Supplementary Material to: "Consumption Risk-sharing in Social Networks"[1]

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This material supplements the paper "Consumption Risk-sharing in Social Networks" with the analysis of four issues. First, we develop game theoretic micro-foundations for our model of community enforcement. Second, we provide background about the mathematical theory of network flows used in the proofs of the paper. Third, we formally develop the extensions of our main results to the case where goods and friendship consumption are imperfect substitutes. Fourth, we explain the numerical methods used to simulate the model and to construct the geographic network representation of the real world Huaraz network.

1 Microfoundations for social sanctions

Consider the following multi-stage game.

Stage 1. An endowment vector e is drawn from a commonly known prior distribution.

Stage 2. Each agent i gives resources $g_i \geq 0$ to a village pool. Then a village elder redistributes this pool by giving each agent i an amount $r_i \geq 0$. We assume $g_W = r_W$, i.e., the village elder cannot steal from the pool. A risk-sharing arrangement $x(e)$ can be implemented using this resource pooling procedure the following way: each agent i gives $g_i = \max[x_i - e_i, 0]$ to the elder and receives $r_i = \min[x_i - e_i, 0]$. Denote the consumption of agent i after redistribution by x_i'.

Stage 3. Agents play friendship games over links. The game over the (i, j) link is

	C	D
C	$c(i,j)$ $c(i,j)$	-1 $c(i,j)/2$
D	$c(i,j)/2$ -1	0 0

which is a coordination game with two pure strategy equilibria, (C, C) and (D, D). Denote the payoff of i from the game with j by $c'(i, j)$.

Stage 4. The realized utility of agent i is $U_i(x_i', c_i')$. The elder does not receive utility from redistribution.

Consider now a pure strategy profile σ. A coalitional deviation by a set of agents F is to choose, at stage 2, a continuation strategy that specifies resource-sharing and the path of play in the friendship games among themselves. Formally, the deviation involves (1) each agent $i \in F$

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giving $\widetilde{g}_i \geq 0$ to an elder in F, who then pays $\widetilde{r}_i \geq 0$ to each i in F; and (2) a path of play in the friendship game for all pair of agents $i, j \in F$. In a coalitional deviation, all other agents behave according the original strategy profile σ. A coalitional deviation is profitable of all agents $i \in F$ at least weakly increase their utility, and some of them increase it strictly. A subgame-perfect equilibrium of the above game that admits no profitable coalitional deviations is a coalition-proof equilibrium.

Our definition of coalition-proof equilibrium does not allow for further coalitional deviations at the friendship game stage. This can be justified if the strategies in the friendship game, once chosen, are irreversibly set; for example, if a friendship link, once broken, is very costly to repair.

Proposition 1 *An allocation $x(e)$ is the outcome of a pure-strategy coalition-proof equilibrium of this game if and only if it is a coalition-proof risk-sharing arrangement of the model in Section 1.*

Proof. Fix a coalition-proof allocation of the model $x(e)$ and consider the following strategy profile σ. In stage 2, each agent makes a payment of $g_i = \max[x_i - e_i, 0]$ to the elder and receives $r_i = \min[x_i - e_i, 0]$. In Stage 3, if there exists a subset of agents F such that for each $i \in F$ we have $g_i < x_i - e_i$, then over all (i, j) links where $i \in F$ and $j \notin F$, agents play (D, D) and over all other links agents play (C, C). This path of play satisfies the best response property in Stage 3, because agents play Nash equilibria of the friendship games. Moreover, it is easy to see that any possible coalitional deviation in the game would be equivalent to a coalitional deviation in the model. It follows that σ is a coalition-proof equilibrium implementing $x(e)$.

Conversely, consider a coalition-proof equilibrium σ, and let $x(e)$ be the consumption allocation it generates. Suppose that in realization e there is coalitional deviation $\widetilde{x}$ by a group F from the risk-sharing arrangement $x(e)$. Then agents in F can also deviate as a group in the multi-stage game. Deviating results in a loss of friendship of at most $c_i - \widetilde{c}_i$ for each agent i, because the worst punishment in Stage 3 is to move from the good to the bad equilibrium, which is what is assumed to happen in the risk-sharing model. The new consumption allocation $\widetilde{x}$ compensates agents in F even for this loss in friendship, and hence results in a profitable coalitional deviation in the game. ■

2 Background on the theory of network flows

The following concepts from the theory of network flows are useful for many of the proofs in the paper. Cormen, Leiserson, Rivest, and Stein (2001) provides a more careful treatment. Fix a finite graph G two nodes s and t (for "source" and "target") and a capacity c.

