



The net present value method versus the option value of waiting: A note on Farzin, Huisman and Kort (1998)[☆]

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Abstract

Using the tools of real-options theory, Farzin, Huisman and Kort (1998) investigate the optimal timing of technology adoption in a model in which there is ongoing technological progress and the firm's investment decision is irreversible. When the firm can switch technologies $n < \infty$ times, Farzin et al. (1998) obtain the surprising result that the criterion for adopting a new technology derived from real-options theory coincides with the net present value criterion for all but the last switch. This note points out a mistake in Farzin et al.'s (1998) derivations that leads to the above result. We show that, compared to the net present value approach, the firm will defer the adoption of a new technology when it takes the option value of waiting into account. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

The net present value (NPV) rule is now widely known to lead to a suboptimal investment decision when part of the investment costs are sunk and the firm has some discretion about the timing of the project. In contrast, real-options theory is explicitly based on the idea that most investment projects embed a series of alternative actions. These alternative actions are akin to financial options in that they can be exercised as a firm's management sees fit. It follows that 'the ability to delay an irreversible investment can profoundly affect the decision to invest' (Dixit and Pindyck, 1994, p. 6).

Using the tools of real-options theory, Farzin et al. (1998) investigate the optimal timing of technology adoption in a model in which there is ongoing technological progress and the firm's investment decision is irreversible. In their model the currently available technology is adopted as soon as an innovation drives technological efficiency θ over a certain threshold. When the firm can switch technologies only once, this threshold is larger than the threshold implied by the NPV approach. However, when the firm can switch technologies $n < \infty$ times, Farzin et al. (1998) obtain the surprising result that the criterion for adopting a new technology derived from real-options theory coincides with the NPV criterion for all but the last switch. They offer the following explanation:

By ignoring the fact that once the firm invests it gives up the option to delay, the net present value rule fails to account for the cost of this lost option and therefore results in a lower trigger level than would be optimal. However, in the present case where the firm has the opportunity to make multiple switches, the firm has *not* given up anything once it has invested for the m th time, if $m < n$, because it can invest again once θ increases sufficiently. (p. 790, their italics)

This result, if it were true, would have far-reaching consequences for both the theoretical treatment and the practical implementation of investment decisions. It not only indicates that the management of a firm can resort to the easy-to-use NPV rule for assessing the viability of most of its investment projects, but it also defies the major insight of real-options theory, namely that irreversibility in combination with the ability to wait overthrows the NPV rule.¹

¹ Moreover, Farzin et al.'s (1998) result fails to carry over to the closely related model proposed by Balcer and Lippman (1984). Farzin et al. (1998) err when they claim that Balcer and Lippman's (1994) model 'is confined to the analysis of the single-switching case and does not consider the case of multiple switchings' (p. 782). In fact, Balcer and Lippman (1984) allow for infinitely many switches as can be seen from their Eq. (3a).

This note points out a mistake in Farzin et al.’s (1998) derivations that leads to the above result and shows that the threshold when the option value of waiting is taken into account exceeds the threshold implied by the NPV approach even though the firm can switch technologies $n < \infty$ times. We pinpoint Farzin et al.’s (1998) mistake to their omission of a state variable in the firm’s dynamic optimization problem. An example illustrates that the firm indeed stands to lose a lot by making a suboptimal adoption decision.

2. The model

A firm’s decision to adopt a new technology is viewed as an irreversible investment decision. Time is continuous and the firm’s horizon is infinite. Technological efficiency at time t is denoted by θ , where θ follows a jump-process. Consider a short interval of time dt . During dt either an innovation occurs or no innovation occurs. From t to $t + dt$ technological efficiency evolves according to

$$d\theta = \begin{cases} U & \text{with probability } \lambda dt, \\ 0 & \text{else,} \end{cases} \tag{1}$$

where $U \sim F(u)$ denotes an innovation. To prevent technological regress, assume $F(0) = 0$.

