

# Not for Publication: Web-based Technical Appendix to Inequality and Unemployment in a Global Economy\*

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## 1 Introduction

This web-based technical appendix contains the full derivations of the expressions in the paper and additional supplementary derivations. Sections 2-6 of the appendix correspond to Sections 2-6 of the paper. Section 7 of the appendix contains additional supplementary derivations. Subsection 7.1 derives the production technology from human capital complementarities or a managerial time constraint. Subsection 7.2 examines the relationship between the screening ability threshold and the marginal product of labor. Subsection 7.3 contains the derivation of the Stole-Zwiebel solution to the bargaining game. Subsection 7.4 contains the derivation of the search cost ( $b$ ) from a constant returns to scale matching technology and a cost of posting vacancies.

## 2 Sectoral Equilibrium

### 2.1 Model Setup

The real consumption index for the sector ( $Q$ ) is:

$$Q = \left[ \int_{j \in J} q(j)^\beta dj \right]^{1/\beta}, \quad 0 < \beta < 1, \quad (1)$$

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\*This web-based technical appendix combines and extends results previously reported separately in Helpman, Itskhoki and Redding (2008a, 2008b).

where  $j$  indexes varieties;  $J$  is the set of varieties within the sector; and  $q(j)$  denotes output of variety  $j$ . The price index dual to  $Q$  is denoted by  $P$ :

$$P = \left[ \int_{j \in J} p(j)^{-\frac{\beta}{1-\beta}} dj \right]^{-\frac{1-\beta}{\beta}},$$

where  $p(j)$  denotes the price of variety  $j$ . As is well known, given this specification of sectoral demand, the inverse CES demand curve for a firm is:

$$p(j) = E^{1-\beta} P^\beta q(j)^{-(1-\beta)}.$$

where  $E = PQ$  denotes sectoral expenditure. Substituting for price in firm revenue we obtain:

$$r(j) = p(j)q(j) = Aq(j)^\beta, \quad (2)$$

$$A \equiv E^{1-\beta} P^\beta = Q^{1-\beta} P$$

Output of each variety ( $y$ ) depends on the productivity of the firm ( $\theta$ ), the measure of workers hired ( $h$ ), and the average ability of these workers ( $\bar{a}$ ):

$$y = \theta h^\gamma \bar{a}, \quad 0 < \gamma < 1, \quad (3)$$

where firm productivity is drawn from a Pareto distribution,  $G_\theta(\theta) = 1 - (\theta_{\min}/\theta)^z$ .

## 2.2 Problem of the Firm

Given a Pareto distribution of worker ability,  $G_a(a) = 1 - (a_{\min}/a)^k$ , a firm that chooses a screening threshold  $a_c$  hires a measure  $h = n(a_{\min}/a_c)^k$  of workers with average ability  $\bar{a} = ka_c/(k-1)$ . Therefore the production technology (3) can be written as:

$$y = \kappa_y \theta n^\gamma a_c^{1-\gamma k}, \quad \kappa_y \equiv \frac{k}{k-1} a_{\min}^{\gamma k}, \quad (4)$$

Since in equilibrium all firms with the same productivity behave symmetrically, we index firms by  $\theta$  from now onwards. Exporters choose output to supply to the domestic market ( $y_d(\theta)$ ) and the export market ( $y_x(\theta)$ ) to equate marginal revenues in the two markets, which from (2) implies:

$$[y_d(\theta)/y_x(\theta)]^{\beta-1} = \tau^{-\beta} (A^*/A). \quad (5)$$

Total firm output is:

$$y(\theta) = y_d(\theta) + y_x(\theta). \quad (6)$$

Together (5) and (6) imply:

$$y_d(\theta) = y(\theta)/\Upsilon(\theta), \quad y_x(\theta) = y(\theta) [\Upsilon(\theta) - 1]/\Upsilon(\theta), \quad (7)$$

where:

$$\Upsilon(\theta) \equiv 1 + I_x(\theta) \tau^{-\frac{\beta}{1-\beta}} \left( \frac{A^*}{A} \right)^{\frac{1}{1-\beta}} \quad (8)$$

and  $I_x(\theta)$  is an indicator variable that equals one if the firm exports and zero otherwise. Note that for non-exporters we have  $y_d(\theta) = y(\theta)$  and  $\Upsilon(\theta) = 1$ , which implies that the allocation rule in (7) also holds for non-exporters. Total firm revenue is:

$$r(\theta) = r_d(\theta) + r_x(\theta). \quad (9)$$

Together (2), (5) and (9) imply:

$$r_d(\theta) = r(\theta) / \Upsilon(\theta), \quad r_x(\theta) = r(\theta) [\Upsilon(\theta) - 1] / \Upsilon(\theta),$$

where again this allocation rule also holds for non-exporters. Additionally, (2), (5) and (7) imply:

$$r(\theta) \equiv r_d(\theta) + r_x(\theta) = \Upsilon(\theta)^{1-\beta} A y(\theta)^\beta, \quad (10)$$

Therefore the firm's problem can be written as:

$$\pi(\theta) \equiv \max_{\substack{n \geq 0, \\ a_c \geq a_{\min}, \\ I_x \in \{0,1\}}} \left\{ \frac{1}{1 + \beta\gamma} \left[ 1 + I_x \tau^{-\frac{\beta}{1-\beta}} \left( \frac{A^*}{A} \right)^{\frac{1}{1-\beta}} \right]^{1-\beta} A (\kappa_y \theta n^\gamma a_c^{1-\gamma k})^\beta - bn - \frac{c}{\delta} a_c^\delta - f_d - I_x f_x \right\},$$

where  $1/(1 + \beta\gamma)$  is the equilibrium share of revenue received by the firm as the outcome of the bargaining game with workers, which is modelled as in Stole and Zweibel (1996a,b). We discuss the bargaining game and derive its solution in Subsection 7.3 below.

## 2.3 Producer Equilibrium

The firm's first-order conditions for the measure of workers sampled ( $n$ ) and the screening ability threshold ( $a_c$ ) are:

$$\frac{\beta\gamma}{1 + \beta\gamma} r(\theta) = bn(\theta), \quad (11)$$

$$\frac{\beta(1 - \gamma k)}{1 + \beta\gamma} r(\theta) = ca_c(\theta)^\delta. \quad (12)$$

Combining the two first-order conditions (11) and (12) we obtain the following relationship between  $n(\theta)$  and  $a_c(\theta)$ :

$$(1 - \gamma k)bn(\theta) = \gamma ca_c(\theta)^\delta.$$

Using the expression for  $r(\theta)$  in (10), we can solve explicitly for

$$\begin{aligned} n(\theta) &= \phi_1 \phi_2^{\beta(1-\gamma k)} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma+\Gamma}{\Gamma}} \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} Q^{-\frac{\beta-\zeta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}, \\ a_c(\theta) &= \phi_1^{1/\delta} \phi_2^{1-\beta\gamma} c^{-\frac{1-\beta\gamma}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\delta\Gamma}} \Upsilon(\theta)^{\frac{1-\beta}{\delta\Gamma}} Q^{-\frac{\beta-\zeta}{\delta\Gamma}} \theta^{\frac{\beta}{\delta\Gamma}}, \end{aligned}$$

where  $\Gamma = 1 - \beta\gamma - \beta/\delta \cdot (1 - \gamma k)$  and we have introduced two constants:

$$\phi_1 \equiv \left[ \frac{\beta\gamma}{1 + \beta\gamma} \left( \frac{ka_{\min}^{\gamma k}}{k-1} \right)^\beta \right]^{\frac{1}{\Gamma}} \quad \text{and} \quad \phi_2 \equiv \left( \frac{1-\gamma k}{\gamma} \right)^{\frac{1}{\delta\Gamma}}$$

and, as in the text of the paper,  $\Upsilon(\theta) = \Upsilon_d = 1$  for  $\theta \in [\theta_d, \theta_x)$  and  $\Upsilon(\theta) = \Upsilon_x = 1 + \tau^{-\beta/(1-\beta)} (Q^*/Q)^{-(\beta-\zeta)/(1-\beta)}$  for  $\theta \geq \theta_x$ . Also we solve for

$$\frac{\beta\gamma}{1+\beta\gamma}r(\theta) = bn(\theta) = \phi_1\phi_2^{\beta(1-\gamma k)} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\Gamma}} \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} Q^{-\frac{\beta-\zeta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}, \quad (13)$$

$$\pi(\theta) + f_d + I_x(\theta)f_x = \frac{\Gamma}{1+\beta\gamma}r(\theta) = \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\Gamma}} \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} Q^{-\frac{\beta-\zeta}{\Gamma}} \theta^{\frac{\beta}{\Gamma}}, \quad (14)$$

$$h(\theta) = n(\theta) \left( \frac{a_{\min}}{a_c(\theta)} \right)^k = a_{\min}^k \phi_1^{(1-k/\delta)} \phi_2^{-(k-\beta)} c^{\frac{k-\beta}{\delta\Gamma}} b^{-\frac{1-\beta/\delta}{\Gamma}} \Upsilon(\theta)^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} Q^{-\frac{(\beta-\zeta)(1-k/\delta)}{\Gamma}} \theta^{\frac{\beta(1-k/\delta)}{\Gamma}},$$

so that  $\kappa_r \equiv \phi_1\phi_2^{\beta(1-\gamma k)}$ . Finally, we solve for the wage rate:

$$\begin{aligned} w(\theta) &= \frac{\beta\gamma}{1+\beta\gamma} \frac{r(\theta)}{h(\theta)} = b \frac{n(\theta)}{h(\theta)} = b \left( \frac{a_c(\theta)}{a_{\min}} \right)^k \\ &= a_{\min}^{-k} \phi_1^{k/\delta} \phi_2^{(1-\beta\gamma)k} c^{-\frac{(1-\beta\gamma)k}{\delta\Gamma}} b^{\frac{1-\beta\gamma-\beta/\delta}{\Gamma}} \Upsilon(\theta)^{\frac{(1-\beta)k}{\Gamma}} Q^{-\frac{(\beta-\zeta)k}{\delta\Gamma}} \theta^{\frac{\beta k}{\delta\Gamma}}. \end{aligned} \quad (15)$$