Definition 2 *An $s \to t$ flow with respect to capacity c is a function $f : G \times G \to \mathbb{R}$ which satisfies*
(i) Skew symmetry: $f(u, v) = -f(v, u)$.
(ii) Capacity constraints: $f(u, v) \leq c(u, v)$.
(iii) Flow conservation: $\sum_w f(u, w) = 0$ unless $u = s$ or $u = t$.

A useful physical analogue is to think about a flow as some liquid flowing through the network from s to t, which must respect the capacity constraints on all links. The value of a flow is the amount that leaves s, given by $|f| = \sum_w f(s, w)$. The maximum flow is the highest feasible flow value in G. Flows are particularly useful in our setting, because the capacity constraints associated with our direct transfer representation are exactly the constraints (ii) in the above definition. In particular, a direct transfer representation that meets the capacity constraints is called a circulation in the computer science literature.

Definition 3 *A cut in G is a disjoint partition of the nodes into two sets $G = S \cup T$ such that $s \in S$ and $t \in T$. The value of the cut is the sum of $c(u, v)$ for all links such that $u \in S$ and $v \in T$.*

It is easy to see that the maximum flow is always less than or equal to the minimum cut value. The following well-known result establishes that these two quantities are equal.

Theorem 2 *[Ford and Fulkerson, 1958] The maximum flow value equals the minimum cut value.*

We rely both on the concept of network flows and the maximum flow - minimum cut theorem in the proofs of the paper.

3 Analysis with imperfect substitutes

3.1 Formal results for Subsection 1.2 of the paper

Here we prove two results for coalitional deviations. The first is that for general preferences, requiring coalitional deviations themselves to be stable with respect to further coalitional deviations does not change the set of coalition-proof agreements. The second is that requiring stability with respect to both ex ante and ex post coalitional deviations yields a subset of the constrained efficient agreements, and in the case of perfect substitutes it is exactly the set of constrained efficient agreements.

Proposition 2 *Requiring coalitions to be robust to further coalitional deviations does not affect the set of coalition-proof allocations.*

Proof. Let F be a deviating coalition, and let $F' \subseteq F$ be a deviating subcoalition. We claim that F' is also a deviating coalition in the original set of agents W. To see why, note that the capacities $\widetilde{c}'$ after the subcoalition F' deviates are exactly those associated with links within F', and hence these are also the capacities that remain when F' deviates in W. Moreover, the allocation $\widetilde{x}'$ of the subcoalition F' only uses the resources in F' and hence is also feasible when F' deviates from W. These observations imply that the same allocation is available to all agents in F' if they consider a coalitional deviation from W. Since these agents are better off with this allocation than they were in the coalition F, where in turn they are better off than in the original allocation, it follows that F' is a profitable coalitional deviation in the original network as well. This logic implies that minimal deviating coalitions are robust to further coalitional deviations. Since any allocation that has a deviating coalition also has a minimal deviating coalition, it follows that requiring no deviating coalitions is equivalent to requiring no deviating coalitions that are robust to further group deviations. ■

We say that a coalition-proof agreement x admits no ex ante coalitional deviations if there is no coalition S and coalition-proof risk-sharing agreement x'_S within S such that all agents in S weakly prefer losing all their links to agents in W/S and having agreement x'_S to keeping all their links and having agreement x, and at least one agent in S strictly prefer the former. Intuitively, an ex ante coalitional deviation implies that the agents of the deviating coalition leave the community (cut their ties with the rest of the community) and agree upon a new risk-sharing agreement among each other (using only their own resources).

Proposition 3 *A coalition-proof agreement that admits no profitable ex ante coalitional deviations is constrained efficient. If goods and friendship are perfect substitutes then the set of coalition-proof agreements that admit no profitable ex ante deviations is equal to the set of constrained efficient agreements.*

Proof. Consider first a coalition-proof agreement x that is not constrained efficient. Then there is another coalition-proof agreement x' that ex ante Pareto-dominates x. But then x' is a profitable ex ante coalitional deviation for coalition W. This concludes the first part of the statement.

