Hierarchircal Cheap Talk - Supplementary Material

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This note discusses how the logic of the Harsanyi purification theorem can be applied directly to the model in the article, so that the mixed strategy equilibria we consider can be interpreted as pure strategy equilibria of a Bayesian game with small uncertainty about payoffs.

1 Incomplete Information About Intermediators' Preferences and Purification

This section addresses two concerns about our model. The first is that the mixed equilibria we describe may sound unappealing, or too complicated to be actually played. In principle it is not clear why the intermediators mix with the exact probablity specified by the equilibrium strategy profile, as they are indifferent between both messages. So it is important to provide a better motivation for why these equilibria would be played.

The second, related concern is that our assumption of perfect information about other player's preferences may not be realistic. While the model assumes that the biases of each player are perfectly known in advance, this need not be so in practice. It would be much more reasonable to assume that there is at least some degree of uncertainty over the biases of other players.

Here, we argue that mixed equilibria can be motivated as equilibria of a game with small amounts of uncertainty over preferences, in the spirit of Harsanyi's purification result. That is, the mixed equilibria of our game are close to pure strategy equilibria of a large class of perturbed games with a small amount of uncertainty over the biases of other players.

Because the action spaces in our model are not finite, the classic theorem in Harsanyi [1973] cannot be applied directly. But notice that the intermediators in our model receive and send messages in a finite set. So tediously following the reasoning in Harsanyi [1973], it can be seen that mixed equilibria in our

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game are approximated by pure strategy equilibria of games in which there is a small amount of uncertainty over the intermediators biases.

Instead of pursuing the general case of this result, we give an example to illustrate the idea. The example makes it clear how a general result would work, and also makes it intuitive how very general specifications of uncertainty over the intermediator's bias can generate the mixed equilibria.

Consider again the case of a 2 message mixed equilibria, characterized in section 4.2.1. Assume that $b^2 > 0$, and that the biases are in the range where a 2 message mixed equilium exists, in which the intermediator mixes with some probability $0 < p < 1$ when receiving the low message m_1^1.

Assume now that there is a small, but arbitrary amount of uncertainty over the intermediator's bias. That is, ex-ante all that the sender and the receiver knows is that the intermediator's bias is $\tilde{b}^2 = b^2 + \epsilon\nu$, where $\epsilon > 0$ is a small real number and ν has an arbitrary distribution F with a continuous density on $[-1, 1]$.

We now argue that, in the spirit of the purification theorem, as $\epsilon \to 0$, the perturbed game has a pure strategy equilibrium that is very close to the 2 message mixed equilbria.

This is easy to see if we use the characterization obtained in section 4.2.1. Consider a candidate equilibrium in which the intermediator always sends m_2^2 when he receives m_1^2, but what he sends when he receives the low message m_1^1 depends on his type. That is, there is some value $b^{*2} = b^2 + \epsilon\nu^*$ such that he sends the low message m_1^1 if his type is lower than this threshold and send the high message m_2^2 if his type is higher. We will show that for small ϵ it is always possible to find b^{*2} that is ϵ close to b^2 and such that this constitutes an equilibrium. So from now on assume that ϵ is small enough such that there are still 2 message mixing equilibria for intermediator's biases in the range $[b^2 - \epsilon, b^2 + \epsilon]$.

In this equilibrium, after receiving message m_1^1, type b^{*2} must be indifferent between both messages (assume that this type is close to b^2). But then, following the argument in section 4.2.1, this completely pins down what the equilibrium messages are. They are the same as those calculated in section 4.2.1 for biases b^1 and b^{*2}. And the reasoning in that section pins down not just the equilibrium messages, but also the mixing probability, which we denote p^*. We must have that

$$p^* = \frac{1}{8}\frac{\left(1 - 4b^{*2}\right)\left(1 - 2\Delta^*\right)}{\Delta^* b^{*2}},$$

where $\Delta^* = b^{*2} - b^1$. But for this to be an equilibrium, we must also have that p^* is the probability that the intermediator's type exceeds b^{*2}. That is

$$p^* = 1 - F(\frac{b^{*2} - b^2}{\epsilon}).$$

Moreover, because all other incentive constraints are satisfied if the messages are given as in section 4.2.1, we have that this is an equilibrium iff this consistency requirement for p^* is satisfied, that is

$$\frac{1}{8}\frac{\left(1-4b^{*2}\right)\left(1-2\Delta^*\right)}{\Delta^* b^{*2}} = 1 - F(\frac{b^{*2}-b^2}{\epsilon}).$$

Now, note that the left side is always between 0 and 1 for b^{*2} in the interval $[b^2-\epsilon, b^2+\epsilon]$, for small ϵ. But the right side equals 1 for $b^{*2}=b^2-\epsilon$ and 0 for $b^{*2}=b^2+\epsilon$. So by the intermediate value theorem there is b^{*2} in $[b^2-\epsilon, b^2+\epsilon]$ that satisfies this equation, and for which our construction is indeed an equilibrium.

Moreover, by the characterization in section 4.2.1, we know that equilibrium messages and the mixing probability depend continuously on the intermediator's bias. So for small ϵ this equilibrium is close to the mixed equilibrium described in section 4.2.1. This example illustrates that a way to interpret and motivate the mixed equilibria is that they correspond to pure strategy equilibria of the game perturbed with a small amount of uncertainty over the intermediator's bias.

References

J.C. Harsanyi. Games with randomly disturbed payoffs: A new rationale for mixed-strategy equilibrium points. *International Journal of Game Theory*, 2 (1):1–23, 1973.