Note that we have the following relationship, which proves useful in further derivations:

$$w(\theta)h(\theta) = bn(\theta) = \frac{\beta\gamma}{1+\beta\gamma}r(\theta).$$

## 2.4 Firm-specific Variables

Now, using the zero-profit cutoff condition,

$$\pi(\theta_d) = \frac{\Gamma}{1+\beta\gamma}r(\theta) = \frac{\Gamma}{\beta\gamma}\phi_1\phi_2^{\beta(1-\gamma k)} c^{-\frac{\beta(1-\gamma k)}{\delta\Gamma}} b^{-\frac{\beta\gamma}{\Gamma}} Q^{-\frac{\beta-\zeta}{\Gamma}} \theta_d^{\frac{\beta}{\Gamma}} - f_d = 0,$$

we can express all firm-level variables solely as the following functions of  $\theta/\theta_d$ ,  $b$  and  $\Upsilon(\theta)$  reported in the paper:

$$\left. \begin{aligned} r(\theta) &= \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} \cdot r_d \cdot \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}}, & r_d &\equiv \frac{1+\beta\gamma}{\Gamma} f_d, \\ n(\theta) &= \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}} \cdot n_d \cdot \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}}, & n_d &\equiv \frac{\beta\gamma}{\Gamma} \frac{f_d}{b}, \\ a_c(\theta) &= \Upsilon(\theta)^{\frac{1-\beta}{\delta\Gamma}} \cdot a_d \cdot \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta}{\delta\Gamma}}, & a_d &\equiv \left[ \frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{c} \right]^{1/\delta}, \\ h(\theta) &= \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}(1-k/\delta)} \cdot h_d \cdot \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta(1-k/\delta)}{\Gamma}}, & h_d &\equiv \frac{\beta\gamma}{\Gamma} \frac{f_d}{b} \left[ \frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca_{\min}^\delta} \right]^{-k/\delta}, \\ w(\theta) &= \Upsilon(\theta)^{\frac{k(1-\beta)}{\delta\Gamma}} \cdot w_d \cdot \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta k}{\delta\Gamma}}, & w_d &\equiv b \left[ \frac{\beta(1-\gamma k)}{\Gamma} \frac{f_d}{ca_{\min}^\delta} \right]^{k/\delta}, \end{aligned} \right\} \quad (16)$$

## 2.5 Sectoral Variables

### 2.5.1 Labor Market Tightness and Hiring Costs

Search costs ( $b$ ) depend on the tightness of the labor market ( $x$ ) as follows:

$$b = \alpha_0 x^{\alpha_1}, \quad \alpha_0 > 1, \quad \alpha_1 > 0. \quad (17)$$

We show in Section 7.4 below that this specification of search costs can be derived from a constant returns to scale matching technology and a cost of posting vacancies following Blanchard and Gali (2008).

Expected income in the differentiated sector is:

$$\omega = xb. \quad (18)$$

Together (17) and (18) imply:

$$b = \alpha_0^{\frac{1}{1+\alpha_1}} \omega^{\frac{\alpha_1}{1+\alpha_1}}, \quad \text{and} \quad x = \left( \frac{\omega}{\alpha_0} \right)^{\frac{1}{1+\alpha_1}}, \quad (19)$$

and we consider parameter values for which  $\alpha_0 > \omega$  so that  $0 < x < 1$ .

### 2.5.2 Productivity Cutoffs and Demand

From (14), the zero-profit productivity is determined by:

$$\frac{\Gamma}{1 + \beta\gamma} \kappa_r \left[ c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A \theta_d^\beta \right]^{1/\Gamma} = f_d. \quad (20)$$

Similarly, from (14), the exporting productivity cutoff is determined by:

$$\frac{\Gamma}{1 + \beta\gamma} \kappa_r \left[ c^{-\frac{\beta(1-\gamma k)}{\delta}} b^{-\beta\gamma} A \theta_x^\beta \right]^{1/\Gamma} \left[ \Upsilon_x^{(1-\beta)/\Gamma} - 1 \right] = f_x. \quad (21)$$

These two conditions imply the following relationship between the productivity cutoffs:

$$\left[ \Upsilon_x^{(1-\beta)/\Gamma} - 1 \right] \left( \frac{\theta_x}{\theta_d} \right)^{\beta/\Gamma} = \frac{f_x}{f_d}, \quad (22)$$

where in a symmetric equilibrium,  $A = A^*$  and hence  $\Upsilon_x = 1 + \tau^{\frac{-\beta}{1-\beta}}$ . Therefore the ratio of the two productivity cutoffs in a symmetric equilibrium is pinned down by (22) alone.

The free entry condition that equates the expected value of entry to the sunk entry cost is:

$$\int_{\theta_d}^{\infty} \pi(\theta) dG(\theta) = f_e.$$

where from (14) and (20)-(21), we have:

$$\pi(\theta) = \pi_d(\theta) + \pi_x(\theta) = f_d \left[ \left( \frac{\theta}{\theta_d} \right)^{\beta/\Gamma} - 1 \right] + I_x(\theta) f_x \left[ \left( \frac{\theta}{\theta_x} \right)^{\beta/\Gamma} - 1 \right],$$

where  $I_x(\theta) = 1$  for  $\theta \geq \theta_x$  and  $I_x(\theta) = 0$  for  $\theta < \theta_x$ . Using these relationships we can rewrite the free entry condition as:

$$f_d \int_{\theta_d}^{\infty} \left[ \left( \frac{\theta}{\theta_d} \right)^{\beta/\Gamma} - 1 \right] dG_\theta + f_x \int_{\theta_x}^{\infty} \left[ \left( \frac{\theta}{\theta_x} \right)^{\beta/\Gamma} - 1 \right] dG_\theta = f_e. \quad (23)$$

### 2.5.3 Expenditure, Mass of Firms and the Labor Force

Given the demand shifter for sector  $i$  ( $A_i$ ), the price index for that sector ( $P_i$ ) can be determined from consumer optimization given prices in all other sectors ( $\mathbf{P}_{-i}$ ) and aggregate income ( $\Omega$ ):

$$A_i = \tilde{A}_i(P_i, \mathbf{P}_{-i}, \Omega), \quad (24)$$

where the functional form of this relationship depends on the way in which the model is closed in general equilibrium, as shown in Section 6 below.

Given the demand shifter ( $A_i$ ) and price index ( $P_i$ ) for sector  $i$ , the real consumption index ( $Q_i$ ) also follows from consumer optimization, which from (2) implies:

$$Q_i = (A_i/P_i)^{\frac{1}{1-\beta}}, \quad (25)$$

which yields total expenditure within the sector  $E = PQ$ . The mass of firms within the sector ( $M$ ) is determined by:

$$E = M \int_{\theta_d}^{\infty} r_d(\theta) dG_{\theta}(\theta) + M^* \int_{\theta_x^*}^{\infty} r_x^*(\theta) dG_{\theta}(\theta). \quad (26)$$

Using the expressions for equilibrium revenue from domestic sales and exports derived above (see (10) and (13)), we can rewrite (26) as:

$$E = \frac{1 + \beta\gamma}{\Gamma} \left[ M f_d \int_{\theta_d}^{\infty} \Upsilon(\theta)^{\frac{1-\beta}{\Gamma}-1} \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) + M^* f_x \frac{\Upsilon_x^{*\frac{1-\beta}{\Gamma}}}{\Upsilon_x^{*\frac{1-\beta}{\Gamma}} - 1} \frac{\Upsilon_x^* - 1}{\Upsilon_x^*} \int_{\theta_x^*}^{\infty} \left( \frac{\theta}{\theta_x^*} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) \right].$$

The sectoral labor force ( $L$ ) can be determined from the equality between the total sectoral wage bill and workers' share of total sectoral revenue:

$$\omega L = M \int_{\theta_d}^{\infty} w(\theta) h(\theta) dG_{\theta}(\theta) = M \frac{\beta\gamma}{1 + \beta\gamma} \int_{\theta_d}^{\infty} r(\theta) dG_{\theta}(\theta), \quad (27)$$

Using the expressions for equilibrium revenue from domestic sales and exports derived above, we can also rewrite (27) as:

$$L = \frac{\beta\gamma}{\Gamma} M \left[ f_d \int_{\theta_d}^{\infty} \left( \frac{\theta}{\theta_d} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) + f_x \int_{\theta_x^*}^{\infty} \left( \frac{\theta}{\theta_x^*} \right)^{\frac{\beta}{\Gamma}} dG_{\theta}(\theta) \right] = z\gamma f_e M,$$

where we have evaluated the integrals in the square brackets using the Pareto distribution and applied the free entry condition (23). This condition implies that  $L/M$  is constant in any equilibrium and  $L$  and  $M$  are equivalent measures of the size of the differentiated sector. Finally, observe that in a symmetric case the expression for  $E$  above can be considerably simplified and becomes identical to the expression for  $L$  above up to a factor of  $\beta\gamma/(1 + \beta\gamma)$ .

## 2.6 Symmetric Countries Closed Form Solutions

In this section, we characterize sectoral equilibrium for the case of symmetric countries, where a number of the expressions above simplify further. Evaluating the integrals in the free entry condition (23) using a

Pareto productivity distribution, we obtain:

$$\left(\frac{\beta}{z\Gamma - \beta}\right) f_d \left(\frac{\theta_{\min}}{\theta_d}\right)^z \left[1 + \frac{f_x}{f_d} \left(\frac{\theta_d}{\theta_x}\right)^z\right] = f_e.$$

Using (22), we can rewrite this as

$$\left(\frac{\beta}{z\Gamma - \beta}\right) f_d \left(\frac{\theta_{\min}}{\theta_d}\right)^z \left[1 + \left(\frac{f_d}{f_x}\right)^{\frac{z\Gamma - \beta}{\beta}} \left[\Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1\right]^{z\Gamma/\beta}\right] = f_e \quad (28)$$

Note that these two expressions do not themselves rely on symmetry. Under the assumption of a symmetric equilibrium,  $A = A^*$ , and hence we have  $\Upsilon_x = 1 + \tau^{-\beta/(1-\beta)}$ . Therefore, in a symmetric equilibrium, (28) defines  $\theta_d$ , and after solving for  $\theta_d$ ,  $\theta_x$  can be obtained from (22).