Assume now that goods and friendship are perfect substitutes and consider a coalition-proof agreement x that is constrained efficient. Suppose there is coalition S and a profitable ex ante deviation x'_S by S. Theorem 1 implies that x can be achieved by a direct-transfer agreement t that respects all capacity constraints. Similarly, x'_S can be achieved by a direct transfer agreement t'_S within S that respects all capacity constraints (within S). Consider now a combined direct transfer agreement (t'_S, t_{-S}) that is equal to t'_S for links within S, and it is equal to t otherwise. Since both t and t'_S respect capacity constraints, so does (t'_S, t_{-S}), hence the resulting consumption profile x'' is coalition-proof. By construction x is equivalent to x'' for agents in $W \backslash S$. Agents in S are at least weakly better off with consumption profile x'' and not losing any of their links than with consumption profile x'_S and losing their links with agents in $W \backslash S$, since x'' is coalition-proof. But this, combined with x'_S being a profitable ex ante coalitional deviation, implies that coalition-proof agreement x'' Pareto-dominates x, which contradicts that x is constrained efficient. ■

3.2 Formal results for Subsection 2.6 of the paper

In Section 2.6 we discuss what happens with imperfect substitutes after an aggregate negative shock. To formalize those ideas, note that for each coalition-proof arrangement x, there exists a direct transfer representation implementing it. Note that we do not require this transfer arrangement to meet any capacity constraints; it is simply a decentralized representation of the payments agents need to make to achieve allocation $x(e)$.

Proposition 4 *Assume that MRS_i is increasing in x_i for all i. Then for any pair of endowment realizations $\underline{e}$ and $\overline{e}$ such that $\underline{e}_i \leq \overline{e}_i$ for all i, a direct transfer arrangement that generates a coalition-proof allocation in $\underline{e}$ also generates a coalition-proof allocation in $\overline{e}$.*

Proof. Let $V(y_i, c_i; s_i) = U_i(y_i + s_i, c_i)$, then $(V_x/V_c)(y_i, c_i; s_i) = (U_x/U_c)(y_i + s_i, c_i)$, and hence the condition that $MRS_i = (U_x/U_c)(x_i, c_i)$ is increasing in x_i implies that $(V_x/V_c)(y_i, c_i; s)$ is increasing in s for any fixed (y_i, c_i), i.e., that $V(y_i, c_i; s)$ satisfies the Spence-Mirrlees single-crossing condition. Since U_i is continuously differentiable and U_x, $U_c > 0$, Theorem 3 in Milgrom and Shannon (1994) implies that V has the single crossing property. In particular, $V(y_i, c_i; 0) \geq V(y'_i, c'_i; 0)$ implies $V(y_i, c_i; s_i) \geq V(y'_i, c'_i; s_i)$ for any $s_i \geq 0$, or equivalently, $U_i(x_i, c_i) \geq U_i(x'_i, c'_i)$ implies $U_i(x_i + s_i, c_i) \geq U_i(x'_i + s_i, c'_i)$. It follows that for any $s_i \geq 0$, the compensating variation satisfies

$$CV_i\left(x_i, c_i, c'_i\right) \leq CV_i\left(x_i + s, c_i, c'_i\right)$$

and hence for any set F, we have $c^x[F] \leq c^{x+s}[F]$. Now denote $\overline{e} - \underline{e} = s \geq 0$; it follows immediately from Lemma 1 of the paper that any coalition-proof transfer arrangement under $\underline{e}$ is coalition-proof under $\overline{e}$ as well. ■

3.3 Formal results for Subsection 3.3 of the paper

A decentralized exchange implementing any constrained efficient arrangement. We show that for any constrained efficient allocation, there exists a simple iterative procedure that only uses local information in each round of the iteration, and converges to the allocation as the number of iterations grow. A simpler version of this procedure, with equal welfare weights and no capacity constraints, was proposed by Bramoulle and Kranton (2006). We develop this procedure using the direct transfer representation of the model. The basic idea is to equalize, subject to the capacity constraints, the marginal utility of every pair of connected agents at each round of iteration. This procedure can be interpreted as a set of rules of thumb for behavior that implements constrained efficiency in a decentralized way.