At time $t = 0$ the firm produces with technological efficiency θ_0 . The firm can switch technologies $n < \infty$ times. Let θ_{i-1} denote the firm’s technological efficiency after the $(i - 1)$ th but before the i th switch. Equipped with a technology of efficiency θ_{i-1} the firm makes instantaneous profits $\pi(\theta_{i-1})$, where π is increasing in its argument.

Let $V_{i-1}(\theta, \theta_{i-1})$ denote the expected discounted profits of a firm that has switched technologies exactly $i - 1$ times and has a technology with efficiency θ_{i-1} in place when the currently available technology is θ . In contrast, Farzin et al. (1998) omit the second argument of the value function and write $V_{i-1}(\theta)$ instead of $V_{i-1}(\theta, \theta_{i-1})$ which conceals their error.

At time t the firm can either decide to adopt the current technology or wait. If it adopts it pays a fixed cost $I > 0$ and exchanges $V_{i-1}(\theta, \theta_{i-1})$ for $V_i(\theta, \theta)$. The Bellman equation is

$$V_{i-1}(\theta, \theta_{i-1}) = \pi(\theta_{i-1}) dt + \frac{1}{1 + r dt} \left\{ \lambda dt \int_0^\infty \max[W^a, W^w] dF(u) + (1 - \lambda dt)V_{i-1}(\theta, \theta_{i-1}) \right\}, \tag{2}$$

where $r > 0$ is the interest rate and

$$W^a \equiv V_i(\theta + u, \theta + u) - I, \tag{3}$$

$$W^w \equiv V_{i-1}(\theta + u, \theta_{i-1}). \tag{4}$$

The Bellman equation adds the current return and the expected future stream of returns, appropriately discounted, under the presumption that future decisions are made optimally. Hence, the term $\pi(\theta_{i-1}) dt$ captures the firm’s profits from t to $t + dt$. The second term of the sum in parentheses describes the evolution of V_{i-1} when no innovation takes place and the first term does so in case of an innovation. The max operator in the first term in turn represents the firm’s choice between adopting and waiting upon the arrival of an innovation, where W^a is the value of adopting and W^w the value of waiting. Substituting for W^a and W^w and letting $dt \rightarrow 0$ the Bellman equation becomes

$$\begin{aligned} &V_{i-1}(\theta, \theta_{i-1}) \\ &= \frac{\pi(\theta_{i-1})}{r + \lambda} + \frac{\lambda}{r + \lambda} \int_0^\infty \max[V_i(\theta + u, \theta + u) - I, V_{i-1}(\theta + u, \theta_{i-1})] dF(u). \end{aligned} \tag{5}$$

Given V_{i-1} and V_i , the firm’s optimal policy is

$$d_i(\theta, \theta_{i-1}) = \begin{cases} 1 & \text{if } V_i(\theta, \theta) - I > V_{i-1}(\theta, \theta_{i-1}), \\ 0 & \text{else,} \end{cases} \tag{6}$$

where $d_i = 1$ indicates that the currently available technology is adopted. Assuming that the optimal policy exhibits the reservation property,² it can be re-written as

$$d_i(\theta, \theta_{i-1}) = \begin{cases} 1 & \text{if } \theta > \theta_i^*(\theta_{i-1}), \\ 0 & \text{else,} \end{cases} \tag{7}$$

where θ_i^* is the threshold when taking the option value of waiting into account and is given by

$$V_i(\theta_i^*, \theta_i^*) - I = V_{i-1}(\theta_i^*, \theta_{i-1}). \tag{8}$$

In contrast, the NPV approach ignores the option value of waiting. Hence, the threshold θ^{NPV} implied by the NPV approach is given by

$$\frac{\pi(\theta^{NPV})}{r} - I = \frac{\pi(\theta_{i-1})}{r} \Leftrightarrow \theta^{NPV}(\theta_{i-1}) = \pi^{-1}(\pi(\theta_{i-1}) + rI), \tag{9}$$

² Farzin et al. (1998) do not prove that the reservation property holds in general.

where the existence of the inverse instantaneous profit function π^{-1} is guaranteed since π is increasing in its argument.

