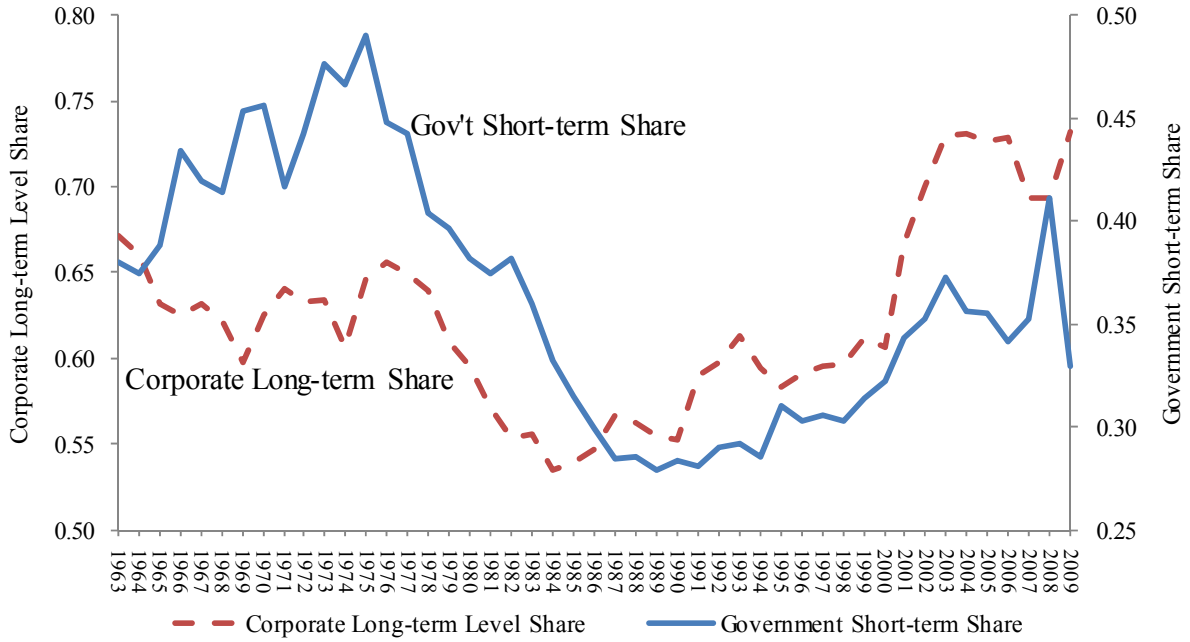
Panel B: 4-week T-bill premium (actual yield minus fitted yield) and the short-term debt-to-GDP ratio



**Figure 2. Debt/GDP and the maturity of government debt, 1952-2009.** The figure plots the weighted average maturity of U.S. government debt against the debt-to-GDP ratio, over the period 1952-2009. Average maturity is based on amounts outstanding as reported by the CRSP bond database. Debt-to-GDP ratio is obtained from Henning Bohn's website.



**Figure 3. Corporate and government debt maturity, 1963-2009.** The figure reproduces and extends Figure 1 from Greenwood, Hanson, and Stein (2010). The dashed line, plotted on the left axis, is the share of long-term corporate debt as a fraction of total debt, based on *Flow of Funds* data. The solid line, plotted on the right axis, is the share of government debt with maturity of one year or less.



**Table 1**  
**Private money creation and the quantity of short-term government debt, 1952-2009.**

Time-series regressions of private money creation on the debt-to-GDP ratio, and the ratio of short-term debt to short-term plus long-term debt:

$$(PrivateMoney / GDP)_t = a + b \cdot (D / GDP)_t + c \cdot (D_S / (D_S + D_L))_t + u_t.$$

The data are annual. Short-term debt  $D_S$  is Treasury debt with a maturity less than one year. Long-term debt  $D_L$  is Treasury debt with a maturity of one year or greater. Private money creation is alternately measured as non-M1 M2 divided by GDP, non-M1 M3 minus retail and institutional money funds divided by GDP, or open-market paper issued by commercial banks, scaled by GDP. Short- and long-term marketable Treasury debt do not sum to total government indebtedness, because savings bonds, inflation protected securities, and various other instruments are not counted. Nominal GDP is from the Bureau of Economic Analysis.  $t$ -statistics based on Newey-West (1987) standard errors, with 3-years of lags, are shown in brackets.

	Panel A: Dep Var = (M2-M1)/GDP		Panel B: Dep Var = (M3-M1)/GDP		Dep Var = Comm Bank Paper/GDP	
$D/GDP$	-0.538	-0.732	-0.554	-1.147	-45.286	-64.338
	[-5.49]	[-7.90]	[-4.62]	[-6.88]	[-3.67]	[-3.53]
$D_S/(D_S + D_L)$		-0.575		-1.122		-56.555
		[-3.89]		[-3.37]		[-1.66]
N	58	58	47	47	58	58
R <sup>2</sup>	0.63	0.73	0.43	0.64	0.29	0.35