Fix an endowment realization e, and denote the efficient allocation corresponding to welfare weights λ_i by x^*. Fix an order of all links in the network: l_1,...,l_L, and let i_k and j_k denote the agents connected by l_k. To initialize the procedure, set $x_i = e_i$ and $t_{ij} = 0$ for all i and j. Then, in every round $m = 1, 2, ...$, go through the links $l_1, ..., l_L$ in this order, and for every l_k, given the current values x_{i_k}, x_{j_k}, and $t_{i_k j_k}$, define the new values x'_{i_k} and x'_{j_k} and $t'_{i_k j_k} = t_{i_k j_k} + x'_{j_k} - x_{j_k}$ such that they satisfy the following two properties: (1) $x'_{i_k} + x'_{j_k} = x_{i_k} + x_{j_k}$. (2) Either $\lambda_{i_k} U'_{i_k}(x'_{i_k}) = \lambda_{j_k} U'(x'_{j_k})$, or $\lambda_{i_k} U'_{i_k}(x'_{i_k}) > \lambda_{j_k} U'_{j_k}(x'_{j_k})$ and $t'_{i_k j_k} = -c(i, j)$, or $\lambda_{i_k} U'_{i_k}(x'_{i_k}) < \lambda_{j_k} U'_{j_k}(x'_{j_k})$ and $t'_{i_k j_k} = c(i, j)$. This amounts to the agent with lower marginal utility helping out his friend up to the point where either their marginal utility is equalized, or the capacity constraint starts to bind. Once this step is completed for link k, we set $x = x'$ and $t = t'$ before moving on to link $k + 1$. For $m = 1, 2, ...$ let x_i^m denote the value of x_i, and let t_{ij}^m denote the value of t_{ij}, at the end of round m. Note that x_m meets the capacity constraints by design for every m.

Proposition 5 *If consumption and friendship are perfect substitutes, then $x^m \to x^*$ as $m \to \infty$.*

Proof. Let $V(x)$ denote the value of the planner's objective in allocation x. The above procedure weakly increases $V(x)$ in every round and for every link l_k. Hence $V(x_1) \leq V(x_2) \leq ..$, and since $V(x) \leq V(x^*)$ for all x that are coalition-proof, we have $\lim_{m\to\infty} V(x_m) = V \leq V(x^*)$. Since the set of coalition-proof allocations is compact, and x_m is coalition-proof for every m, there exists a convergent subsequence of x_m, with limit x and associated transfers t. Clearly, $V(x) = V$. If $V = V^*$ then $x = x^*$ since the optimum is unique. If $V < V^*$, then x is not optimal, and hence does not satisfy the first order condition over all links. Let l_k be the first link in the above order for which the first order condition fails in x and t. Then there is a transfer meeting the capacity constraints at x that increases the planner's objective by a strictly positive amount δ. But this means that for every x_m far along the convergent subsequence, the planner's objective increases by at least $\delta/2$ at that round, which implies that $V(x_m)$ is divergent, a contradiction. Hence $\lim x_m = x^*$ along all convergent subsequences, which implies that x_m itself converges to x^*. ■

First-order conditions with general preferences. To present our characterization result for general preferences, first we define a measure of marginal social welfare gain of transfers to agents. Fix a coalition-proof arrangement x. For any $S \subset W$ and $i \in S$, let $\widehat{t}_i(S, x)$ be the extra amount of transfer that would compensate i for losing all his links to agents outside S, so that $\widehat{t}_i(S, x) = CV_i\left(x_i, c_i, \widetilde{c}_i^S\right)$ where $\widetilde{c}^S$ denotes the remaining capacities after S deviates. We say that the coalitional incentive constraint binds for $S \subset W$ if equation (1) in the paper holds with equality for $F = S$. Let $\mathcal{C}(x)$ be the collection of sets for which the coalitional incentive constraints bind in allocation x. We say that the coalitional constraint from i to j binds if there is $S \in \mathcal{C}(x)$ such that $i \in S$ and $j \notin S$.

Consider now the following iterative construction. In the first round of the procedure, define $\Delta_i^1 = \lambda_i U_{i,x}(x_i, c_i)$ for all $i \in W$. Suppose now that values Δ_i^l are defined for $l = 1, ..., k-1$. To define Δ_i^k, first introduce some new notation. Fix i, and for every $j \in W$ such that the coalitional constraint from i to j binds, and for every $S \in \mathcal{C}(x)$ for which $i \in S$ and $j \notin S$, let $\widehat{x}_{i,j}(S) = x_i + \widehat{t}_i(S, x)$, and let $\widehat{c}_i(S) = c_i - \widetilde{c}_i^S$. Next, let

$$\widetilde{c}_{ij} = \max_{S\in\mathcal{C}:\ i\in S, j\notin S} \frac{U_{i,x}(\widehat{x}_{i,j}(S), \widehat{c}_i(S))}{U_{i,x}(x_i, c_i)}$$

and

$$\delta_{ij}^{k} = \lambda_i U_{i,x}(x_i, c_i) \cdot \widetilde{c}_{ij} + \Delta_j^{k-1} \cdot [1 - \widetilde{c}_{ij}]\,.$$