Using (22) and (28), we can now derive the conditions on the parameters that ensure  $\theta_x > \theta_d > \theta_{\min}$  in a symmetric equilibrium. Note that the square bracket in (28) is always greater than 1. Therefore, it is enough to require that

$$f_d > f_e \frac{z\Gamma - \beta}{\beta}$$

to ensure that  $\theta_d > \theta_{\min}$  in any symmetric equilibrium. Therefore a high enough  $f_d$  ensures  $\theta_d > \theta_{\min}$  in a symmetric equilibrium. Next note from (22) that since  $\left[\Upsilon_x^{(1-\beta)/\Gamma} - 1\right] < 1$ , it is enough to require that  $f_x \geq f_d$  to ensure  $\theta_x > \theta_d$  in any symmetric equilibrium. Therefore a high enough  $f_x$  ensures  $\theta_x > \theta_d$  in a symmetric equilibrium. Note that the same condition applies in Melitz (2003). Numerical simulations suggest that a much weaker condition is generally sufficient in this model.

Once we have established the equilibrium value of  $\theta_d$ , we can determine the equilibrium demand shifter  $A$  from the domestic productivity cutoff condition (20):

$$A = \left(\frac{1 + \beta\gamma}{\kappa_r\Gamma} f_d\right)^\Gamma c^{\beta(1-\gamma k)/\delta} b^{\beta\gamma} \theta_d^{-\beta}. \quad (29)$$

Note that  $b$  and  $c$  do not affect  $\theta_d$  in the symmetric equilibrium. However, these parameters do affect the demand shifter  $A$ . In contrast, trade costs do not alter the relationship between  $A$  and  $\theta_d$  in (29), but higher trade costs do reduce  $\theta_d$  and hence increase  $A$  (see (28)).

Finally, knowing  $A$  and following the steps outlined in Section 2.5.3, we can solve for sectoral price index  $P$ , real consumption index  $Q$  and expenditure  $E = PQ$ . Then the mass of firm entrants  $M$  and the measure of workers searching for a job in the differentiated sector  $L$  should satisfy:

$$\frac{\beta\gamma}{1 + \beta\gamma} E = L = z\gamma f_e M.$$

This completes the solutions for the case of symmetric countries.

### 3 Sectoral Wage Inequality

#### 3.1 Wage Distribution among Workers in Exporting and Non-exporting Firms

The share of workers employed by firms that serve only the domestic market is from (16) and the Pareto productivity distribution:

$$S_{h,d} = 1 - \frac{\int_{\theta_x}^{\infty} h(\theta) dG_{\theta}(\theta)}{\int_{\theta_d}^{\infty} h(\theta) dG_{\theta}(\theta)} = \frac{1 - \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}}}{1 + \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}} \left[ \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1 \right]}, \quad (30)$$

where  $\rho \equiv \theta_d/\theta_x$ . To compute the distribution of wages across workers employed by non-exporting firms, note that the fraction of workers receiving a particular wage  $w(\theta) \in [w_d, w_d/\rho^{\beta k/\delta\Gamma}]$  is proportional to  $h(\theta)dG_{\theta}(\theta)$ . In other words, we have:

$$G_{w,d}(w) = \frac{M \int_{\theta_d}^{\theta_{w,d}(w)} h(\theta) dG_{\theta}(\theta)}{M \int_{\theta_d}^{\theta_x} h(\theta) dG_{\theta}(\theta)} = 1 - \frac{\int_{\theta_{w,d}(w)}^{\theta_x} h(\theta) dG_{\theta}(\theta)}{\int_{\theta_d}^{\theta_x} h(\theta) dG_{\theta}(\theta)} \quad \text{for } w \in [w_d, w_d/\rho^{\beta k/\delta\Gamma}],$$

where  $\theta_{w,d}(\cdot)$  is the inverse of  $w(\cdot)$  and equal to  $\theta_{w,d}(w) = \theta_d(w/w_d)^{\delta\Gamma/(\beta k)}$ . Finally, for  $w < w_d$ ,  $G_{w,d}(w) = 0$ , and for  $w > w_d/\rho^{\beta k/\delta\Gamma}$ ,  $G_{w,d}(w) = 1$ . Using the Pareto productivity distribution, the distribution of wages across workers employed by domestic firms is the following truncated Pareto distribution:

$$G_{w,d}(w) = \frac{1 - \left( \frac{\theta_d}{\theta_{w,d}(w)} \right)^{z - \frac{\beta(1-k/\delta)}{\Gamma}}}{1 - \left( \frac{\theta_d}{\theta_x} \right)^{z - \frac{\beta(1-k/\delta)}{\Gamma}}} = \frac{1 - \left( \frac{w_d}{w} \right)^{1+1/\mu}}{1 - \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}}}, \quad \text{for } w \in [w_d, w_d/\rho^{\beta k/\delta\Gamma}],$$

where  $\mu \equiv \beta k / [\delta(z\Gamma - \beta)]$ .

The distribution of wages across workers employed by exporters can be computed in the same way:

$$G_{w,x}(w) = \frac{M \int_{\theta_x}^{\theta_{w,x}(w)} h(\theta) dG_{\theta}(\theta)}{M \int_{\theta_x}^{\infty} h(\theta) dG_{\theta}(\theta)} = 1 - \frac{\int_{\theta_{w,x}(w)}^{\infty} h(\theta) dG_{\theta}(\theta)}{\int_{\theta_x}^{\infty} h(\theta) dG_{\theta}(\theta)} \quad \text{for } w \in \left[ w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\beta k/\delta\Gamma}, \infty \right),$$

where  $\theta_{w,x}(\cdot)$  is the inverse of  $w(\cdot)$  and equal to  $\theta_{w,x}(w) = \theta_d(w/w_d)^{\delta\Gamma/(\beta k)} \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\beta k/\delta\Gamma}$ . Finally, for  $w < w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\beta k/\delta\Gamma}$ ,  $G_{w,x}(w) = 0$ . Using the Pareto productivity distribution, the distribution of wages across workers employed by exporters is the following untruncated Pareto distribution:

$$G_{w,x}(w) = 1 - \left( \frac{\theta_x}{\theta_w(w)} \right)^{z - \frac{\beta(1-k/\delta)}{\Gamma}} = 1 - \left( \frac{w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} \rho^{-\frac{\beta k}{\delta\Gamma}}}{w} \right)^{1+1/\mu}, \quad \text{for } w \in \left[ w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\beta k/\delta\Gamma}, \infty \right).$$

Combining  $S_{h,d}$ ,  $G_{w,d}(\cdot)$  and  $G_{w,x}(\cdot)$  together we obtain the unconditional wage distribution among workers employed in the differentiated sector,  $G_w(w)$ , as defined in the paper:

$$G_w(w) = \begin{cases} S_{h,d} G_{w,d}(w) & \text{for } w_d \leq w \leq w_d / \rho^{\frac{\beta k}{\delta\Gamma}}, \\ S_{h,d} & \text{for } w_d / \rho^{\frac{\beta k}{\delta\Gamma}} \leq w \leq w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\frac{\beta k}{\delta\Gamma}}, \\ S_{h,d} + (1 - S_{h,d}) G_{w,x}(w) & \text{for } w \geq w_d \Upsilon_x^{\frac{k(1-\beta)}{\delta\Gamma}} / \rho^{\frac{\beta k}{\delta\Gamma}}, \end{cases} \quad (31)$$

### 3.2 Proof of Proposition 1

Consider the closed economy wage distribution:

$$G_w^a(w) = 1 - \left(\frac{w_d}{w}\right)^{1+1/\mu}, \quad \text{for } w \geq w_d.$$

It is a Pareto distribution with a shape parameter  $(1 + 1/\mu) > 2$  and lower bound  $w_d$  defined in (16). We now show that with a Pareto distribution a large class of inequality measures depends only on the shape parameter and not on the lower bound of the distribution.

(i) With this distribution, the mean and variance of wages are given by:

$$\bar{w}^a = (1 + \mu)w_d \quad \text{and} \quad \mathbb{V}^a(w) = \frac{1 + \mu}{1 - \mu} \mu^2 w_d^2.$$

Recall that we require  $\mu \in (0, 1)$  so that the variance of the wage distribution is finite. Therefore, the coefficient of variation is given by:

$$CV_w^a = \frac{\sqrt{\mathbb{V}^a(w)}}{\bar{w}^a} = \frac{\mu}{\sqrt{1 - \mu^2}}.$$

Clearly,  $\mu$  is a sufficient statistic for the coefficient variation.

(ii) We now characterize the Lorenz Curve for the sectoral wage distribution. In order to do so, we compute the share in sectoral employment and the share in the sectoral wage bill of firms with productivity below  $\theta'$ :

$$s_h(\theta) = \frac{M \int_{\theta_d}^{\theta'} h(\theta) dG_\theta(\theta)}{M \int_{\theta_d}^{\infty} h(\theta) dG_\theta(\theta)} = 1 - \left(\frac{\theta_d}{\theta'}\right)^{z - \beta(1 - k/\delta)/\Gamma},$$

$$s_w(\theta) = \frac{M \int_{\theta_d}^{\theta'} w(\theta) h(\theta) dG_\theta(\theta)}{M \int_{\theta_d}^{\infty} w(\theta) h(\theta) dG_\theta(\theta)} = 1 - \left(\frac{\theta_d}{\theta'}\right)^{z - \beta/\Gamma},$$

where we have used the solution for firm-level variables (16) and the Pareto productivity distribution. From these expressions we can solve for the wage share  $s_w$  as a function of the employment share  $s_h$ :

$$s_w = \mathcal{L}^a(s_h) = 1 - (1 - s_h)^{1/(1+\mu)}, \quad \mu = \frac{\beta k}{\delta(z\Gamma - \beta)}, \quad s_h, s_w \in [0, 1]. \quad (32)$$

This relationship represents the Lorenz Curve, because workers' wages are increasing in firm productivity and hence this procedure ranks workers by their wages. Note that  $\mu$  is the only parameter which determines the position of the Lorenz Curve. A higher  $\mu$  makes the Lorenz Curve more convex which implies greater wage inequality.