3. The option value of waiting

If the firm can switch technologies $n < \infty$ times, it is stuck forever with whatever technology it adopted at the n th switch. Hence, the firm’s payoff upon adopting technology $\theta + u$ is

$$\frac{\pi(\theta + u)}{r}, \tag{10}$$

which replaces $V_n(\theta + u, \theta + u)$ in Eq. (5). In principle, the value functions $\{V_i\}_{i=0}^{n-1}$ can now be solved for by backward recursion on Eq. (5).

We are now ready to point out the mistake in Farzin et al.’s (1998) derivations. Suppose that U has compact support $[0, \bar{u}]$ and that $\theta_{i-1} + \bar{u} \leq \theta_i^*(\theta_{i-1})$ for all θ_{i-1} , so that immediate consecutive switches are ruled out. Provided the optimal policy exhibits the reservation property, Eq. (5) can be re-written as

$$V_{i-1}(\theta, \theta_{i-1}) = \frac{\pi(\theta_{i-1})}{r + \lambda} + \frac{\lambda}{r + \lambda} \left\{ \int_{\theta_i^* - \theta}^{\bar{u}} V_i(\theta + u, \theta + u) - I dF(u) + \int_0^{\theta_i^* - \theta} V_{i-1}(\theta + u, \theta_{i-1}) dF(u) \right\}. \tag{11}$$

Consider first a firm that will switch technologies after the next improvement occurs. Setting $\theta = \theta_i^*$ Eq. (11) becomes

$$V_{i-1}(\theta_i^*, \theta_{i-1}) = \frac{\pi(\theta_{i-1})}{r + \lambda} + \frac{\lambda}{r + \lambda} \int_0^{\bar{u}} V_i(\theta_i^* + u, \theta_i^* + u) - I dF(u). \tag{12}$$

Consider next a firm that has just switched technologies and thus will not switch technologies after the next improvement occurs. Replacing i by $i + 1$ and setting $\theta = \theta_i^*$ Eq. (11) becomes

$$V_i(\theta_i^*, \theta_i) = \frac{\pi(\theta_i)}{r + \lambda} + \frac{\lambda}{r + \lambda} \int_0^{\bar{u}} V_i(\theta_i^* + u, \theta_i) dF(u). \tag{13}$$

Substituting Eqs. (12) and (13) into Eq. (8) which defines θ_i^* yields after some manipulation

$$\frac{\pi(\theta_i^*)}{r} - I = \frac{\pi(\theta_{i-1})}{r} + \frac{\lambda}{r} \int_0^{\bar{u}} V_i(\theta_i^* + u, \theta_i^* + u) - V_i(\theta_i^* + u, \theta_i^*) dF(u). \tag{14}$$

Since Farzin et al. (1998) omit the second argument of the value function, they mistakenly cancel the terms inside the integral and Eq. (14) appears to reduce to

Eq. (9) which in turn defines θ^{NPV} . Hence, Farzin et al. (1998) are in error when they conclude that ‘the first $(n - 1)$ optimal trigger values are in fact the ones which would be obtained by applying the net present value method’ (p. 790).

Comparing Eq. (14) to Eq. (9), it can be seen that the threshold when the option value of waiting is taken into account exceeds the threshold implied by the NPV approach, $\theta_i^*(\theta_{i-1}) > \theta^{\text{NPV}}(\theta_{i-1})$ for all θ_{i-1} , whenever the integrand in Eq. (14) is positive. But this is indeed the case since V_i is increasing in its second argument because the firm can only gain from having a more efficient technology in place.

Using the functional forms and parameter values suggested by Farzin et al. (1998) we computed the optimal policy and the NPV rule when the firm can switch technologies twice.³ In Fig. 1 the optimal policy θ_i^* for the first technology switch is plotted as a solid line and the NPV rule θ^{NPV} as a broken line.