As we will show below, δ_{ij} measures the marginal social gain of an additional dollar to i, under the assumption that i optimally transfers some of the dollar to j. Intuitively, to transfer to j, i has to increase her own consumption somewhat to maintain incentive compatibility. More formally, we show below that a share $\widetilde{c}_{ij}$ of a marginal unit of consumption good must be kept by i, but the remaining share can be transferred to j, where it has a welfare impact of Δ_j^{k-1}. Denote $\delta_{ii} = \lambda_i U_{i,x}(x_i, c_i)$, and to account for the softening of the coalitional incentive constraints, let

$$\Delta_i^k = \max\left\{\delta_{ij}^k \mid j : \text{the IC constraint from } i \text{ to } j \text{ binds or } j = i\right\}.$$

The values Δ_i^k defined above converge for each $i \in W$ as $k \to \infty$, since Δ_i^k is weakly increasing in k, and the sequence is bounded from above by $\max\limits_{j \in W} \Delta_j^1$ (since in every round the values are defined as convex combinations of values defined in the previous round). Let $\Delta_i = \lim\limits_{k \to \infty} \Delta_i^k$.

With this recursive definition, the marginal social welfare of an additional dollar takes into account both the marginal increase in i's consumption, and the softening of coalitional incentive constraints which allow transfers to further agents.

Proposition 6 *[Constrained efficiency with imperfect substitutes] Assume that MRS_i is increasing in x_i for every i. A transfer arrangement t is constrained efficient iff there exist positive $(\lambda_i)_{i \in W}$ such that for every $i, j \in W$ one of the following conditions holds:*
1) $\Delta_j = \Delta_i$
2) $\Delta_j > \Delta_i$ and the IC constraint from i to j binds
3) $\Delta_j < \Delta_i$ and the IC constraint j to i binds.

Proof. We begin with some preliminary observations. Suppose that the IC constraint from i to j binds, and i receives an additional dollar. Suppose that i keeps a share α of the dollar and transfers the remaining $1 - \alpha$ such that the resulting arrangement is still coalition-proof and the coalitional incentive constraint continues to bind for some $S \ni i$ for which $j \notin S$. Then it must be that $\alpha U_{i,x}(x_i, c_i) \geq U_{i,x}(\widehat{x}_{i,j}(S), \widehat{c}_i(S))$ for (or equivalently, $\alpha \geq U_{i,x}(x_i, c_i)/U_{i,x}(\widehat{x}_{i,j}(S), \widehat{c}_i(S))$) for all such S, and the inequality holds with equality for some S. To maintain coalition proofness, this share of the dollar has to be consumed by i, and only the remainder can be transferred to j.

Now we establish the necessity part of the proposition. Fix a constrained efficient arrangement, and let λ_i be the associated planner weights. Consider realization e. For any $i \in W$, the marginal value to the planner of an additional amount of consumption good to an agent i is at least Δ_i. To see this, note that i can transfer at most $1 - \widetilde{c}_{ij}$ of the additional unit to j, hence the marginal welfare gain if he chooses to transfer to j will be δ_{ij}. Hence if i chooses to send an additional transfer to the agent where it is most productive, the marginal social gain will be the maximum of δ_{ij} over $j \in W$, which is exactly Δ_i.

It follows easily that if $\Delta_j > \Delta_i$ for some i, j, then the IC constraint from i to j has to bind: otherwise social welfare could be improved by marginally increasing t_{ij}. This establishes that in a constrained efficient allocation, for any endowment realization and any pair of agents one of conditions (1)-(3) from the theorem have to hold.

For sufficiency, let now x denote the unique welfare maximizing coalition-proof allocation, let t be a acyclical transfer scheme achieving this allocation, and let $\widehat{\Delta}_i$ denote the marginal social value of an extra unit of transfer to i, given x and t, as defined above. Assume now that there exists another coalition-proof consumption vector $x' \neq x$ achieved by acyclical transfer scheme t' such that x' satisfies conditions (1)-(3). Let Δ_i' denote the marginal social value of an extra unit of transfer to i, given x' and t'. Then there exists an acyclical nonzero transfer scheme t^d that achieves x from x'. By definition of x, t^d from x' improves social welfare: $\sum_{i \in W} \lambda_i U_i(x_i' + \sum_{j \in W/\{i\}} t_{ij}^d, c_i) > \sum_{i \in W} \lambda_i U_i(x_i', c_i)$. Because utility functions are concave, transfer αt^d improves social welfare for any $\alpha \in (0,1)$. Moreover, as established in Proposition 4, $x' + \alpha t^d$ is a coalition-proof allocation. Hence, social welfare can be improved from x' by a set of marginal transfers such that the relative magnitude of marginal transfers are given by t^d, without violating any coalitional incentive constraints. This means that there is at least one sequence of marginal transfers $i_1 \to i_k$ which is welfare improving and does not violate coalitional constraints. But this contradicts that conditions (1)-(3) hold for x'. ■