(iii) The Gini Coefficient is determined uniquely by the shape of the Lorenz Curve and, therefore,  $\mu$  is also a sufficient statistic for the Gini Coefficient. Specifically, the Gini Coefficient is defined as:

$$\mathcal{G}_w^a \equiv 1 - 2 \int_0^1 \mathcal{L}^a(s_h) ds_h = \frac{\mu}{2 + \mu}.$$

(iv) The Theil Index of wage inequality is defined as:

$$\mathcal{T}_w^a = \int_{w_d}^{\infty} \frac{w}{\bar{w}^a} \ln\left(\frac{w}{\bar{w}^a}\right) dG_w^a(w),$$

where the properties of the Theil Index are discussed in Bourguignon (1979). Using the autarky wage distribution, we can compute this integral to obtain:

$$\mathcal{T}_w^a = \mu - \ln(1 + \mu).$$

Note that  $\mu$  is a sufficient statistic for the Theil Index, which is decreasing in  $\mu$  and equals to zero when  $\mu = 0$ .

Note that all discussed measures of inequality depend on the parameters of the model only through their effect on  $\mu$ ; all measures of inequality are increasing in  $\mu$ . This completes the proof of Proposition 1.

### 3.3 Proof of Proposition 2

From Proposition 1 we know that  $\mu$  is a sufficient statistic for sectoral wage inequality in the closed economy. Recall that

$$\mu = \frac{\beta k}{\delta(z\Gamma - \beta)}, \quad \Gamma = 1 - \beta\gamma - \frac{\beta}{\delta}(1 - \gamma k).$$

Evidently,  $\partial\mu/\partial z < 0$ . The effect of  $k$  is more subtle as it increases both the numerator and denominator in the expression for  $\mu$ . Taking the derivative with respect to  $k$  and rearranging we obtain:

$$\text{sign} \left\{ \frac{\partial\mu}{\partial k} \right\} = \text{sign}\{1 - z\gamma\mu\} = \text{sign}\{\beta^{-1} - \gamma - \delta^{-1} - z^{-1}\}.$$

This finishes the proof of Proposition 2.

### 3.4 Proof of Proposition 3

We first prove the second part of the proposition. The two limiting cases for the open economy wage distribution (31) are (1) autarky (as  $\rho = \theta_d/\theta_x \rightarrow 0$  and no firms export) and (2) when all firms export ( $\rho \rightarrow 1$ ). In the first case, the wage distribution  $G_w^a(w)$  is an untruncated Pareto with shape parameter  $(1 + 1/\mu)$  and lower bound  $w_d$ . In the second case, the wage distribution is again an untruncated Pareto with shape parameter  $(1 + 1/\mu)$ , but now with a higher lower bound given by  $w_d \Upsilon_x^{k(1-\beta)/(\delta\Gamma)}$ . However, from Proposition 1, a broad class of inequality measures depend only on the shape parameter of the Pareto distribution and do not depend on its lower bound. Therefore, wage inequality is the same in autarky and when all firms export.

Now consider the first part of the proposition. Define the following notation:

$$\begin{aligned} \eta_1 &\equiv \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1, & \eta_2 &\equiv \Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1, \\ \vartheta_1 &\equiv z - \frac{\beta(1-k/\delta)}{\Gamma}, & \vartheta_2 &\equiv z - \frac{\beta}{\Gamma}. \end{aligned}$$

Using this notation, the lowest wage paid by exporters and the highest wage paid by domestic firms can be written as:

$$w(\theta_x^+) = w(\theta_x^-) \frac{1 + \eta_2}{1 + \eta_1} \quad \text{and} \quad w(\theta_x^-) = w_d \rho^{\vartheta_2 - \vartheta_1}.$$

Similarly, using this notation, the actual wage distribution (31) can be written as:

$$G_w(w) = \begin{cases} \frac{1}{1+\eta_1\rho^{\vartheta_1}} [1 - (w_d/w)^{1+1/\mu}], & w_d \leq w \leq w(\theta_x^-), \\ (1 - \rho^{\vartheta_1})/(1 + \eta_1\rho^{\vartheta_1}), & w(\theta_x^-) \leq w \leq w(\theta_x^+), \\ \frac{1-\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} + \frac{(1+\eta_1)\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} [1 - (w(\theta_x^+)/w)^{1+1/\mu}], & w \geq w(\theta_x^+) \end{cases} \quad (33)$$

and the mean of this distribution can be written as

$$\bar{w} = (1 + \mu)w_d \frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_1\rho^{\vartheta_1}}.$$

The counterfactual wage distribution is defined as:

$$G_w^c(w) = 1 - (w_d^c/w)^{1+1/\mu}, \quad w \geq w_d^c, \quad (34)$$

where, in order for the mean of the counterfactual distribution to equal  $\bar{w}$ , its lower limit must satisfy:

$$w_d^c = \frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_1\rho^{\vartheta_1}} w_d.$$

Therefore we can establish the following result:

$$w_d^c > w_d \quad \text{since} \quad \frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_1\rho^{\vartheta_1}} > 1 \quad \text{for} \quad 0 < \rho < 1,$$

as  $0 < \eta_1 < \eta_2 < 1$  and  $1 < z/2 < \vartheta_2 < \vartheta_1 < z$ . Similarly, we can establish:

$$w_d^c < w(\theta_x^+) \quad \text{since} \quad \frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_1\rho^{\vartheta_1}} < \frac{1 + \eta_2}{1 + \eta_1} \rho^{\vartheta_2 - \vartheta_1} = \frac{(1 + \eta_2)\rho^{\vartheta_2}}{(1 + \eta_1)\rho^{\vartheta_1}},$$

with the inequality being satisfied since:

$$\frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_1\rho^{\vartheta_1}} = \frac{(1 + \eta_2)\rho^{\vartheta_2} + (1 - \rho^{\vartheta_2})}{(1 + \eta_1)\rho^{\vartheta_1} + (1 - \rho^{\vartheta_1})} = \frac{(1 + \eta_2)\rho^{\vartheta_2}}{(1 + \eta_1)\rho^{\vartheta_1}} \cdot \frac{1 + \frac{1-\rho^{\vartheta_2}}{(1+\eta_2)\rho^{\vartheta_2}}}{1 + \frac{1-\rho^{\vartheta_1}}{(1+\eta_1)\rho^{\vartheta_1}}} < \frac{(1 + \eta_2)\rho^{\vartheta_2}}{(1 + \eta_1)\rho^{\vartheta_1}},$$

as  $\rho^{\vartheta_1} < \rho^{\vartheta_2} < 1$  and  $(1 + \eta_2) > (1 + \eta_1)$ . Note that in general we can have either  $w_d^c > w(\theta_x^-)$  or  $w_d^c < w(\theta_x^-)$ , but the same arguments apply in each case.

We can also show that the slope of the counterfactual wage distribution is smaller than the slope of the actual wage distribution at  $w(\theta_x^+)$ :  $g_w(w(\theta_x^+)) > g_w^c(w(\theta_x^+))$ . Since the truncations of  $G_w(w)$  and  $G_w^c(w)$  at  $w(\theta_x^+)$  are both Pareto with shape parameter  $(1 + 1/\mu)$ , we can show that  $g_w(w(\theta_x^+)) > g_w^c(w(\theta_x^+))$  by establishing that  $1 - G_w(w(\theta_x^+)) > 1 - G_w^c(w(\theta_x^+))$ . From (33) and (34), this implies:

$$\begin{aligned} 1 - \frac{1 - \rho^{\vartheta_1}}{1 + \eta_1\rho^{\vartheta_1}} &> \left( \frac{w_d^c}{w(\theta_x^+)} \right)^{1+1/\mu} \Leftrightarrow \frac{(1 + \eta_1)\rho^{\vartheta_1}}{1 + \eta_1\rho^{\vartheta_1}} > \left( \frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_1\rho^{\vartheta_1}} \frac{1 + \eta_1}{1 + \eta_2} \right)^{\frac{\vartheta_1}{\vartheta_1 - \vartheta_2}} \rho^{\vartheta_1} \\ &\Leftrightarrow \phi(\rho) \equiv \left( \frac{1 + \eta_1\rho^{\vartheta_1}}{1 + \eta_1} \right)^{\vartheta_2} - \left( \frac{1 + \eta_2\rho^{\vartheta_2}}{1 + \eta_2} \right)^{\vartheta_1} > 0. \end{aligned}$$

To show that  $\phi(\rho) > 0$  for all  $\rho \in [0, 1)$ , note that:

$$\phi(0) \equiv \left(\frac{1}{1+\eta_1}\right)^{\vartheta_2} - \left(\frac{1}{1+\eta_2}\right)^{\vartheta_1} > 0,$$

as  $\eta_1 < \eta_2$  and  $\vartheta_1 > \vartheta_2$ . Note also that  $\phi(1) = 1 - 1 = 0$ . Consider now the derivative of  $\phi(\rho)$  for  $\rho \in (0, 1]$ :

$$\phi'(\rho) = \frac{\eta_1\eta_2}{\rho} \left[ \left(\frac{1+\eta_1\rho^{\vartheta_1}}{1+\eta_1}\right)^{\vartheta_2} \frac{\eta_1\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}} - \left(\frac{1+\eta_2\rho^{\vartheta_2}}{1+\eta_2}\right)^{\vartheta_1} \frac{\eta_2\rho^{\vartheta_2}}{1+\eta_2\rho^{\vartheta_2}} \right].$$

Note that

$$\frac{\eta_2\rho^{\vartheta_2}}{1+\eta_2\rho^{\vartheta_2}} > \frac{\eta_1\rho^{\vartheta_1}}{1+\eta_1\rho^{\vartheta_1}},$$

since  $\eta_1 < \eta_2$  and  $\rho^{\vartheta_1} < \rho^{\vartheta_2}$ . As a result, whenever  $\phi(\rho) \leq 0$ , we also necessarily have  $\phi'(\rho) < 0$ . Therefore, if there exists  $\rho'$  such that  $\phi(\rho') = 0$ , then  $\phi(\rho) < 0$  for all  $\rho > \rho'$ . But since  $\phi(1) = 0$ , this implies that  $\phi(\rho) > 0$  for all  $\rho \in (0, 1)$ .