Evidently, $\theta_1^*(\theta_0) > \theta^{\text{NPV}}(\theta_0)$ for all θ_0 . Consequently, under the NPV approach a firm adopts a new technology too early. The reason is that the NPV approach understates the investment cost: When the firm exercises its option to invest in a new technology, it loses the option value of waiting any further. While the firm has not committed itself irrevocably to the new technology, in the presence of positive sunk costs it is not optimal for the firm to exchange the technology it has in place with a new technology whenever a new technology becomes available. Instead, the firm is stuck with whatever technology it adopts at least for some period of time.

The example also illustrates that a firm stands to lose a lot by making a suboptimal adoption decision. Consider the situation in which a firm has a technology with efficiency $\theta_0 = 1$ in place when the currently available technology is $\theta = 1$ and the firm can switch technologies twice. The firm’s expected discounted profits are $V_0(1, 1) = 5561$ provided it switches technologies optimally. A simulation experiment shows that when the firm switches technologies as prescribed by the NPV rule, its expected discounted profits are a mere 4125.⁴

4. Conclusion

This note pointed out a mistake in Farzin et al.’s (1998) derivations and showed that the threshold when the option value of waiting is taken into

³The details of the computations are available from the author upon request.

⁴More precisely the firm applies the NPV rule for the first technology switch and decides optimally for the second one as suggested by Farzin et al. (1998). Were the firm to apply the NPV rule for both technology switches, its expected discounted profits would decline further to 2476.

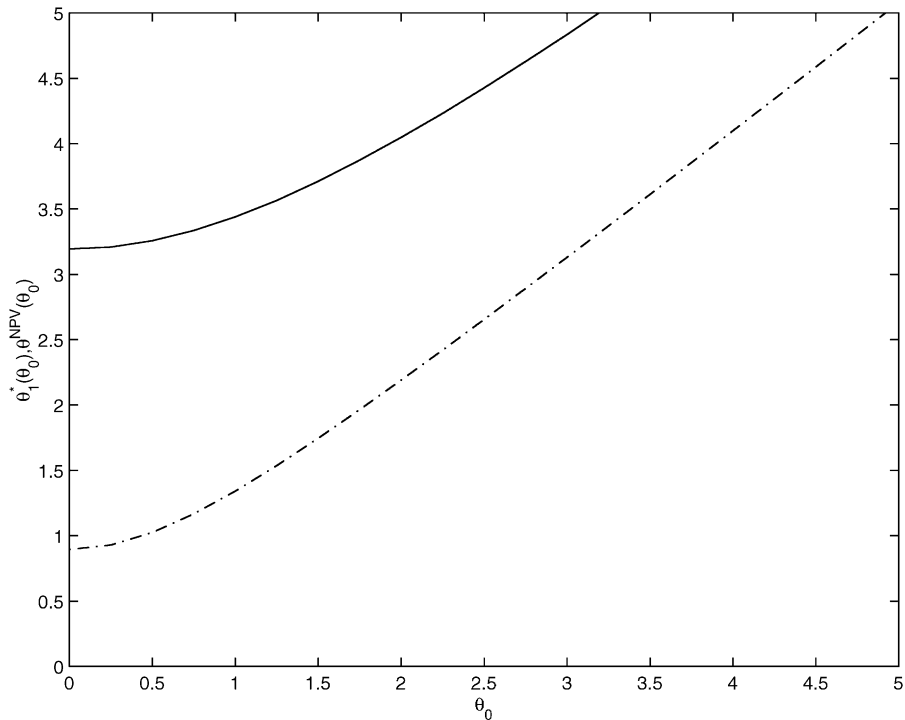


Fig. 1. Optimal policy θ_1^* for the first of two technology switches (solid line) and the NPV rule θ_1^{NPV} (broken line).

account exceeds the threshold implied by the NPV approach even though the firm can switch technologies $n < \infty$ times. It appears that the combined assumptions of ongoing technological progress and sunk investment costs are quite powerful in that they invariably give rise to an option value of waiting. While real-options theory takes the option value of waiting into account, the NPV rule ignores it, and thus leads to a suboptimal adoption decision.

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