Extending Proposition 7 of the paper with imperfect substitutes. Fix a constrained efficient arrangement, and let e and e' be two endowment realizations such that $e_i > e_i'$ for some $i \in W$, and $e_j = e_j'$ $\forall$ $j \in W \backslash \{i\}$. Let $x^*(e)$ and $x^*(e')$ be the constrained efficient allocations after endowment realizations e and e', respectively. Analogously to the perfect substitutes case, let $\widehat{W}(i)$ be the largest set of connected agents containing i such that all coalitional incentive constraints for sets strictly smaller than $\widehat{W}(i)$ are slack, given $x^*(e)$. Let $\Delta_j(e)$ and $\Delta_j(e')$ be the marginal social utility of a transfer to j, as defined above, given $x^*(e)$ and $x^*(e')$, respectively.

Proposition 7 *[Spillovers with imperfect substitutes] Assume that MRS_i is concave. Then:*

(i) [Monotonicity] $\Delta_j(e') \leq \Delta_j(e)$ for all j, and if $j \in \widehat{W}(i)$ then $\Delta_j(e') > \Delta_j(e)$.

(ii) [Local sharing] There exists $\delta > 0$ such that $|e_i - e_i'| < \delta$ implies $\Delta_i(e') = \Delta_j(e')$ for all $j \in \widehat{W}(i)$.

(iii) [More sharing with close friends] For any $j \neq i$, there exists a path $i \to j$ such that for any agent l along the path, $\Delta_l(e') \geq \Delta_j(e')$.

The proof of this result is analogous to the perfect substitutes case and hence omitted. Note that (ii) is weaker than in Proposition 7 of the paper, because even small shocks can spill over the boundaries of the risk-sharing islands of agent hit by the shocks. Also note that since $\Delta_i = \lambda_i U_{i,x}$ for any agent not on the boundary of an island, (i) implies that consumption is monotonic in the endowment realization for such agents.

4 Numerical methods

Risk-sharing simulations. We use the following numerical approach for the simulations underlying Figures 4 and 6. We assume throughout that endowment shocks are uniformly distributed with support $[-1, 1]$. We build on Theorem 1 and express a SDISP-minimizing direct-transfer arrangement as a cost-minimizing flow as follows. (1) Create two artificial nodes s and t as in the proof of Theorem 1. (2) Divide the shock support into K equal intervals. For each agent i, denote the subinterval into which i's endowment falls by k_i (treating $[-1, -1 + 2/K]$ as the first interval and $[1 - 2/K, 1]$ as the Kth interval). Create k_i links between s and i such that each link has capacity $2/K$ in the direction from s to i and zero in reverse direction. Define the "cost" of a flow going from s to i across any of these links to be j for the jth link of out k_i links. Similarly, create $K - k_i$ links between t and i. such that each link has capacity $2/K$ in the direction from i to t and zero in reverse direction. Define the cost of a flow going from i to t across any of these links to be j

for the jth link of out k_i links. (3) Use Edmonds and Karp's (1972) algorithm to calculate a cost-minimizing flow in this augmented network. This solution induces a direct-transfer arrangement that maximizes a piecewise linear approximation to the quadratic utility function assumed in the definition of $SDISP$, where the marginal utility of consumption for any agent is constant within each of the K intervals. Simulations (not reported) show that this approximation generates highly accurate predictions for $K = 20$. For the results presented in the text we set $K = 100$.

Geographic network representation. The algorithm used in the geographic representation constructed in Figure 5 is the following. For each household i, we first construct vectors v_j to every other households j in the unit square using households' initial (re-scaled) geographic coordinates. We also calculate the length d_i of each of these vectors. Note, that the maximum distance between two households is $\sqrt{2}$. We then calculate a shift vector as the weighted sum $-\sum(\sqrt{2} - d_i)v_j / \|v_j\|$ and move each household in the direction of this shift vector. Shifts are larger if a household is closely surrounded by other households and the shift will push the household away from its neighbors. This procedure is repeated 23 times to obtain the representation in Figure 5E.