We now establish that  $G_w^c(w)$  second-order stochastically dominates  $G_w(w)$  for  $\rho \in (0, 1)$ . Using the fact that the truncations of  $G_w(w)$  and  $G_w^c(w)$  at  $w(\theta_x^+)$  are both Pareto with shape parameter  $(1 + 1/\mu)$ , and the result above that  $g_w(w(\theta_x^+)) > g_w^c(w(\theta_x^+))$ , we know that this inequality holds for all  $w > w(\theta_x^+)$ . We have two cases:

1.  $w(\theta_x^-) \leq w_d^c < w(\theta_x^+)$ :

$$g_w(w) - g_w^c(w) = \begin{cases} > 0, & w_d \leq w < w(\theta_x^-), \\ = 0, & w(\theta_x^-) \leq w < w_d^c, \\ < 0, & w_d^c \leq w < w(\theta_x^+), \\ > 0, & w \geq w(\theta_x^+). \end{cases}$$

2.  $w_d^c < w(\theta_x^-)$ :

$$g_w(w) - g_w^c(w) = \begin{cases} > 0, & w_d \leq w < w_d^c, \\ \leq 0, & w_d^c \leq w < w(\theta_x^-), \\ < 0, & w(\theta_x^-) \leq w < w(\theta_x^+), \\ > 0, & w \geq w(\theta_x^+). \end{cases}$$

Importantly,  $g_w(w) - g_w^c(w)$  takes either only positive or only negative values in the range  $[w_d^c, w(\theta_x^-))$ , since for this range

$$g_w(w) - g_w^c(w) = (C - C^c) w^{-(2+1/\mu)},$$

where  $C$  and  $C^c$  are positive constants.

Note that in both cases the above characterization of  $g_w(w) - g_w^c(w)$  implies that this difference of density functions is positive for low values of  $w$ , negative for intermediate values of  $w$ , and again positive for larger values of  $w$ . This immediately implies that the cumulative distribution functions intersect only once in the range where the difference of density functions is negative (see Figure 2 in the text), which is a sufficient condition to establish that indeed  $G_w^c(w)$  second-order stochastically dominates  $G_w(w)$  (see, for example, Mas-Colell, Whinston and Green 1995, p.195).

Therefore, for all measures of inequality that respect second-order stochastic dominance, wage inequality in the open economy when some but not all firms export is strictly greater than in the closed economy. This

finishes the proof of Proposition 3.

## 4 Sectoral Unemployment

The sectoral unemployment rate is given by

$$u = \frac{L - H}{L} = 1 - \frac{H}{N} \frac{N}{L} = 1 - \sigma x,$$

where  $\sigma = H/N$  is the hiring rate and  $x = N/L$  is labor market tightness. From (19), labor market tightness is given by  $x = (\omega/\alpha_0)^{1/(1+\alpha_1)}$ .

We now derive the expression for the sectoral hiring rate:

$$\sigma = \frac{M \int_{\theta_d}^{\infty} h(\theta) dG_{\theta}(\theta)}{M \int_{\theta_d}^{\infty} n(\theta) dG_{\theta}(\theta)} = \frac{h_d \int_{\theta_d}^{\theta_x} \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}} dG_{\theta}(\theta) + h_x \int_{\theta_x}^{\infty} \left(\frac{\theta}{\theta_x}\right)^{\frac{\beta(1-k/\delta)}{\Gamma}} dG_{\theta}(\theta)}{n_d \int_{\theta_d}^{\theta_x} \left(\frac{\theta}{\theta_d}\right)^{\beta/\Gamma} dG_{\theta}(\theta) + n_x \int_{\theta_x}^{\infty} \left(\frac{\theta}{\theta_x}\right)^{\beta/\Gamma} dG_{\theta}(\theta)},$$

where  $h_x = \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} (\theta_x/\theta_d)^{\frac{\beta(1-k/\delta)}{\Gamma}} h_d$ ,  $n_x = \Upsilon_x^{\frac{1-\beta}{\Gamma}} (\theta_x/\theta_d)^{\frac{\beta}{\Gamma}} n_d$  and we have used the solution for firm-specific variables (16). Evaluating the integrals using the Pareto productivity distribution yields

$$\sigma = \sigma^a \cdot \varphi(\rho, \Upsilon_x),$$

where

$$\sigma^a \equiv \frac{1}{1 + \mu} \left[ \frac{\Gamma}{\beta(1 - \gamma k)} \frac{c a_{\min}^{\delta}}{f_d} \right]^{k/\delta}$$

is the autarky hiring rate and

$$\varphi(\rho, \Upsilon_x) \equiv \frac{1 + \left[ \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1 \right] \rho^{z - \beta(1-k/\delta)/\Gamma}}{1 + \left[ \Upsilon_x^{\frac{(1-\beta)}{\Gamma}} - 1 \right] \rho^{z - \beta/\Gamma}} = \frac{1 + \eta_1 \rho^{\vartheta_1}}{1 + \eta_2 \rho^{\vartheta_2}},$$

where the last equality uses the notation introduced in Section 3.4 of this appendix. Note that  $\varphi(0, \Upsilon_x) = 1$  and  $\varphi(\rho, \Upsilon_x) < 1$  for  $0 < \rho \leq 1$  since  $\eta_1 < \eta_2$  and  $\vartheta_1 > \vartheta_2$ . Therefore, the hiring rate is lower in any open economy equilibrium than in autarky.

Opening up to trade does not affect sectoral labor market tightness  $x$  directly, but does affect it indirectly through  $\omega$ . In case when  $\omega$  is invariant to trade, sectoral unemployment is affected by trade only through hiring rate. As a result, in this case sectoral unemployment is strictly higher in any open economy equilibrium than in autarky. This constitutes a proof of Proposition 5.

Finally, we look at the determinants of the hiring rate and sectoral unemployment in the autarky equilibrium. From the expression for  $u$ ,  $x$  and  $\sigma^a$ , it is evident that holding  $\omega$  constant  $c$ ,  $z$  and  $k$  affect the unemployment rate only through their effects on the hiring rate  $\sigma^a$ , while  $\alpha_0$  affects the unemployment rate only through its effect on labor market tightness  $x$ . Specifically,  $\alpha_0$  decreases  $x$  and increases  $u$ ;  $z$  and  $c$  increase  $\sigma^a$  and decrease  $u$ ; the effect of  $k$  on  $\sigma^a$  and hence  $u$  is ambiguous. This constitutes a proof of Proposition 4.

## 5 Sectoral Income Inequality

First, consider the Lorenz Curve for the sectoral income distribution. When the Lorenz Curve for the wage distribution is given by  $s_w = \mathcal{L}_w(s_h)$  and the unemployment rate is  $u$ , the Lorenz Curve for the income distribution can be written as:

$$s_\iota = \mathcal{L}_\iota(s_\ell) = \begin{cases} 0, & 0 \leq s_\ell \leq u, \\ \mathcal{L}_w\left(\frac{s_\ell - u}{1 - u}\right), & u \leq s_\ell \leq 1 \end{cases}$$

and from (32)  $\mathcal{L}_w(s) = 1 - (1 - s)^{1/(1+\mu)}$ . We can now compute the Gini Coefficient of sectoral income inequality:

$$\mathcal{G}_\iota = 1 - 2 \int_0^1 \mathcal{L}_\iota(s_\ell) ds_\ell = u + (1 - u)\mathcal{G}_w = u + (1 - u) \left[ 1 - \frac{2}{1 - u} \int_u^1 \mathcal{L}_w\left(\frac{s_\ell - u}{1 - u}\right) ds_\ell \right].$$

Substitution of variables  $s_h = (s_\ell - u)/(1 - u)$  results in

$$\mathcal{G}_\iota = u + (1 - u)\mathcal{G}_w,$$

where  $\mathcal{G}_w$  is the Gini Coefficient of sectoral wage inequality as stated in the text of the paper. This is the expression we state in the text. Therefore, sectoral income inequality as measured by the Gini coefficient is increasing in the sectoral unemployment rate and sectoral wage inequality.

Now consider the Theil Index of sectoral income inequality. The general definition of the Theil index is

$$\mathcal{T}_\varpi = \int \frac{\varpi}{\bar{\varpi}} \ln \frac{\varpi}{\bar{\varpi}} dG_\varpi(\varpi),$$

where  $\varpi$  is a measure of income distributed according to the cumulative distribution function  $G_\varpi(\varpi)$  and  $\bar{\varpi}$  is the mean of this measure. Note that  $\varpi \ln \varpi = 0$  at  $\varpi = 0$ . Then for the sectoral income distribution we have  $G_\iota(\iota) = u$  for  $\iota \in [0, w_d]$  and  $G_\iota(\iota) = u + (1 - u)G_w(\iota)$  for  $\iota > w_d$ , where  $G_w(w)$  is the sectoral wage distribution. The mean of the sectoral income distribution is  $\bar{\iota} = (1 - u)\bar{w} + u \cdot 0 = (1 - u)\bar{w}$ , where  $\bar{w}$  is the mean of the sectoral wage distribution. We can now compute the Theil Index of sectoral income inequality:

$$\mathcal{T}_\iota = u \cdot 0 + (1 - u) \int_{w_d}^\infty \frac{w}{\bar{\iota}} \ln \frac{w}{\bar{\iota}} dG_w(w) = \frac{(1 - u)\bar{w}}{\bar{\iota}} \int_{w_d}^\infty \frac{w}{\bar{w}} \left[ \ln \frac{w}{\bar{w}} - \ln \frac{\bar{\iota}}{\bar{w}} \right] dG_w(w).$$

Since  $\bar{\iota}/\bar{w} = (1 - u)$ , we can rewrite this as:

$$\mathcal{T}_\iota = \mathcal{T}_w - \ln(1 - u),$$

where  $\mathcal{T}_w$  is the Theil Index of sectoral wage inequality as stated in the text of the paper. The same expression can be derived using the decomposition of the Theil Index into within and between-group components, as studied in Bourguignon (1979) and applied to this setting in Helpman, Itskhoki and Redding (2008a,b). Finally, note that sectoral income inequality as measured by Theil Index is increasing in the sectoral unemployment rate and sectoral wage inequality, consistent with the results for the Gini Coefficient above. Combining these results with the Propositions in Sections 3 and 4 we arrive at Propositions 6 and 7.

## 6 General Equilibrium

### 6.1 Outside Sector and Risk Neutrality

As noted in the paper, the Harris-Todaro condition that equates *ex ante* expected indirect utility in the two sectors takes the following form:

$$\mathbb{V} = \frac{1}{1-\eta} \mathbb{E} \left( \frac{w}{\mathcal{P}} \right)^{1-\eta} = \frac{\mathbb{E}w}{\mathcal{P}} = \frac{1}{\mathcal{P}}, \quad \text{for } \eta = 0, \quad (35)$$

which, given the probability of matching  $x$  and the expected wage conditional on matching of  $w(\theta)h(\theta)/n(\theta) = b$ , implies:

$$xb = \omega = 1.$$

From this expression, a sufficient condition for  $\alpha_0 > \omega$ , and hence  $0 < x < 1$  in (19), is  $\alpha_0 > 1$ .

*Ex post* indirect utility depends on the aggregate price index ( $\mathcal{P}$ ) and *ex post* wages, which depend on a worker's sector and firm of employment:

$$\mathbb{V} = \begin{cases} 1/\mathcal{P} & \text{if employed in the outside sector} \\ w(\theta)/\mathcal{P} & \text{if employed by a } \theta\text{-firm in the differentiated sector} \\ 0 & \text{if unemployed} \end{cases} \quad (36)$$

where  $\mathcal{P}$  is the dual of the aggregate consumption index ( $\mathcal{C}$ ):

$$\mathcal{P} = \left[ \vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}} \right]^{-\frac{1-\zeta}{\zeta}}, \quad (37)$$

and depends on the differentiated sector price index ( $P$ ), which can be determined from the expression for the demand-shifter ( $A$ ) with CES preferences:

$$A^{\frac{1}{1-\beta}} = \frac{(1-\vartheta) P^{\frac{\beta-\zeta}{(1-\beta)(1-\zeta)}} \bar{L}}{\vartheta + (1-\vartheta) P^{\frac{-\zeta}{1-\zeta}}} \quad (38)$$

where we have used  $\Omega = \bar{L}$ . From the solutions for firm-specific variables (16), *ex post* wages in the differentiated sector in the closed and open economies are:

$$w^a(\theta) = w_d \left( \frac{\theta}{\theta_d^a} \right)^{\frac{\beta k}{\delta \Gamma}} \quad \text{for } \theta \geq \theta_d^a,$$

$$w^t(\theta) = \begin{cases} w_d \left( \frac{\theta}{\theta_d^t} \right)^{\frac{\beta k}{\delta \Gamma}} & \text{for } \theta \in [\theta_d^t, \theta_x^t) \\ \Upsilon_x^{\frac{k(1-\beta)}{\delta \Gamma}} w_d \left( \frac{\theta}{\theta_d^t} \right)^{\frac{\beta k}{\delta \Gamma}} & \text{for } \theta \geq \theta_x^t \end{cases},$$

where the superscripts  $a$  and  $t$  denote closed and open economy variables respectively.

To characterize the impact of the opening of trade on *ex post* wages at domestic and exporting firms, note that  $w^t(\theta) < w^a(\theta)$  for  $\theta \in [\theta_d^t, \theta_x^t)$  because  $\theta_d^t > \theta_d^a$ . Note also that  $w^t(\theta) > w^a(\theta)$  for  $\theta \geq \theta_x^t$  whenever  $\Upsilon_x^{1-\beta} > (\theta_d^t/\theta_d^a)^\beta$ . To show that this inequality must be satisfied, note that the free entry conditions in the

closed and open economy (28) together imply:

$$\left(\frac{\theta_d^t}{\theta_d^a}\right)^z = \left[1 + \left(\frac{f_d}{f_x}\right)^{\frac{z\Gamma-\beta}{\beta}} \left[\Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1\right]^{z\Gamma/\beta}\right].$$

We can rewrite this condition as:

$$\left(\frac{\theta_d^t}{\theta_d^a}\right)^z - 1 = \left[\Upsilon_x^{\frac{1-\beta}{\Gamma}} - 1\right]^{\frac{z\Gamma}{\beta}} \left(\frac{f_d}{f_x}\right)^{\frac{z\Gamma}{\beta}-1} < \left[\Upsilon_x^{\frac{1-\beta}{\Gamma}}\right]^{\frac{z\Gamma}{\beta}} - 1,$$

where the above inequality holds because  $\Upsilon_x > 1$ ,  $z\Gamma > \beta$  and  $f_d/f_x < 1$ . This immediately implies  $(\theta_d^t/\theta_d^a)^\beta < \Upsilon_x^{1-\beta}$ .

Proposition 8 can be proved as follows. The Harris-Todaro condition under risk neutrality implies  $xb = \omega = 1$ , which together with the search technology (17) implies that search costs and labor market tightness are invariant to trade:  $b = \alpha_0^{1/(1+\alpha_1)}$  and  $x = \alpha_0^{-1/(1+\alpha_1)}$ . From the free entry condition (28),  $\theta_d$  is higher in the open economy than in the closed economy. From the zero-profit productivity cutoff condition (20), a higher value of  $\theta_d$  and an unchanged value of  $b$  implies a lower value of  $A$  in the open economy than in the closed economy. From (38) above, a lower value of  $A$  implies a lower value of  $P$  in the open economy than in the closed economy. From (37), this reduction in  $P$  in turn implies a lower value of  $\mathcal{P}$  and higher *ex ante* expected welfare (35) in the open economy than in the closed economy, which establishes part (i) of the proposition. Parts (ii)-(iii) of the proposition follow immediately from the lower value of  $\mathcal{P}$  in the open economy than in the closed economy, the changes in *ex post* wages and welfare (established in (36) above), and the analysis of unemployment in Section 4 (noting that in this general equilibrium environment  $\omega$  is invariant to trade).

## 6.2 Single Differentiated Sector and Risk Neutrality

The conditions for sectoral equilibrium remain the same as in (17)-(27). With a single differentiated sector, the expression for the demand shifter in (24) becomes:

$$A \equiv Q^{1-\beta} P = Q^{1-\beta}, \quad (39)$$

where we have used the choice of numeraire:  $P = 1$ . Therefore, total revenues are  $E = Q$ .

The solutions for  $(\theta_d, \theta_x)$  under the assumption of country symmetry follow from (28) and (22), as discussed above. Using symmetric countries, the equilibrium labor force (27) and labor market clearing ( $L = \bar{L}$ ), we obtain:

$$\frac{\beta\gamma}{1+\beta\gamma} Q = \bar{L}\omega.$$

Using the search technology (17), expected worker income (18) and the zero-profit cutoff condition (20), we obtain:

$$Q = \left(\frac{f_d}{\kappa_r} \frac{1+\beta\gamma}{\Gamma}\right)^{\frac{\Gamma}{1-\beta}} c^{\frac{\beta(1-\gamma k)}{(1-\beta)\delta}} \alpha_0^{\frac{\beta\gamma}{(1-\beta)(1+\alpha_1)}} \theta_d^{-\frac{\beta}{1-\beta}} \omega^{\frac{\beta\gamma}{1-\beta} \frac{\alpha_1}{1+\alpha_1}}$$

These equations define two upward-sloping relationships in  $(Q, \omega)$  space that determine the equilibrium values of  $Q$  and  $\omega$ . One can show that a necessary and sufficient condition for the stability of the equilibrium

is:

$$\frac{\beta\gamma}{1-\beta} \frac{\alpha_1}{1+\alpha_1} > 1, \quad (40)$$

which is satisfied for sufficiently large  $\alpha_1$  (sufficiently convex hiring costs) and sufficiently large  $\beta$  (a sufficiently high elasticity of substitution between varieties). As  $0 < \alpha_1/(1+\alpha_1) < 1$ , a necessary condition for this parameter restriction to hold is:

$$\frac{\beta\gamma}{1-\beta} > 1,$$

which is satisfied for values of  $\beta$  sufficiently close to but less than 1. Assuming parameter values satisfying these inequalities, the conditions for sectoral equilibrium in (17)-(27) (where (39) replaces (24)) can be solved to yield the following solutions for the other endogenous variables of the model:

$$\begin{aligned} Q &= Q^* = \frac{1+\beta\gamma}{\beta\gamma} \alpha_0^{\frac{\gamma\beta}{(1-\beta)\Delta}} c^{\frac{\beta(1-\gamma k)(1+\alpha_1)}{\delta(1-\beta)\Delta}} \theta_d^{-\frac{\beta(1+\alpha_1)}{(1-\beta)\Delta}} \bar{L}^{-\frac{\gamma\beta}{(1-\beta)\Delta}} \kappa_b^{\frac{(1+\alpha_1)}{\Delta}}, \\ \omega &= \omega^* = \alpha_0^{\frac{\gamma\beta}{(1-\beta)\Delta}} c^{\frac{\beta(1-\gamma k)(1+\alpha_1)}{\delta(1-\beta)\Delta}} \theta_d^{-\frac{\beta(1+\alpha_1)}{(1-\beta)\Delta}} \bar{L}^{-\frac{(1+\alpha_1)}{\Delta}} \kappa_b^{\frac{(1+\alpha_1)}{\Delta}}, \\ b &= b^* = \alpha_0^{\frac{1}{\Delta}} c^{\frac{\beta(1-\gamma k)\alpha_1}{\delta(1-\beta)\Delta}} \theta_d^{-\frac{\beta\alpha_1}{(1-\beta)\Delta}} \bar{L}^{-\frac{\alpha_1}{\Delta}} \kappa_b^{\frac{\alpha_1}{\Delta}}, \\ x &= x^* = \alpha_0^{\frac{(\frac{\gamma\beta}{(1-\beta)\Delta}-1)}{\Delta}} c^{\frac{\beta(1-\gamma k)}{\delta(1-\beta)\Delta}} \theta_d^{-\frac{\beta}{(1-\beta)\Delta}} \bar{L}^{-\frac{1}{\Delta}} \kappa_b^{\frac{1}{\Delta}}, \\ M &= M^* = \left( \frac{z\Gamma - \beta}{z} \right) \frac{\theta_{\min}^{-z}}{f_d \beta \gamma} \alpha_0^{\frac{\gamma\beta}{(1-\beta)\Delta}} c^{\frac{\beta(1-\gamma k)(1+\alpha_1)}{\delta(1-\beta)\Delta}} \theta_d^{z-\frac{\beta(1+\alpha_1)}{(1-\beta)\Delta}} \bar{L}^{-\frac{\gamma\beta}{(1-\beta)\Delta}} \kappa_b^{\frac{(1+\alpha_1)}{\Delta}}, \end{aligned} \quad (41)$$

where

$$\begin{aligned} \Delta &\equiv \frac{(1-\beta)(1+\alpha_1) - \beta\gamma\alpha_1}{(1-\beta)} < 0, \\ \kappa_b &\equiv \frac{\beta\gamma}{(1+\beta\gamma)} \left[ \frac{f_d(1+\beta\gamma)}{\kappa_r \Gamma} \right]^{\frac{\Gamma}{1-\beta}} > 0. \end{aligned}$$

From the above, a sufficient condition for  $\alpha_0 > \omega$ , and hence  $0 < x < 1$  in (19), is:

$$\alpha_0^{\frac{(1+\alpha_1)[(1-\beta)-\beta\gamma]}{(1+\alpha_1)(1-\beta)-\alpha_1\beta\gamma}} > c^{\frac{\beta(1-\gamma k)(1+\alpha_1)}{\delta(1-\beta)\Delta}} \theta_d^{-\frac{\beta(1+\alpha_1)}{(1-\beta)\Delta}} \bar{L}^{-\frac{(1+\alpha_1)}{\Delta}} \kappa_b^{\frac{(1+\alpha_1)}{\Delta}}$$

Finally, under the assumption of risk neutrality, expected indirect utility is:

$$\mathbb{V} = \mathbb{E} \left( \frac{w}{\mathcal{P}} \right) = \omega \quad \text{for } \eta = 0. \quad (42)$$

The proof of Proposition 9 follows immediately from the closed form solutions for  $(\theta_d, \theta_x)$  in (28) and (22) and from the closed form solutions for  $(Q, \omega, b, x, M, \mathbb{V})$  in (41) and (42).

### 6.3 Outside Sector and Risk Aversion

We first show how the introduction of worker risk aversion affects the equilibrium share of revenue received by workers in the bargaining game. Specifically, with CRRA-CES preferences, the solution to the bargaining game under risk aversion takes a similar form as when there are differences in bargaining weight between the firm and its workers. Given workers' revenue share, we next show that the determination of sectoral equilibrium remains unchanged except for the Harris-Todaro condition equating expected utility in the two

sectors.

### 6.3.1 Stole and Zweibel Bargaining

The bargaining game takes the same form as discussed in the paper for risk neutrality, except that workers are assumed to be risk averse ( $0 < \eta < 1$ ), and instead of assuming equal bargaining weights we allow for differences in bargaining weight between the firm and its workers. The bargaining power of workers is denoted by  $\lambda \in (0, 1)$  and their outside option of unemployment involves zero income.

We start with the case of a discrete number of workers  $h$  and then take the limit of continuous divisibility of the workforce. Denote by  $R(h)$  the revenue function when a firm employs  $h$  workers and denote by  $w(h)$  the equilibrium wage schedule that the firm pays to each worker when the employment level is  $h$ . Now consider that the firm is separated with  $\delta > 0$  workers (the special case is  $\delta = 1$  and we will later take the limit  $\delta \rightarrow 0$ ). This will reduce revenue to  $R(h - \delta)$  and the wage rate to  $w(h - \delta)$ .

Now consider the firm's bargaining with the marginal  $\delta$  workers when employment is  $h$ . Denote by  $t$  the wage that the firm pays to these  $\delta$  workers, while other workers receive  $w(h)$ . The utility of these marginal workers is  $t^{1-\eta}/(1-\eta)$ . If the bargaining breaks down and they leave, the revenues will fall by  $R(h) - R(h - \delta)$  and the firm will be paying the remaining  $h - \delta$  workers the wage rate  $w(h - \delta)$ . Therefore, the incremental payoff to the firm from employing these marginal  $\delta$  workers is:

$$[R(h) - R(h - \delta)] - (h - \delta)[w(h) - w(h - \delta)] - \delta t.$$

Assuming zero outside option for workers, the payoff from employment to each of the marginal workers is equal to  $t^{1-\eta}/(1-\eta)$ . Therefore, we can write Stole and Zweibel Bargaining solution as:

$$t(h) = \arg \max_t \left( [R(h) - R(h - \delta)] - (h - \delta)[w(h) - w(h - \delta)] - \delta t \right)^{1-\lambda} \left( \frac{1}{1-\eta} t^{1-\eta} \right)^\lambda$$

The equilibrium requirement is that  $t(h) \equiv w(h)$ . That is, for every employment level  $h$ , as a result of bargaining with marginal workers the firm pays them accordingly to the equilibrium wage schedule.

Denote by

$$\phi \equiv \frac{1 - \lambda}{\lambda(1 - \eta)}$$

the effective relative bargaining weight of the firm. Then we can implicitly write the bargaining solution, taking into account the equilibrium requirement, as:

$$(1 + \phi)\delta w(h) = [R(h) - R(h - \delta)] - (h - \delta)[w(h) - w(h - \delta)].$$

This holds for every  $h > 0$ . We now take the limit as  $\delta \rightarrow 0$  to obtain the differential equation for the wage schedule:

$$(1 + \phi)w(h) = R'(h) - w'(h)h \quad \Rightarrow \quad w(h) = \frac{1}{h^{1+\phi}} \int_0^h R'(\xi)\xi^\phi d\xi.$$

This is the generalized Stole and Zweibel (1996a,b) condition for the case of asymmetric bargaining weights. Therefore risk aversion is equivalent to an adjustment in the bargaining weights (greater risk aversion of workers reduces their bargaining weight). When revenue is a power function of employment with power  $\beta\gamma$

( $R(h) = Ah^{\beta\gamma}$ ), the solution to this differential equation is given by:

$$w(h) = \frac{\beta\gamma}{\phi + \beta\gamma} \frac{R(h)}{h}.$$

Note that the wage rate is again a constant fraction of average revenues. The fraction of revenues accruing to workers is increasing in  $\beta\gamma$  (decreasing in the concavity of revenues) and decreasing in  $\phi$  (the relative effective bargaining power of the firm). In turn,  $\phi$  is decreasing in  $\lambda$  (the primitive bargaining power of workers) and increasing in  $\eta$  (the degree of risk aversion of workers). The case of symmetric bargaining weights and no risk aversion is a special case of this more general formulation. Specifically, when  $\lambda = 1$  and  $\eta = 0$ , we have  $\phi = 1$  and arrive at our baseline formulation in the paper.

### 6.3.2 Sectoral Equilibrium

After taking into account the change in workers' revenue share, the only equilibrium condition that changes when risk aversion is introduced is the Harris-Todaro condition of worker indifference between searching for employment in the two sectors. Specifically, this condition now equates the utility from a certain income of one in the homogenous sector with the expected utility from being hired and receiving a wage drawn from the equilibrium wage distribution in the differentiated sector:

$$x\sigma\mathbb{E}w^{1-\eta} = x\sigma \int_{w_d}^{\infty} w^{1-\eta} dG_w(w) = 1. \quad (43)$$

Evaluating the integral using the open economy wage distribution (31), this condition becomes:

$$x\sigma\mathbb{E}w^{1-\eta} = x\sigma \frac{1+\mu}{1+\mu\eta} w_d^{1-\eta} \left[ \frac{1 + \rho^{z - \frac{\beta(1-\eta k/\delta)}{\Gamma}} \left[ \Upsilon_x^{\frac{(1-\beta)(1-\eta k/\delta)}{\Gamma}} - 1 \right]}{1 + \rho^{z - \frac{\beta(1-k/\delta)}{\Gamma}} \left[ \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1 \right]} \right] = 1$$

Recall that the open economy hiring rate can be expressed as:

$$\sigma = \frac{1 + \left[ \Upsilon_x^{\frac{(1-\beta)(1-k/\delta)}{\Gamma}} - 1 \right] \rho^{z - \beta(1-k/\delta)/\Gamma}}{1 + \left[ \Upsilon_x^{\frac{(1-\beta)}{\Gamma}} - 1 \right] \rho^{z - \beta/\Gamma}} \cdot \frac{1}{1+\mu} \cdot \frac{1}{\phi_w},$$

where

$$\phi_w \equiv \left[ \frac{\Gamma}{\beta(1-\gamma k)} \frac{ca_{\min}^{\delta}}{f_d} \right]^{k/\delta}.$$

In addition, the lowest wage in the differentiated sector can be written as  $w_d = b\phi_w$ . Therefore, we can rewrite the Harris-Todaro condition in the form given in the paper:

$$\Lambda(\rho, \Upsilon_x) \frac{b^{1-\eta} x}{\phi_w^{\eta} (1+\mu\eta)} = 1, \quad (44)$$

where

$$\Lambda(\rho, \Upsilon_x) \equiv \frac{1 + \rho^{z - \frac{\beta(1-\eta)k/\delta}{\Gamma}} \left[ \Upsilon_x^{\frac{(1-\beta)(1-\eta)k/\delta}{\Gamma}} - 1 \right]}{1 + \rho^{z - \frac{\beta}{\Gamma}} \left[ \Upsilon_x^{\frac{(1-\beta)}{\Gamma}} - 1 \right]}.$$

Evidently, we have  $\Lambda(0, \Upsilon_x) = 1$  and  $0 < \Lambda(\rho, \Upsilon_x) < 1$  for  $0 < \rho \leq 1$ .

Using the Harris-Todaro condition (44) and the hiring cost function (17), we obtain the following expressions for  $(x, b)$ :

$$x = \left( \frac{(1 + \mu\eta) \phi_w^\eta}{\alpha_0^{1-\eta} \Lambda(\rho, \Upsilon_x)} \right)^{\frac{1}{1+(1-\eta)\alpha_1}}, \quad (45)$$

$$b = \alpha_0^{\frac{1}{1+(1-\eta)\alpha_1}} \left( \frac{(1 + \mu\eta) \phi_w^\eta}{\Lambda(\rho, \Upsilon_x)} \right)^{\frac{\alpha_1}{1+(1-\eta)\alpha_1}}. \quad (46)$$

Expected worker income is therefore:

$$\omega = xb = \alpha_0^{\frac{\eta}{1+(1-\eta)\alpha_1}} \left( \frac{(1 + \mu\eta) \phi_w^\eta}{\Lambda(\rho, \Upsilon_x)} \right)^{\frac{1+\alpha_1}{1+(1-\eta)\alpha_1}}. \quad (47)$$

A sufficient condition for  $0 < x < 1$  is now given by:

$$\alpha_0^{1-\eta} > \frac{(1 + \mu\eta) \phi_w^\eta}{\Lambda(\rho, \Upsilon_x)}.$$

The proof of Proposition 10 follows from the above. In the closed economy  $\rho = 0$  and therefore  $\Lambda(0, \Upsilon_x) = 1$ . Opening economy to trade leads to  $\rho = \theta_d/\theta_x > 0$  and therefore  $\Lambda(\rho, \Upsilon_x) < 1$ . Consequently, equations (45)-(47) imply that  $x, b$  and  $\omega$  are higher in the open economy than in autarky.

## 7 Supplementary Derivations

### 7.1 Derivation of the Production Technology

We assume the following production function:

$$y = \theta h^\gamma \bar{a} = \theta \left( \frac{1}{h} \right)^{1-\gamma} \int_0^h a_i di, \quad (48)$$

where  $i \in [0, h]$  indexes the workers employed by the firm. One way to think about this technology is the following: A manager with productivity  $\theta$  has one unit of time which he allocates equally among his employees. Thus, the manager allocates  $1/h$  of his time to each worker and, as a result, a worker with match-specific ability  $a$  can contribute  $\theta(1/h)^{1-\gamma}a$  to the total output of the firm, where  $(1-\gamma)$  measures the importance of the managerial time input. Aggregating across workers yields the assumed production function. We further assume, following a large literature on moral hazard in teams, that the contributions of individual workers to total output are unobservable as production is done in teams and the production process is non-separable. As a result, the match-specific ability of workers cannot be deduced from observing output. This justifies the assumption that the manager splits his time equally among the workers since they all appear homogeneous because of the unobservable and non-verifiable nature of their match-specific ability. Alternatively, an equal managerial time allocation among workers can be rationalized by assuming that the

productivity of each worker depends on average worker ability as a result of human capital externalities across workers within firms.<sup>1</sup>

## 7.2 Marginal Product of Labor

Given the production technology (48), the marginal product of a worker with match-specific ability  $a$  is:<sup>2</sup>

$$MP(a) \equiv MP(a|\bar{a}, h; \theta) = \theta h^{-(1-\gamma)} [a - (1 - \gamma)\bar{a}].$$

Let match-specific ability in the pool of candidate employees be distributed according to cumulative distribution function  $G_a(a)$ . Then, if the firm only hires workers with ability above  $a_c$ , the mean ability of its workers is

$$\bar{a}(a_c) = \frac{1}{1 - G_a(a_c)} \int_{a_c}^{\infty} a dG_a(a).$$

The marginal product of the threshold-ability worker is thus

$$MP(a_c) = \theta h^{-(1-\gamma)} [a_c - (1 - \gamma)\bar{a}(a_c)],$$

which is negative whenever  $(1 - \gamma)\bar{a}(a_c) > a_c$ . Since  $\bar{a}(a_c) > a_c$  for any non-degenerate distribution,  $MP(a_c) < 0$  can be guaranteed by choosing  $\gamma$  small enough.

Specifically, consider the case of Pareto-distributed ability as we assume in the paper. In this case  $\bar{a}(a_c) = ka_c/(k - 1)$  and the marginal product is

$$MP(a_c) = -\theta h^{-(1-\gamma)} \frac{1 - \gamma k}{k - 1} a_c,$$

which is negative whenever  $\gamma k < 1$ . Recall that this is the same parameter restriction which ensures that output is increasing in the cutoff ability. It also guarantees that the workers not hired by the firm have negative marginal product. The firm will not want to retain these workers even at a zero wage, because the resulting reduction in output from lower average worker ability would dominate the increase in output from a greater measure of hired workers.

Note further that given  $\gamma k < 1$  the firm hires some workers with ability in the range  $[a_c, \hat{a})$ , where

$$\hat{a} = (1 - \gamma)\bar{a} = \frac{(1 - \gamma)k}{k - 1} a_c > a_c,$$

and these workers have a negative marginal product. The firm would prefer not to hire these workers, but costly search and screening make it optimal to hire all workers with match-specific ability above  $a_c$ . Finally,

<sup>1</sup>For empirical evidence on human capital externalities within plants, see for example Moretti (2004). See also related work on O-ring production technologies following Kremer (1993).

<sup>2</sup>To define the marginal product rewrite the production function as

$$y = \theta \left[ \int_0^h di \right]^{-(1-\gamma)} \int_0^h a_i di.$$

Then the marginal product of adding worker  $h$  with productivity  $a_h$  is

$$MP_h \equiv \frac{dy}{dh} = \theta h^{-(1-\gamma)} [a_h - (1 - \gamma)\bar{a}].$$

Note that the production function does not depend on the ordering of workers and hence the marginal product depends only on the ability of the worker in the sense that  $MP_h = MP(a_h) = MP(a_h|\bar{a}, h; \theta)$ .

note that the average marginal product of workers employed by a firm with productivity  $\theta$  is always positive:

$$\overline{MP}(\theta) = \gamma\theta h(\theta)^{-(1-\gamma)}\bar{a}(a_c(\theta)) > 0.$$

### 7.3 Division of Revenue in the Bargaining Game

In our model, the firm and workers bargain about the division of revenues, as the outside option of workers is normalized to zero. As in Stole and Zweibel (1996a,b), the firm bargains with every worker taking into account the effect of his departure on the bargaining game with remaining workers. All the firm's other decisions – sampling, screening, production, exporting – are sunk by the bargaining stage. Therefore, from (10) and (3), revenues  $r(\theta, h)$  is a continuous, increasing and concave function of employment  $h$ , and all other arguments of firm revenue ( $\theta$ ,  $\bar{a}(\theta)$ ,  $\Upsilon(\theta)$ ,  $A$ ) are fixed. Let  $w(\theta, h)$  be the bargained wage rate that a  $\theta$ -firm pays as a function of the measure of workers hired  $h$ . This function has to satisfy the differential equation:

$$\frac{\partial}{\partial h} [r(\theta, h) - w(\theta, h)h] = w(\theta, h) \quad (49)$$

so that the surplus of worker from employment (the wage rate) is equal to the marginal surplus of the firm from employing the worker.<sup>3</sup> Using the assumed functional forms for revenues this differential equation yields the solution

$$w(\theta, h) = \frac{\beta\gamma}{1 + \beta\gamma} \frac{r(\theta, h)}{h},$$

so that each workers gets a fraction  $\beta\gamma/(1 + \beta\gamma)$  of average revenues and the firm gets the remaining  $1/(1 + \beta\gamma)$  share of revenues. The worker's share of surplus is increasing in  $\beta\gamma$ , where  $\beta$  captures the concavity of demand and  $\gamma$  captures the concavity of the production technology. Therefore, the worker's share of surplus is decreasing in the concavity of the revenue function in  $h$ , because a more concave revenue function implies a smaller effect of the departure of any given worker on firm revenue.

### 7.4 Derivation of the Search Cost (b)

Following Blanchard and Gali (2008), search costs (17) can be derived from a constant returns to scale matching technology and a cost of posting vacancies. Suppose there is a cost of posting a vacancy of  $\psi_0$  units of the numeraire. Suppose also that the measure of matched workers is a Cobb-Douglas function of vacancies and workers searching for employment:

$$N = \psi_1 V^{\psi_2} L^{1-\psi_2}, \quad \psi_1 > 0, \quad 0 < \psi_2 < 1.$$

Given this search technology, a firm choosing to match with  $n$  workers has to post  $v > n$  vacancies:

$$v = \psi_1^{-1/\psi_2} x^{(1-\psi_2)/\psi_2} n, \quad x \equiv N/L.$$

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<sup>3</sup>Since individual abilities are unobservable, the expressions on both sides of (49) are evaluated holding  $\bar{a} = \bar{a}(\theta)$  constant for all  $h$ . Indeed, when a marginal worker departs, the productivity of the remaining workers is still  $\bar{a}$  in expectation. The firm evaluates the expected loss from the departure of a worker as:

$$\frac{\partial r}{\partial h} + \frac{\partial r}{\partial \bar{a}} \frac{\partial \bar{a}}{\partial h},$$

where  $\partial \bar{a}/\partial h$  equals zero in expectation and hence there remains only the direct effect of  $h$  on revenues. As a result, the bargaining solution in this environment with symmetric uncertainty is the same as if all workers had a productivity of  $\bar{a}$ .

Given the cost of posting each vacancy, a firm choosing to match with  $n$  workers incurs a search cost of:

$$b = \alpha_0 x^{\alpha_1}, \quad \alpha_0 \equiv \left( \frac{\psi_0}{\psi_1^{1/\psi_2}} \right), \quad \alpha_1 \equiv \frac{(1 - \psi_2)}{\psi_2}.$$

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